

Original Research Article**Simulation of Heat Conduction using An Hybrid Floating Walk and Markov Chain Technique****Abstract**

Markov chain technique had been developed for numerical simulation of heat conduction. However the technique could only be used to handle domain with regular boundaries. An hybrid of floating walk and markov chain techniques had therefore been developed.

The technique was used to simulate temperature distribution in arbitrary shaped domain with mixed boundary conditions. The results obtained were compared with that obtained using finite difference and as well as using floating walk technique. Results were statistically analysed using ANOVA ($\alpha = 0.05$) and the computer execution for all the three cases considered compared.

The results from the developed hybrid technique were not significantly different from those from finite difference and floating walk techniques. The hybrid technique execution time was longer than that of finite difference technique but shorter than the floating walk technique.

The study established the suitability of the hybrid floating walk markov chain technique for analysis of heat conduction of arbitrary shaped domain.

Keywords: Heat conduction, Arbitrary Domain, Numerical technique

1.0 Introduction

Several numerical techniques to model heat conduction have been documented. Finite difference technique was used by Anjo[1] to simulate solidification of aluminium in green sand mould while Dhawan and Kumar[2] reported the use of finite element for heat transfer analysis. The study investigated the effect of different combinations of meshes and numerical schemes for modelling the cooling of an aluminium plate. Takemori et al[3] carried out a numerical simulation of welding process in the development of a new compressor. The numerical results were supported with experimental studies. Yavuzturk et al[4] modelled U Tube Ground heat exchanger using finite volume technique while Motley and Provost[5] developed a mapped

infinite mapped element approach for a one dimensional transient heat conduction. Shen et al[6] used a finite difference technique to model heat transfer during a grinding process. An optimization technique was developed by Mehta[7] to calculate for temperature in a finite conducting slab. The results was reported to compared well with finite difference solution.

Monte Carlo technique which is a probability technique was first reported by Haji-Sheikh and Sparrow[8] for modelling heat conduction. The method has not received much attention due to its slow execution time although with capability to determine solution at a point much easily.. Other probability approaches include heat conduction analysis by Grigorin[9] and thermal studies of a superconductivity current limiter by Leveque and Rezzong[10]. Two main versions of Monte carlo technique namely fixed and floating walk have been developed. The fixed walk has a very slow execution time and could not easily handle irregular boundary cases. Floating walk has comparatively fast execution time and could easily handle irregular boundary..

A Markov chain technique was developed by Sadiku et al[11]; it was however applied to simple heat conduction cases such as a slab with isothermal boundaries. The objective of this work is to develop an hybrid of floating walk and Markov chain techniques for analysis of heat conduction. It is premised that the developed technique would be able to generate solution at a fast rate while benefitting from the low memory demand of the Monte Carlo(probability method) technique. The technique will be used to analyse heat conduction in an arbitrary shaped slab and as well as rectangular slab with non isothermal boundaries.

2.0 Description of the Methodology

In this section, the floating walk and regional Markov Chain techniques are first discussed. Thereafter the developed hybrid technique is described.

2.1 Floating Walk Technique

In a given domain, a walk is commenced at the point where the solution is being sought. The walk is regarded as floating as the length depends on the closeness of the current position of the particle to the boundary. The walk is terminated whenever an absorbing boundary is encountered, and the boundary value of such recorded. When the particles reaches an adiabatic boundary it is reflected back into the domain at a specified reflection distance. For convective

boundary, a random number is first selected. If the selected random number is less or equal to then the boundary is treated as being absorbing otherwise it is regarded as being reflective. The average of all the scores is used to estimate the solution at the desired point. A modified representative flow chart of the procedure as obtained from Ogundare[12](1990) is presented in Figure 1.

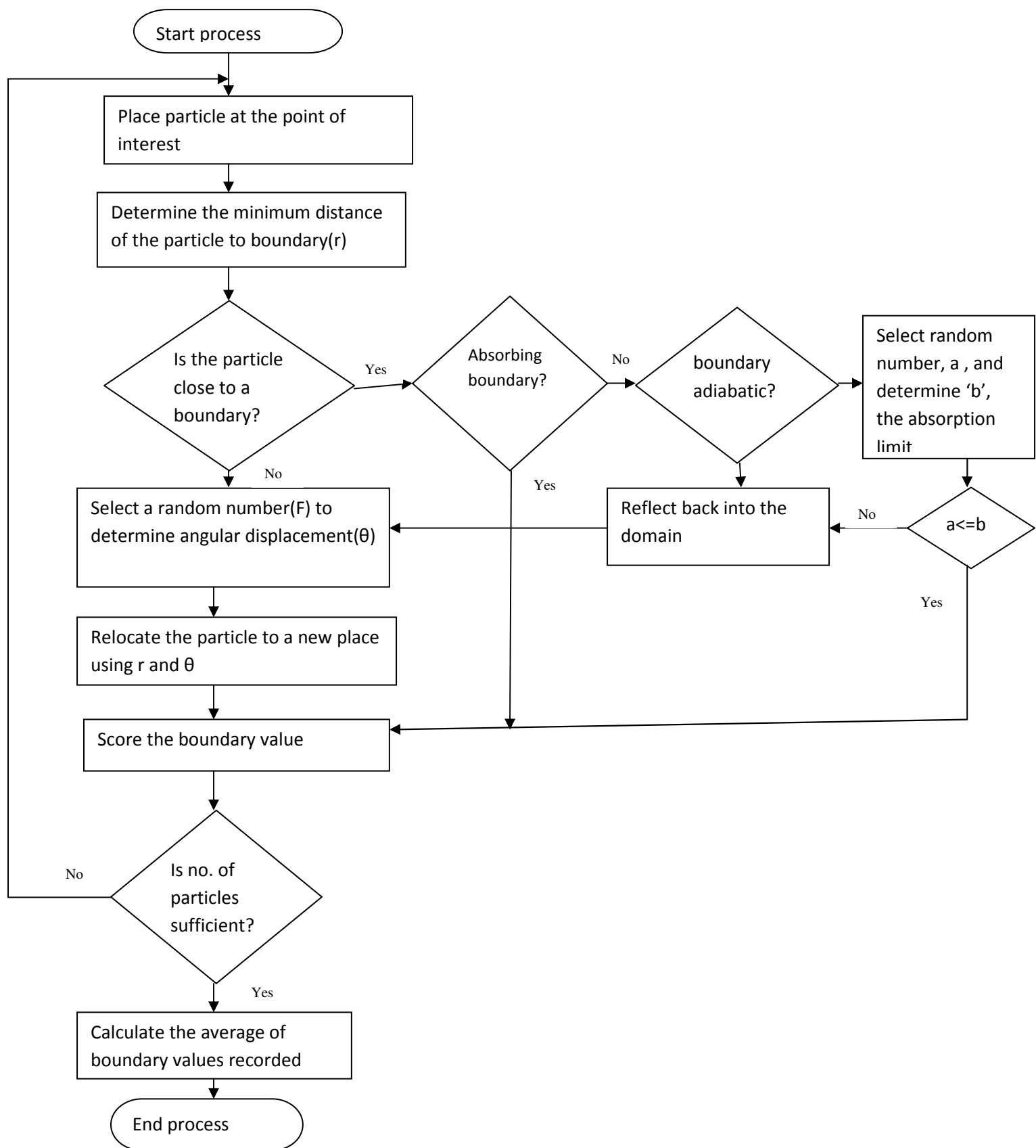


Fig. 1: Flow Chart for Floating Walk Technique

2.1.1 Special Features in Application of Floating Walk to an Arbitrary Shaped Boundary

When the boundary has arbitrary shape, determination of the minimum distance between the particle and the boundary requires special consideration. A boundary is fully described by set of nodes on it. When a particle commences walk at a solution point, its minimum distance to the boundary is determined by identifying the boundary node(i) closest to it. Other neighbouring boundary nodes, $i+1$ and $i-1$ are also identified. The possible locations of the particle relative to the three nodes are as shown in Figure 2. Then the actual distance to the boundary is determined using Table 1.

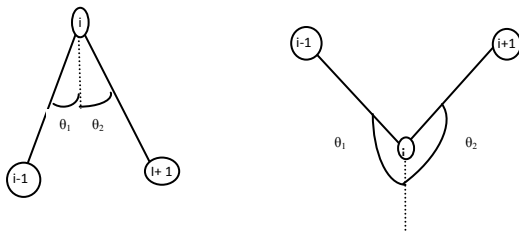


Fig. 2: Possible Particle Position Close to the Boundary

Table 1: Particle Distance Determination Guide

Case	Particle Location	Minimum Distance
1	if both θ_1 and θ_2 are acute angles	$r_{\min} = r \sin \theta_1 (\theta_1 < \theta_2)$ $r_{\min} = r \sin \theta_2 (\theta_2 < \theta_1)$
2	if θ_1 and θ_2 are equal or are both obtuse angles	$r_{\min} = r$
3	if θ_1 is acute and θ_2 is obtuse	$r_{\min} = r \sin \theta_1$
4	if θ_2 is acute and θ_1 is obtuse	$r_{\min} = r \sin \theta_2$

2.2 Markov Chain Technique

Sadiku et al[11] discussed the application of Markov chain for analysis of heat conduction.

A given domain was divided into grids and numbered. Nodes with known temperatures in the domain were first numbered and regarded as absorbing nodes while other nodes are not.

A transition probability P_{ij} which was defined as the probability that a random-walking particle at node 'i' moves to node 'j' was obtained. The probability matrix P formed was recast as

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}$$

where R is the probabilities of moving from non-absorbing nodes to absorbing nodes:

Q - the probability of moving from non absorbing node to another.

I - is an identity matrix

O - is a null matrix

$$\text{and } Q_{ij} = \begin{cases} 0.25 & \text{if } i \text{ is directly connected to } j \\ 0 & \text{if } i = j \text{ or } i \text{ is not directly connected to } j \end{cases}$$

The absorbing probability matrix B was defined as

$$B = NR \text{ -----(1)}$$

With $N = (I - Q)$

$$\text{and } T_f = B T_p \text{ ----(2)}$$

T_f = nodal values at non-absorbing nodes

T_p = nodal values at absorbing nodes

The unknown variables T_f were solve using Eqn.(2).

2.3 Hybrid Floating Walk and Markov Chain(HFWMC) Techniques

In this technique, internal nodes that are very close to the boundary are first estimated using the floating walk technique. They are then regarded as absorbing nodes when the markov chain technique is subsequently applied. Implementation flow chart for the technique is shown in Fig. 3.

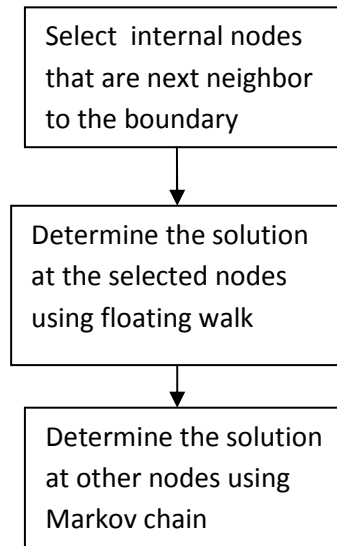


Fig. 3: Implementation Flow Chart for the Hybrid Floating Walk Markov Chain Technique

3.0 Application of HFWMC technique to some selected cases

The developed hybrid floating walk markov chain technique was applied to some selected cases which are hereby discussed.

(i) **Heat Conduction Analysis in a Rectangular Slab with Convective and Adiabatic Boundaries**

A rectangular slab with convective and adiabatic boundaries is shown in Fig. 4.

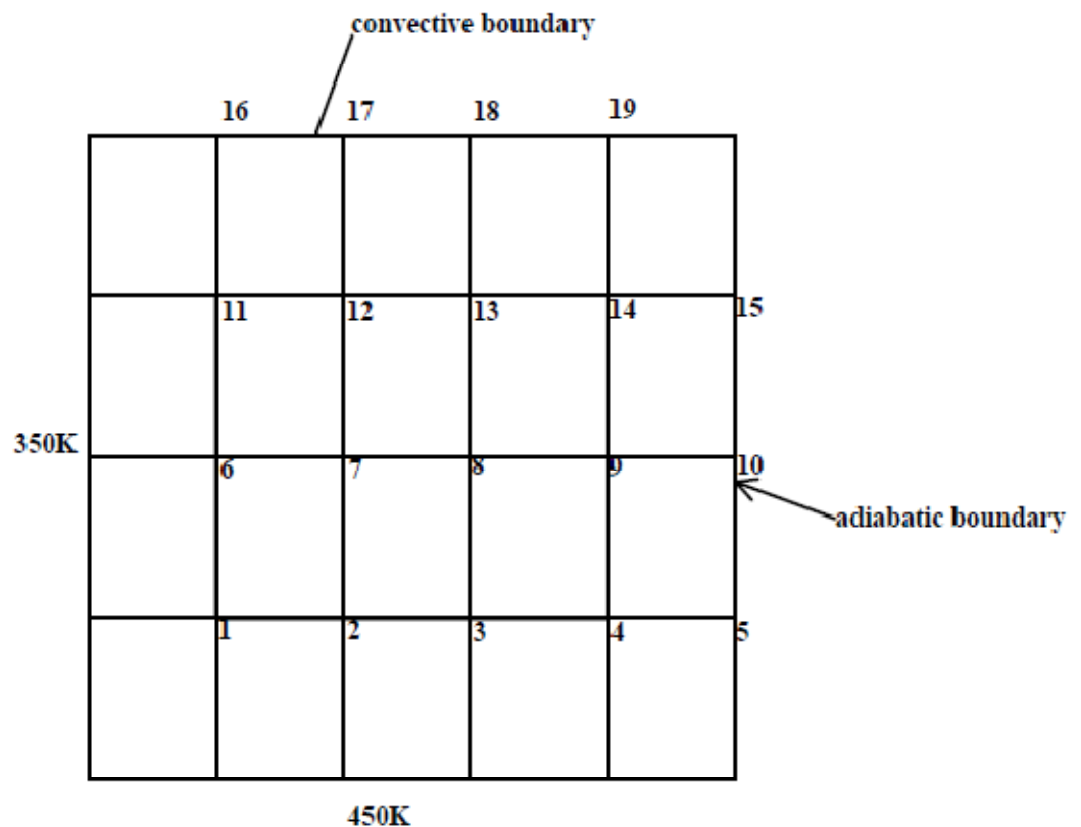


Fig. 4: Rectangular Slab with Mixed Boundaries

The temperatures at nodes 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18 and 19 were first obtained using a floating walk technique and thereafter Markov Chain technique was used to obtain results for other solution points. The temperature estimates using finite difference technique as well as HFWMC are presented in Tables 2 and 3 respectively. Both the finite difference and hybrid HFWMC solutions were subjected to statistical analysis using ANOVA ($\alpha=0.05$). ANOVA yielded a probability value of 0.673 implying that the results from the two techniques are not significantly different. ANOVA box plot is shown in Fig. 5. The computer execution time using finite difference was 3.74 seconds while that of the HFWMC was 30.2 seconds.

Table 2: Temperature Distribution in the Rectangular Slab using Finite Difference Technique

363.8	377.6	393.6	405.6	412.6	414.9
350.0	376.5	395.5	408.1	415.0	417.2
350.0	382.9	404.0	416.2	422.2	423.8
350.0	401.1	421.4	430.4	434.0	433.4
400.0	450.0	450.0	450.0	450.0	441.7

Table 3: Temperature Distribution in the Rectangular Slab using Hybrid Floating Walk Markov Chain Technique

372.5	395.0	409.0	415.0	412.0	415.5
350.0	380.0	402.6	413.5	417.6	419.0
350.0	386.0	407.8	418.8	425.8	429.0
350.0	403.0	424.0	428.0	438.0	434.0
400.0	450.0	450.0	450.0	450.0	442.0

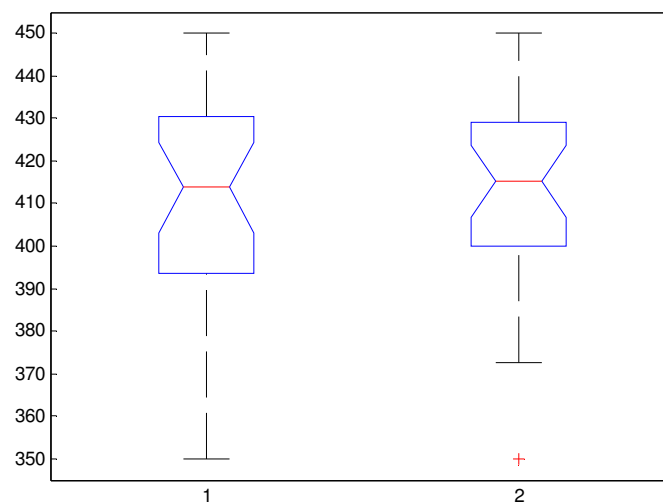


Fig. 5: ANOVA Box plot for Rectangular Slab with Mixed Boundaries

(ii) **Heat Conduction Analysis in an Arbitrary Shaped Slab with Isothermal Boundaries**

Figure 6 is an arbitrary slab with isothermal boundary conditions. As in the previous case, the temperature at neighbouring nodes were first obtained using floating walk technique before the Markov chain technique was applied.

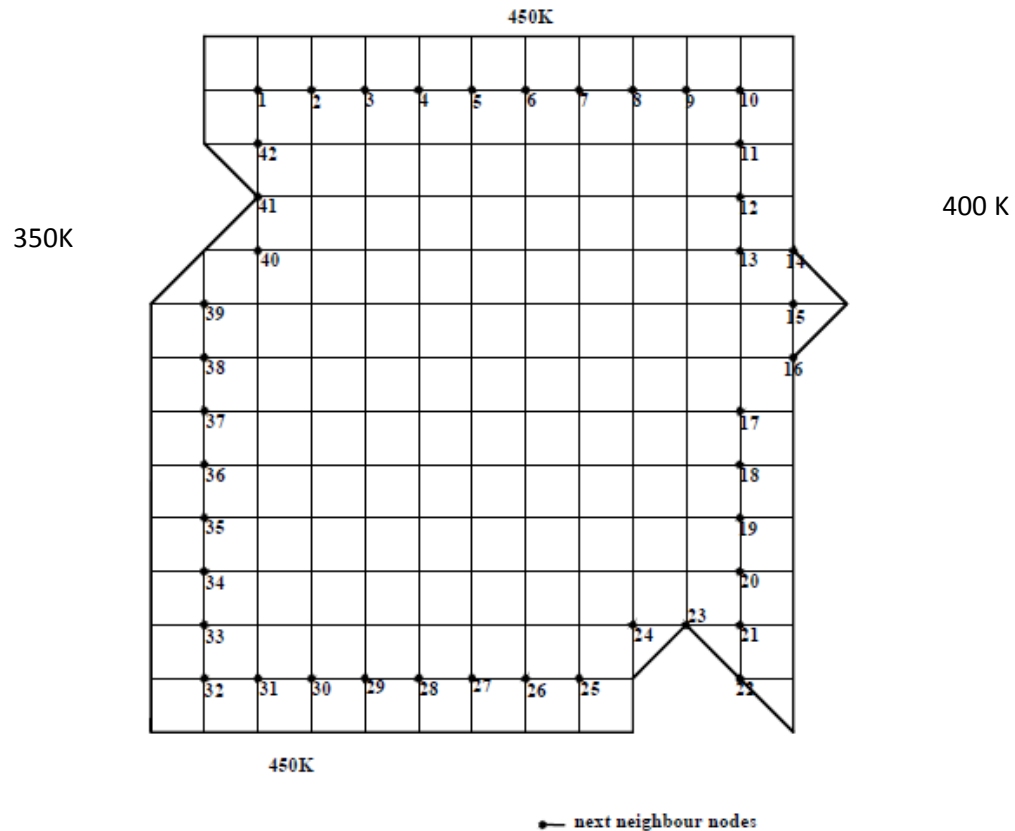


Fig. 6: Arbitrary Shaped Slab with Selected Next Neighbour nodes

A plot of obtained temperature values using only floating walk technique and HFWMC technique for all solution points is shown in Figure 7. As could be observed, values from the two techniques were very close. Statistical analysis using ANOVA($\alpha = 0.05$) yielded a probability value of 0.978 further confirming that the two techniques yielded almost the same results. ANOVA box plot in Figure 8 also showed that the two techniques results have similar shapes.

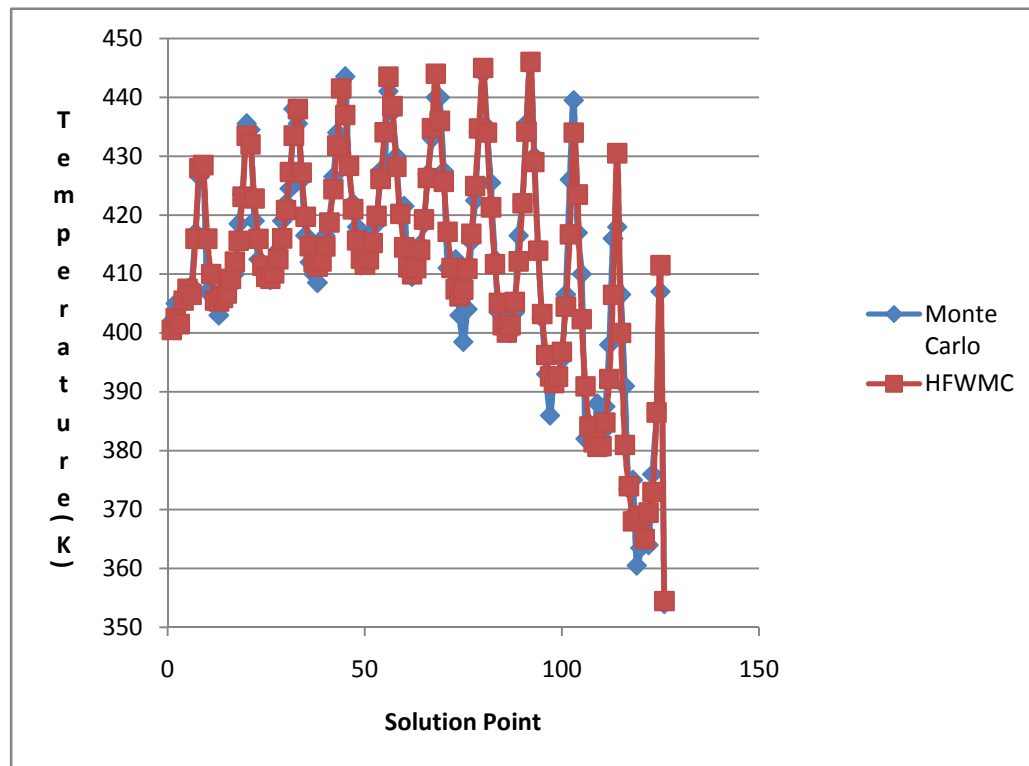


Fig. 7: Plot of temperature for various solution points using floating walk and HFWMC techniques for Arbitrary Slab with Isothermal Boundaries

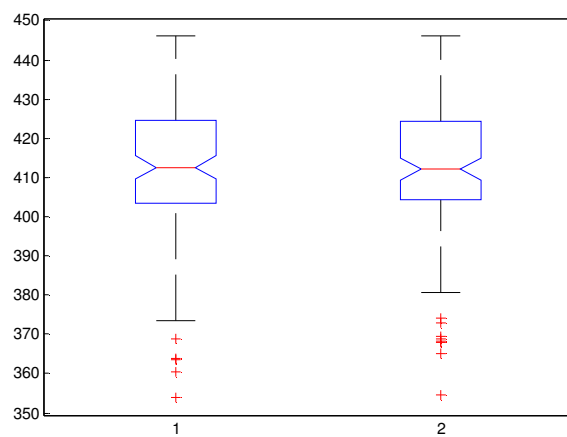


Fig. 8: ANOVA Box plot with Floating walk and HFWMC techniques

(iii) **Heat Conduction Analysis in an Arbitrary Shaped Slab with Mixed Boundaries**

Both floating walk and hybrid floating walk markov chain techniques were used to study the heat conduction in an arbitrary shaped slab with mixed boundaries(see Fig. 9). The results at all solution points are presented in Tables 4 and 5. Comparison of the values on cell to cell basis showed that they are mostly the same. Statistical analysis using ANOVA($\alpha = 0.05$) yielded probability value of 0.884 implying that the results from the two techniques are not significantly different. The ANOVA box plot as can be observed in Figure 10 showed similar shapes for the two techniques.

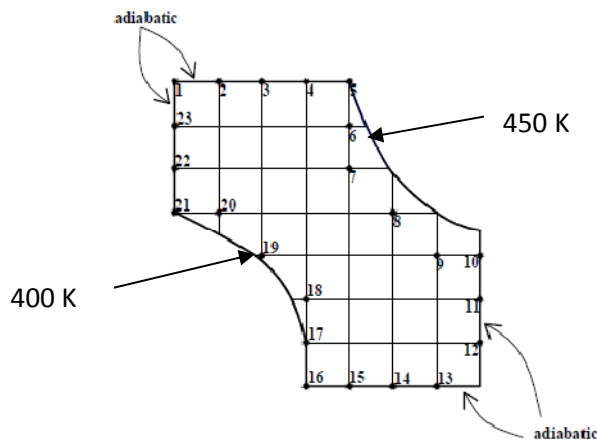


Fig. 9 : Grid Section of a Arbitrary Slab with Mixed Boundaries
Arbitrary Slab with Mixed Boundaries

Table 4: Temperature Values at Grid Points using Floating Walk Technique for Arbitrary Slab with Mixed Boundaries

435.6	436.2	441.6	442.8	442.2			
434.4	434.4	436.2	445.2	447.6			
432.6	433.8	435.6	439.8	444.6			
	425.4	426.6	429.6	437.4	445.8		
		424.2	424.8	431.4	436.2	441	438.6
			424.2	429.6	431.4	435.6	431.4
			421.8	427.8	431.4	433.8	433.2
			423	428.4	433.2	434.4	433.8

Table 5: Temperature Values at Grid Points using HFWMC Technique for Arbitrary Slab with Mixed Boundaries

436.2	437.7	440.1	444.6	445.2			
432.0	434.8	437.4	440.8	444.3			
434.7	432.1	433.9	436.9	439.8			
	425.1	429.1	433.0	437.4	443.4		
		424.5	428.6	433.4	437.7	440.1	441.6
			423.6	429.8	433.8	435.9	436.2
			424.5	428.2	432.0	433.4	432.9
			425.1	426.6	432.6	432.9	432.9

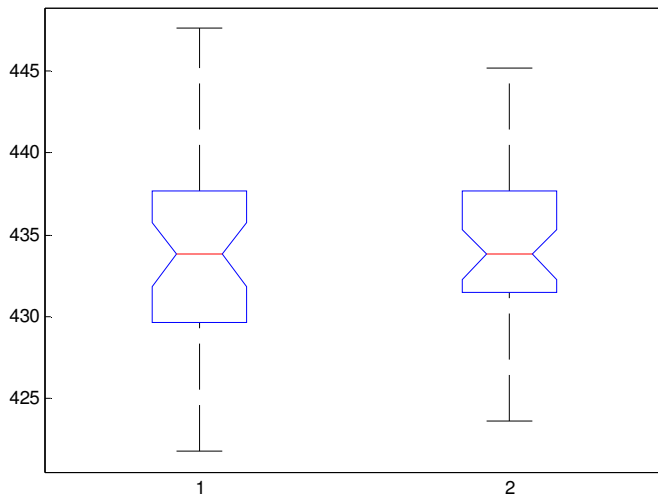


Fig. 10: ANOVA Box plot with Floating walk and HFWMC techniques for Arbitrary Slab with Mixed Boundaries

4.0 Conclusion

An hybrid technique for numerical simulation of heat conduction has been developed. The technique gave good results when compared with finite difference solution and suitably handled irregular shaped body. The computational time is however longer as compared with the finite difference technique. However in view of the general difficulty in applying finite difference

technique for analysis of irregular boundary, the developed technique may be well suited to handle such instances.

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