

# A numerical procedure for coupled consolidation in saturated soils

J. M. Horta Rangel<sup>1\*</sup>, R. Galaviz Gonzalez<sup>1</sup>, E. Rojas Gonzalez<sup>1</sup>, L. Pérez Rea<sup>1</sup>, T. Lopez Lara<sup>1</sup>, J.B. Hernandez Zaragoza<sup>1</sup>

<sup>1</sup>Graduate Engineering Department, Universidad Autónoma de Querétaro, Cerro de las Campanas s/n, Querétaro, Qro. C.P. 76010, Mexico

---

## ABSTRACT

Consolidation is the reduction of soil volume with time. This phenomenon generates a stresses transfer process beginning with the application of load and subsequent increase in pore water pressure. This increase in pore water pressure generates a flow of water that reduces the volume of the soil. Thus, consolidation represents a hydro-mechanical coupled problem. Different models have been developed to simulate this phenomenon properly. This paper presents a coupled hydro-mechanical consolidation model for saturated materials. It can also be extended to the case of unsaturated materials when effective stresses are used. This model considers different drainage, displacement and loading conditions. It properly simulates the phenomenon of consolidation and is consistent with other models. It also correctly reproduce laboratory tests.

*Keywords: coupled model; consolidation theory; saturated soil, dynamic solution, fluid-solid fields*

## 1. BACKGROUND

A saturated porous medium, such as soils, can be considered as a deformable solid skeleton with all pores filled with fluid [1]. There are many geotechnical problems related to the consolidation of saturated soils. A very common one refers to the settlements of a foundation slab placed on the surface of a saturated material. The soil suffers volume decrease induced by the applied load. These volume changes may result in differential settlements producing the cracking of the foundation [2,3]. It has established that these damages are more expensive than those associated to natural disasters [4]. Additionally, volume changes are induced by the increase in pore water pressure generating a flow of water. This process is called consolidation [5]. The consolidation process involves the flow of water, the change in effective stresses and a volumetric reduction, hence it is considered as a coupled hydro-mechanical problem [7]. Unsaturated soils may reach the saturated condition when heavy rains occur or water pipe leaks [5,6].

Some coupled models have been developed to simulate the saturated consolidation, such as: Magaña and Romo [5], Bentler [7], Manzolillo et al. [8], Di-Rado et al. [9] y Krishnamoorthy [10]. These models are based on the following hypothesis: a) the soil is saturated, b) the solid and water phase are incompressible c) Darcy's law governs the behavior of flow through the soil. In addition, other authors have made improvements by

incorporating different theories as for example: the theory of small deformations used by Bentler [7]. The principle of virtual work used by Manzolillo et al. [8] and the principle of mass conservation used by Manzolillo et al. [8] y Di-Rado et al. [9]. These latter employed the Terzaghi's effective stress and the Galerkin's solution method of weighted residuals, respectively. Besides, Krishnamoorthy [10] took into account the nonlinear behavior of soil and used the hyperbolic relationship of Duncan and Chang (1970). These models show consistent results with experimental tests. All these models have their advantages and disadvantages. However, the last model can be improved by combining some of the theories previously mentioned.

This has motivated the development of a new numerical procedure of coupled hydro-mechanical consolidation for saturated materials. This formulation considers the principles of the balance of momentum, the transient flow and effective stresses where Terzaghi's principle is introduced. Initially, an inner product between the vector functions that takes into account the virtual work principle associated to the variational principle of the minimum potential energy is considered. In addition, the Finite Element Method (FEM) in combination with Galerkin's method that includes a time increment in which the evolution of displacements and pore water pressure are determined using an approximate solution.

As there are no analytical solutions for the hydro-mechanical coupling in unsaturated soils, numerical techniques, such as the Finite Element method needs to be used. This it can be done when the mechanical consolidation equation, the soil-water retention curve (SWRC) and the permeability of the soil are combined [11]. Therefore, this procedure of coupled hydro-mechanical consolidation of saturated soils, can be extended to the unsaturated case. In such a case, the model uses Bishop's effective stress equation for unsaturated soils which includes the net stress  $(\sigma - u_a)$  and the matric stress  $\chi\psi$ , represented by the product of parameter  $\chi$  and soil suction. Both parameter  $\chi$  and the hydraulic conductivity can be obtained from the SWRC as a function of suction.

## 2. GOVERNING EQUATIONS

Due to seasonal changes, natural soils are subjected to cyclic wetting-drying cycles. When the water content is high, applied loads on the soil causes the flow of water and reduction of the soil volume. Thus, mechanical soil deformation and water flow represents a coupled field problem. Some assumptions include: a) the soil is fully saturated, b) the water and solid phase are incompressible, c) Darcy's law governs the flow of water. In addition, small deformations theory is used. So, according to the continuum mechanics theory the governing equations are as follow:

### i. Equilibrium Equations:

According to the Balance Law of Continuum Mechanics, and introducing the Piola-Kirchhoff Stress Tensor  $\underline{S}$  in Lagrangean Configuration; this tensor gives the force measured per unit area in the configuration of reference [12].

$$Div \underline{S} + \underline{b}_o = \underline{\chi} \underline{\rho}_o \quad (1)$$

Body forces commonly are considered as nil,  $\underline{b}_o = \underline{0}$ . Also acceleration field is considered

nil,  $\underline{\chi} = \underline{0}$ , since the settlement process is considered as quasi-static.

101 ii. Continuity Equations

102

$$\text{div} \underline{v} - \dot{\epsilon} = 0 \quad (2)$$

103

104 where  $\underline{v}$  is the velocity field and  $\dot{\epsilon}$  is the derivative of the field strain.

105

106 iii. Darcy's Law

107

$$v_i = k_{ij} (\gamma_w h_i)_{,j} / \gamma_w \quad (3)$$

108

109 where  $k$  the coefficient of permeability of the soil,  $\gamma$  is the specific weight. Piezometric level  $h$   
110 is evaluated according to the reference level  $z$ :  $h = p_w / \gamma_w + z$ .

111

112 iv. Boundary Conditions:

113

114 These conditions are related to the values of pore water pressure and stress at the  
115 boundaries. Also the surface inflow or outflow due to seasonal wetting-drying is considered.  
116 The total boundary  $\Omega$  conditions include both Dirichlet  $\Omega_D$  boundary and Newman's boundary  
117  $\Omega_N$  conditions. Then, we can write these conditions in terms of the prescribed values:

118

119 a) Excess pore pressure conditions:

120

$$u = \hat{u} \quad \text{on} \quad \partial\Omega_D \quad (4)$$

121

122 b) Surface stresses:

123

$$\underline{S} \underline{n} = \hat{\underline{t}} \quad \text{on} \quad \partial\Omega_N \quad (5)$$

124

125 c) Surface Inflow:

126

$$Q = \hat{Q} \quad \text{on} \quad \partial\Omega_i \quad (6)$$

127

128 Another condition is related to the inflow in the soil. This case can be observed as an induced  
129 water flow due to failure of sewers or water pipes.

130

131 d) Inflow inside the soil:

132

$$Q_s = \hat{Q}_s \quad \text{on} \quad \partial\Omega_i \quad (7)$$

133

134 **3. CONSTITUTIVE EQUATION**

135

136 The relationship between stresses and soil deformation follows the Hooke's Law. Stress  
137 Tensor  $\underline{S}$  involves stresses on the soil skeleton as on the pore water

138

$$\underline{S} = \underline{\bar{S}} + p_w \underline{I} \quad (8)$$

139

140 where:  $\tilde{S}$  is the Tensor of stresses,  $p_w$  is the pore water pressure (considered as  
 141 hydrostatic) and  $\tilde{I}$  is the Identity Tensor  $= \sum_i e_i \otimes e_i$  (Tensor Nomenclature [12])

142  
 143 The procedure for establishing coupled equations is based on the virtual work principle.  
 144 Discrete coupled equations are written as follows:  
 145

$$[K][U] + [K_v]^T \{p\} = \{F\} \quad (9)$$

146  
 147 where  $[K]$  is the soil stiffness matrix,  $[K_v]^T$  is the coupled matrix transpose,  $\{U\}$  is the  
 148 nodal displacements vector,  $\{p\}$  is the nodal water excess pore pressure,  $\{F\}$  is the external  
 149 forces vector.  
 150

$$[K_v] \left\{ \dot{U} \right\} + \frac{1}{\gamma_w} [K_h] \{p\} = \{Q\} \quad (10)$$

151  
 152 where  $[K_v]$  is the coupled matrix,  $[K_h]$  is the drained matrix,  $\{Q\}$  external/internal flow  
 153 vector,  $\left\{ \dot{U} \right\}$  velocity vector of nodal displacements.

154  
 155 Resulting matrices involve the following integral forms:  
 156

$$\begin{aligned} [K] &= \int_V [B_\phi]^T [\bar{D}] [B_u] dV \\ [K_v]^T &= \int_V [B_\phi]^T \langle 1 \ 1 \ 0 \rangle^T [N_p] dV \\ [K_h] &= \int_V [B_\theta]^T [k] [B_p] dV \end{aligned} \quad (11)$$

157  
 158 Matrices  $[B]$  contain derivatives of the shape function;  $[k]$  is the permeability matrix, and the  
 159 vector  $\langle 1 \ 0 \ 0 \rangle^T$  is required to couple the degree of freedom of the two involved fields.

160  
 161 The resulting matrix equation associates the coupled hydro-mechanical consolidation model  
 162 of saturated soils. Equations (9) and (10) can be written as a coupled equation:  
 163

$$\begin{bmatrix} 0 & 0 \\ K_v & 0 \end{bmatrix} \begin{Bmatrix} \dot{U} \\ p \end{Bmatrix} + \begin{bmatrix} K & K_v^T \\ 0 & \frac{1}{\gamma_w} K_h \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix} \quad (12)$$

164  
 165 Eq. (12) can be solved using a dynamic procedure. The variables are the soil displacements  
 166 and the excess of pore water pressure with time. One of the procedures used in this work is  
 167 the application of the finite difference method with time along with Galerkin's method for theta  
 168 values. This approximate quantities are referred as the external forces on soil surface  $F$ ; the  
 169 induced inflow charges  $Q$ ; the excess of pore water pressure on the soil  $p$ ; the displacements  
 170 of soil mass, as well as the rate of movement in terms of velocity.  
 171

$$\begin{aligned}
\{F\} &= \{F_a\} + \theta\{\{F_b\} - \{F_a\}\} \\
\{Q\} &= \{Q_a\} + \theta\{\{Q_b\} - \{Q_a\}\} \\
\{p\} &= (1 - \theta)\{p_b\} + \theta\{p_a\} \\
\{U\} &= (1 - \theta)\{U_b\} + \theta\{U_a\} \\
\{\dot{U}\} &= \{U_b\} - \{U_a\} / \Delta t
\end{aligned} \tag{13}$$

172  
173 where:  $\{F_a\}$ ,  $\{Q_a\}$ ,  $\{p_a\}$  and  $\{U_a\}$  are the initial vectors,  $\{F_b\}$ ,  $\{Q_b\}$ ,  $\{p_b\}$  and  $\{U_b\}$  are  
174 the vectors at certain time,  $\theta$  is the value of Galerkin's parameter on the numerical solution  
175 of evolution over time,  $\Delta t$  is the increment of time that avoid numerical oscillations according  
176 to the following relationship (for triangular finite elements).  
177

$$\Delta t \leq \frac{\alpha}{1 - \theta} \quad \alpha_{\min} = \frac{2A}{9C_v} \tag{14}$$

178  
179 where:  $A$  is the area of the linear triangular finite element,  $C_v$  is the coefficient of  
180 consolidation for the soil.  
181

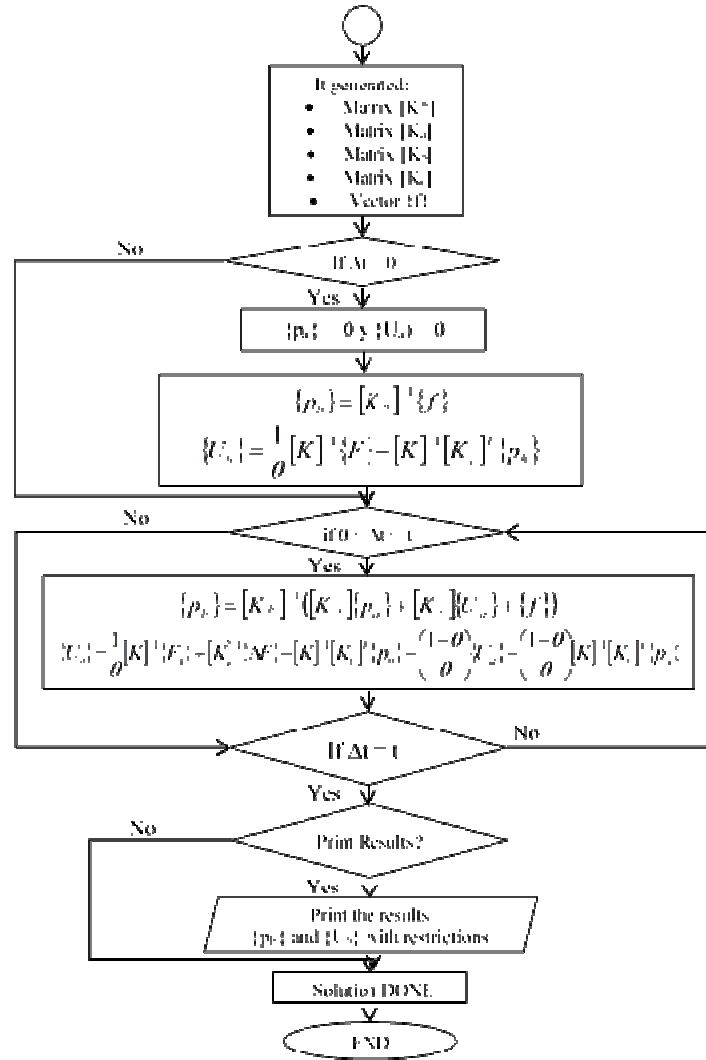


**Fig. 1. Flowchart of the dynamic process of saturated consolidation coupled model.**

A Fortran code has been developed based on the proposed algorithm along with some visualization tools. Fig. 1 shows the flow chart of the coupled model.

#### 4. NUMERICAL EXAMPLE

A rectangular slab of  $L=5.0$  m, with unitary width (plane strain condition) and a distributed surface stress  $F = -40$  kPa (the negative sign denotes downwards) on a layer of clay  $H=3.0$  m thick (see Fig. 2).

Displacements  $U_x$  and  $U_y$  are nil at the bottom and lateral boundaries of the soil. Furthermore, the clay layer is placed on a thin sand layer which allows outflow of water from the lower boundary. So, on this boundary the excess of pore water pressure is zero. In addition, water flow is not allowed on the upper boundary at the contact between the soil and the slab. Finally, water outflow is not allowed on lateral boundaries. The size of the sides of triangular elements in the mesh is 0.25 m.

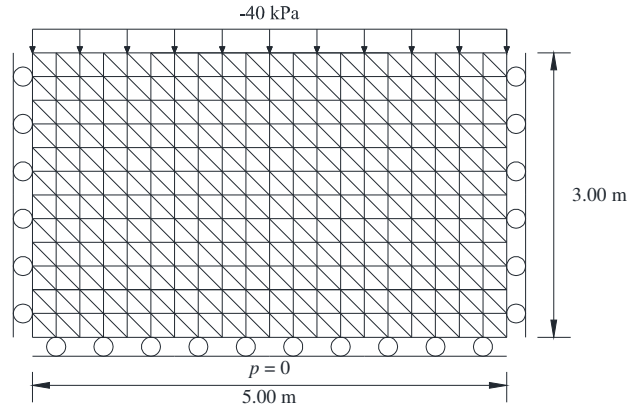


**Fig. 2. Soil's domain with slab foundation applied on the surface.**

During the dynamic analysis, the time step was taken as 50 days. The soil properties required for mechanical analysis are shown in TABLE 1. They represent medium values for a saturated clay [12,13].

**Table 1. Properties of clayey soil stratum for consolidation analysis.**

| Soil Properties                           | Symbol | Magnitude      |
|-------------------------------------------|--------|----------------|
| Elastic Modulus                           | E      | 20000 kPa      |
| Poisson's ratio                           | $\nu$  | 0.35           |
| Coefficient of permeability (x direction) | $k_x$  | 0.000117 m/day |
| Coefficient of permeability (y direction) | $k_y$  | 0.000117 m/day |

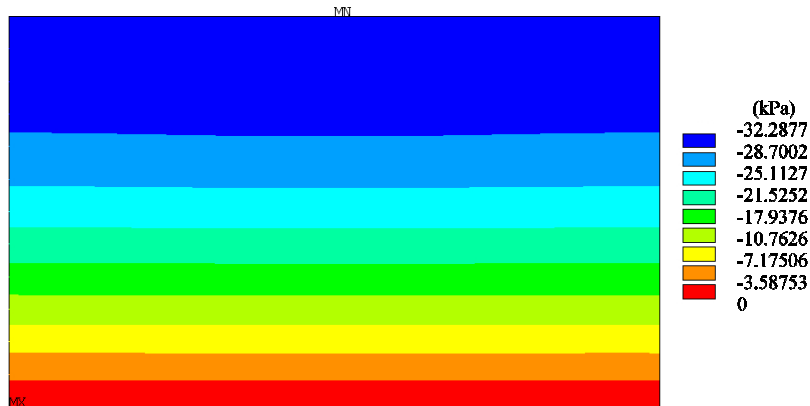


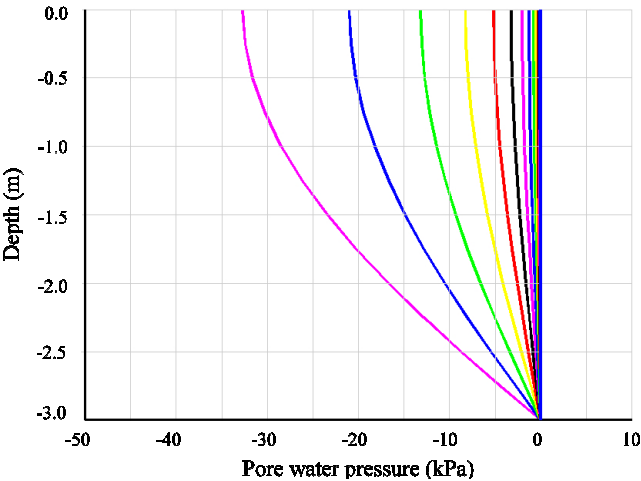
**Fig. 3. Pore water pressure distribution at 50 days.**

Pore water pressure tend to dissipate when time increases. Table 2, shows the distribution of pore water pressure at times of 50, 450, and 900 days. Fig. 3 shows the distribution of the increment of pore water pressure 50 days after the load was applied. The initial pore water pressure (-32.29 kPa) below the slab is still close to the stress applied by the foundation (-40 kPa).

225 **Table 2. Pore water pressure distribution.**  
 226

| Pore water pressure distribution (kPa) |         |                        |                         |
|----------------------------------------|---------|------------------------|-------------------------|
| Color Scale                            | 50 days | 450 days               | 900 days                |
|                                        | -32.29  | $-4.63 \times 10^{-4}$ | $-1.97 \times 10^{-9}$  |
|                                        | -28.70  | $-4.12 \times 10^{-4}$ | $-1.75 \times 10^{-9}$  |
|                                        | -25.11  | $-3.60 \times 10^{-4}$ | $-1.53 \times 10^{-9}$  |
|                                        | -21.53  | $-3.09 \times 10^{-4}$ | $-1.31 \times 10^{-9}$  |
|                                        | -17.94  | $-2.57 \times 10^{-4}$ | $-1.09 \times 10^{-9}$  |
|                                        | -14.35  | $-2.06 \times 10^{-4}$ | $-8.75 \times 10^{-10}$ |
|                                        | -10.76  | $-1.54 \times 10^{-4}$ | $-6.56 \times 10^{-10}$ |
|                                        | -7.18   | $-1.03 \times 10^{-4}$ | $-4.37 \times 10^{-10}$ |
|                                        | -3.59   | $-5.15 \times 10^{-5}$ | $-2.19 \times 10^{-10}$ |

227  
 228 Fig. 4 shows a vertical profile of the evolution of the pore water pressure with time. It is  
 229 plotted on the axis of excess in pore water pressures against depth. Notice that all values  
 230 tend to zero as time increases.  
 231



232  
 233  
 234 **Fig. 4. Evolution of pore water pressure.**  
 235

236 Another main result refers to the vertical displacements. The vertical displacements due to  
 237 applied loads tend to diminish with increasing time. Fig. 5 shows the vertical displacements  
 238 50 days after the load was applied. In this figure displacements are around -0.0168 m  
 239 (directed downwards) at the edge of the foundation, while at the bottom they are close to  
 240 zero. The behavior of the evolution of vertical displacement is consistent with those observed  
 241 in the laboratory and in the field. This is evidenced in Fig. 5 and Table 3.  
 242

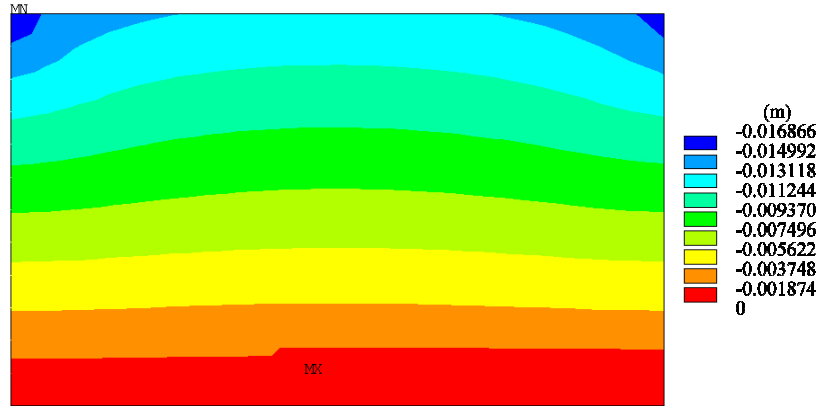


**Fig. 5. Distribution of the vertical displacements  $U_y$ .**

Table 3 shows the values of the displacements at 50, 450, and 900 days. Furthermore, according to Table 3, the maximum displacement observed on the surface is reached at 450 days. These displacements represent the final settlements caused by the loads transmitted by the slab to the soil as at this time pore water pressure has almost completely dissipated. This assertion is verified through Fig. 6 which shows the evolution of displacements with time.

**Table 3. Distribution of vertical displacements  $U_y$ .**

| Vertical displacements $U_y$ (m) |         |          |          |
|----------------------------------|---------|----------|----------|
| Color Scale                      | 50 days | 450 days | 900 days |
|                                  | -0.0169 | -0.0189  | -0.0189  |
|                                  | -0.0149 | -0.0168  | -0.0168  |
|                                  | -0.0131 | -0.0147  | -0.0147  |
|                                  | -0.0112 | -0.0126  | -0.0126  |
|                                  | -0.0094 | -0.0105  | -0.0105  |
|                                  | -0.0075 | -0.0084  | -0.0084  |
|                                  | -0.0056 | -0.0063  | -0.0063  |
|                                  | -0.0037 | -0.0042  | -0.0042  |
|                                  | -0.0018 | -0.0021  | -0.0021  |

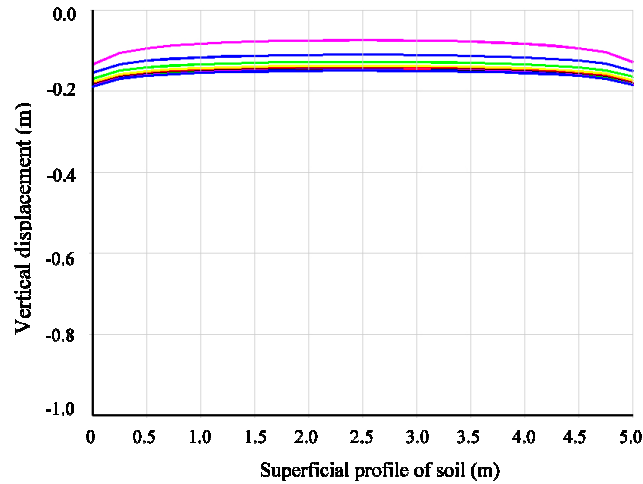


**Fig. 6. Superficial profile of soil.**

Another case of interest is when an isolated foundation is placed on a saturated soil as shown in Fig. 7. For the following exercise the same domain with  $L=5.0$  m was used, but only one isolated footing is placed to the center and transfers a distributed load of  $-40$  kPa over  $2.5$  m. In this way those parts of the superior boundary that are not in contact with the slab, allow the water flow ( $1.25$  m on each side of the foundation). It means that the excess of pore water pressure remains null in these zones. In addition, the lower boundary also allows water flow. The displacements are restricted in the lateral and lower boundaries (see Fig. 7). Table 4 shows the properties of the soil, the increment of time and the final time for the dynamic analysis.

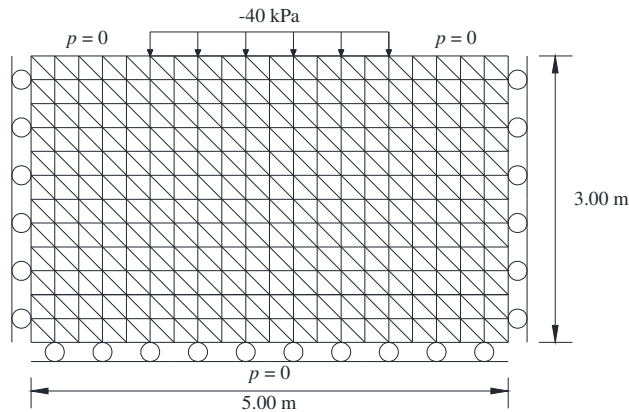


**Fig. 7. Soil's domain with isolated footing applied on the surface.**

Table 4. Data required for the dynamic analysis.

| Data for dynamic analysis                 | Symbol     | Magnitude     |
|-------------------------------------------|------------|---------------|
| Elastic Modulus                           | E          | 20000 kPa     |
| Poisson's ratio                           | $\nu$      | 0.35          |
| Coefficient of permeability (x direction) | $k_x$      | 0.00117 m/day |
| Coefficient of permeability (y direction) | $k_y$      | 0.00117 m/day |
| Increment of time                         | $\Delta t$ | 30 days       |
| Time analysis                             | t          | 900 days      |

Fig. 8 shows the distribution of the increment of pore water pressure at a time of 30 days. Notice that the excess of pore water pressure at 30 days is still very close to the value of the applied stress on the surface. The distribution of pore water pressures is similar to this figure for 450 and 870 days and only the values of pore water pressures change as shown in

Error! Not a valid bookmark self-reference. shows again that pore water pressure dissipates with time and it tends to become null at time infinity.

Table 5.

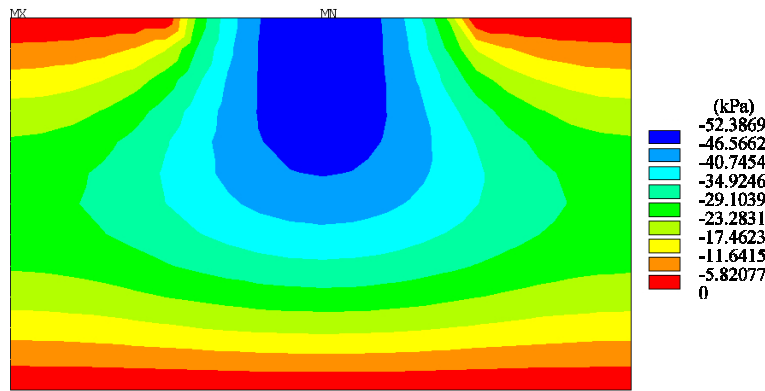


Fig. 8. Distribution of pore water pressure for isolated footing at 30 days.

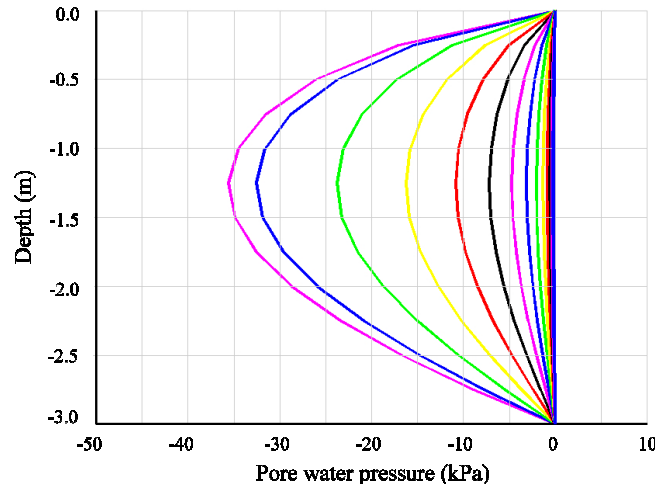
Error! Not a valid bookmark self-reference. shows again that pore water pressure dissipates with time and it tends to become null at time infinity.

Table 5. Distribution of pore water pressure for isolated footing.

| Pore water pressure distribution (kPa) |         |          |          |
|----------------------------------------|---------|----------|----------|
| Color Scale                            | 30 days | 450 days | 870 days |
|                                        | -52.38  | -4.41    | -0.23    |
|                                        | -46.56  | -3.92    | -0.21    |
|                                        | -40.74  | -3.43    | -0.18    |
|                                        | -34.92  | -2.94    | -0.16    |
|                                        | -29.10  | -2.45    | -0.13    |
|                                        | -23.28  | -1.96    | -0.10    |
|                                        | -17.46  | -1.47    | -0.08    |
|                                        | -11.64  | -0.98    | -0.05    |
|                                        | -5.82   | -0.49    | -0.03    |

297  
298  
299  
300  
301  
302  
303

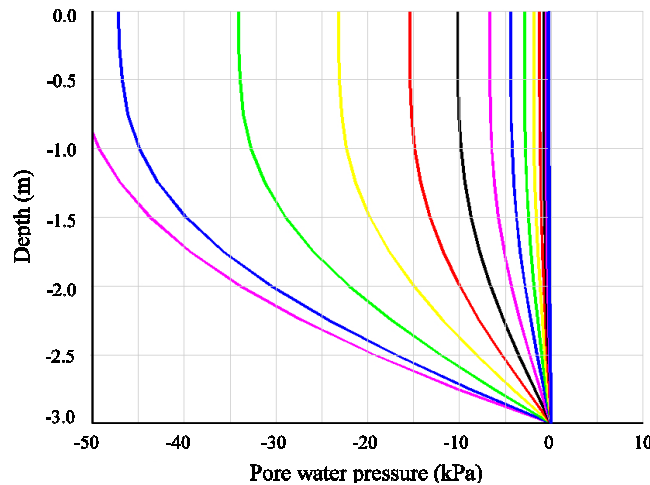
Another result is the vertical profile of the pore water pressure shown in Fig. 9. This figure shows the distribution of pore water pressure with depth at 1.25 m from the left boundary. This point represents the limit of the free surface where the flow of water is allowed. Thus, the excess of pore water pressure remains null at the surface and lower boundary, while inside the domain it shows the classic evolution of double drained soils.



304  
305  
306  
307  
308  
309  
310  
311  
312

**Fig. 9. Evolution of the vertical profiles of pore water pressure at 1.25 m.**

Moreover, Fig. 10 shows the vertical profile of the pore water pressure at 2.50 m (center of the domain). It can be observed that pore water pressure initiates at a value close to the applied stress on the surface and, as depth increases, it reduces. In both figures, the values of pore water pressures tend to become zero at time infinity.



313  
314  
315  
316  
317  
318

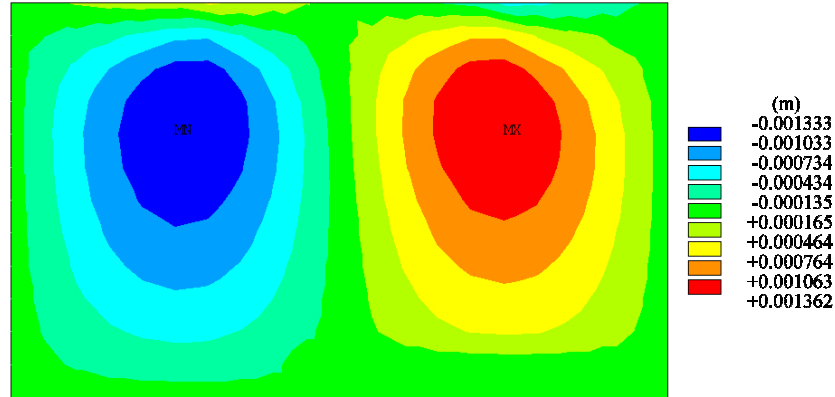
**Fig. 10. Evolution of the vertical profiles of pore water pressure at 2.50 m.**

The model also provides diagrams of the horizontal  $U_x$  and vertical  $U_y$  displacement. They are presented in Fig. 11 and Fig. 12. Additionally,

319 Table 6 and Table 7 show the distribution of displacements  $U_x$  and  $U_y$  at different times (30,  
320 450, and 870 days).

321  
322 In Fig. 11 and

323 Table 6 a characteristic phenomenon of symmetry due to loads applied at the central zone is  
324 observed. From the center line to the left, values have a negative sign. This is due to the  
325 direction in which they occur, as in the coordinate system, the y-axis is located at the center  
326 line. In contrast, the displacements on the right side from the center line are positive.



327  
328  
329 **Fig. 11. Horizontal displacements  $U_x$  for isolated footing.**

330  
331 **Table 6. Distribution of  $U_x$  displacements for isolated footing.**  
332

| Horizontal displacements $U_x$ (m) |                        |                        |                        |
|------------------------------------|------------------------|------------------------|------------------------|
| Color Scale                        | 30 days                | 450 days               | 870 days               |
|                                    | $-1.33 \times 10^{-3}$ | $-1.52 \times 10^{-3}$ | $-1.51 \times 10^{-3}$ |
|                                    | $-1.03 \times 10^{-4}$ | $-1.18 \times 10^{-4}$ | $-1.17 \times 10^{-4}$ |
|                                    | $-7.34 \times 10^{-4}$ | $-8.35 \times 10^{-4}$ | $-8.26 \times 10^{-4}$ |
|                                    | $-4.34 \times 10^{-4}$ | $-4.91 \times 10^{-4}$ | $-4.86 \times 10^{-4}$ |
|                                    | $-1.35 \times 10^{-4}$ | $-1.48 \times 10^{-4}$ | $-1.46 \times 10^{-4}$ |
|                                    | $1.65 \times 10^{-4}$  | $1.96 \times 10^{-4}$  | $1.95 \times 10^{-4}$  |
|                                    | $4.64 \times 10^{-4}$  | $5.40 \times 10^{-4}$  | $5.35 \times 10^{-4}$  |
|                                    | $7.64 \times 10^{-4}$  | $8.83 \times 10^{-4}$  | $8.75 \times 10^{-4}$  |
|                                    | $1.06 \times 10^{-4}$  | $1.23 \times 10^{-4}$  | $1.22 \times 10^{-4}$  |

333  
334 Fig. 12 and Table 7 show the evolution of the vertical displacements  $U_y$ . Furthermore, through  
335 this figure displacements are about -0.0069 m on the edge of the foundation, while near the  
336 bottom they are close to zero. Here, the negative sign indicates the direction in which the  
337 displacements evolve. Vertical displacements are produced by the load applied by the slab.  
338 Notice that displacements show a symmetrical diagram. They are also appropriate and  
339 consistent with the results reported by Magaña y Romo [5], Bentler [7], Manzoillo et al. [8],  
340 Di-Rado et al. [9] and Krishnamoorthy [10].  
341

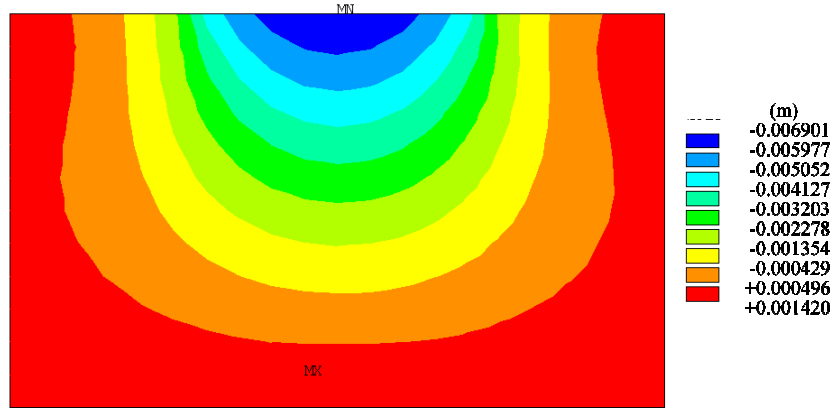


**Fig. 12. Vertical displacements  $U_y$  for isolated footing.**

Fig. 13 shows the surface profile of vertical displacements. Again, notice that they are negative (directed downwards) and the maximum value ( $-0.0130$  m) occurs at the center of the domain, i.e., at the center of the slab. In addition, in the free surface areas (where stresses are not applied) they show positive values. This means that in these areas the soil moves upwards while the slab sinks.

**Table 7.  $U_y$  displacements for isolated footing.**

| Vertical displacements $U_y$ (m) |                        |                        |                        |
|----------------------------------|------------------------|------------------------|------------------------|
| Color Scale                      | 30 days                | 450 days               | 870 days               |
|                                  | $-6.90 \times 10^{-3}$ | $-1.30 \times 10^{-2}$ | $-1.30 \times 10^{-2}$ |
|                                  | $-5.97 \times 10^{-3}$ | $-1.16 \times 10^{-2}$ | $-1.16 \times 10^{-2}$ |
|                                  | $-5.05 \times 10^{-3}$ | $-1.10 \times 10^{-2}$ | $-1.10 \times 10^{-2}$ |
|                                  | $-4.13 \times 10^{-3}$ | $-8.68 \times 10^{-3}$ | $-8.68 \times 10^{-3}$ |
|                                  | $-3.20 \times 10^{-3}$ | $-7.23 \times 10^{-3}$ | $-7.23 \times 10^{-3}$ |
|                                  | $-2.28 \times 10^{-3}$ | $-5.79 \times 10^{-3}$ | $-5.79 \times 10^{-3}$ |
|                                  | $-1.35 \times 10^{-3}$ | $-4.34 \times 10^{-3}$ | $-4.34 \times 10^{-3}$ |
|                                  | $-4.29 \times 10^{-4}$ | $-2.89 \times 10^{-3}$ | $-2.89 \times 10^{-3}$ |
|                                  | $+4.96 \times 10^{-4}$ | $+1.45 \times 10^{-3}$ | $+1.45 \times 10^{-3}$ |

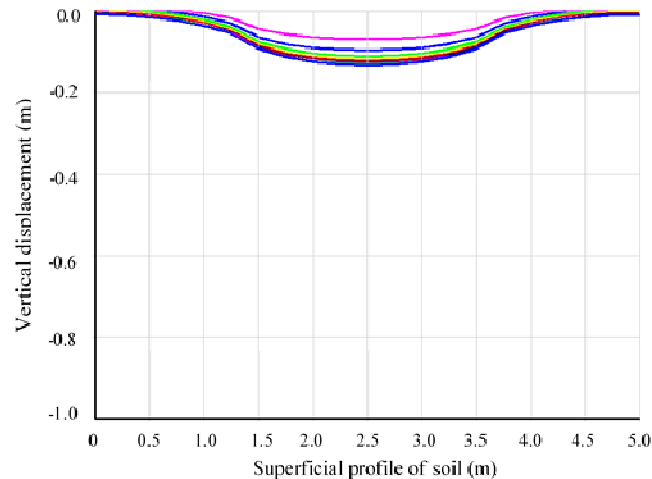


**Fig. 13. Superficial profile of soil for isolated footing.**

## 5. CONCLUSIONS

The coupled model algorithm presented herein, shows appropriate results and uses few basic soil parameters. It is flexible, allowing infinite combinations of load, displacements and pore water pressure at the boundaries. Therefore it can reproduce different field or laboratory conditions. It may consider water intake or outtake through the flow vector  $\{Q\}$ . The evolution of pore water pressure as well as horizontal and vertical displacements can be observed. Furthermore, the results show an appropriate behavior of the phenomenon of consolidation of saturated soils when compared to other models and experimental results.

Another advantage of this model is that it can be extended for the case of unsaturated soils. This is possible only by replacing the effective stress equation of saturated soils to that of unsaturated materials and by introducing a model for the retention curve during wetting-drying cycles. In such a case the effective stress parameter  $\chi$  and the hydraulic permeability of the soil can be determined for any value of suction. In the future, efforts will be aimed towards unsaturated consolidation through this model.

## ACKNOWLEDGEMENT

The authors would like to thank to the Consejo Nacional de Ciencia y Tecnología (CONACYT) and the Universidad Autónoma de Querétaro for their founding to the project "Consolidación en suelos saturados mediante acoplamiento de campos usando el Método del Elemento Finito (FOFI-FIN201606)".

## REFERENCES

1. Inoue N, Romanel C, Roehl D, Análisis por el método de los elementos finitos de la consolidación de suelos no saturados, 4to Congreso Internacional, 2do Congreso Nacional de Métodos Numéricos en Ingeniería y Ciencias Aplicadas. 2007. México.
2. Ali MM, Identifying and analyzing problematic soils. Geotech. Geol. Eng. 2011;29, 343-350.
3. Al-Shamrani MA, Dhownian AW, Experimental study of lateral restraint effects on the potential of expansive soils. Engineering Geology, 2003;69, 63-81.

- 393 4. Jones LD, Jefferson I, Expansive soils. In: Burland, J. Institution of Civil Engineers (ICE)  
394 manual of geotechnical engineering. 2012. ed. London, UK.: ICE Publishing.
- 395 5. Magaña R, Romo MP, Modelo numérico para estudiar el movimiento de agua en suelos  
396 no saturados. Instituto de Ingeniería, Universidad Nacional Autónoma de México.  
397 1980;1-23.
- 398 6. Wong TT, Fredlund DG, Krahn J, A numerical study of coupled consolidation in  
399 unsaturated soils. Canadian Geotechnical Journal. 1998;35:926-937.
- 400 7. Bentler DJ, Coupled consolidation, Finite element analysis of deep excavations.  
401 1998;83-102.
- 402 8. Manzanillo JE, Di-Rado HA, Awruch AM, Simulación numérica del comportamiento de  
403 suelos saturados bajo cargas de fundaciones. Comunicaciones Científicas y  
404 Tecnológicas, Universidad Nacional del Nordeste. 2000.
- 405 9. Di-Rado HA, Beneyto PA, Mroginski JL, Manzanillo JE, Awruch AM, Análisis  
406 tridimensional de la consolidación de suelos saturados utilizando el MEF, Mecánica  
407 Computacional. 2004;23:607-617.
- 408 10. Krishnamoorthy MS, Consolidation analysis using finite element method. The 12th  
409 International Conference of International Association for Computer Methods and  
410 Advances in Geomechanics (IACMAG). 2008;1157-1161.
- 411 11. Tsiamposi A, Smith P, Potts D, Coupled consolidation in unsaturated soils: From a  
412 conceptual model to applications in boundary value problems. Computers and  
413 Geotechnics. 2016;In press, <http://dx.doi.org/10.1016/j.compgeo.2016.10.008>.
- 414 12. Gurtin ME, An introduction to continuum mechanics, Mathematics in Sciences and  
415 Engineering. 2003. Academic Press Elsevier.
- 416 13. Pérez-Rea ML, Predicción de propiedades mecánicas y reológicas de suelos usando  
417 teoría de percolación. Ph. D. thesis. Instituto Tecnológico y de Estudios Superiores de  
418 Monterrey. 2005. Estado de México, México.
- 419 14. Vázquez-Nogal I, Impacto en la conductividad hidráulica en la modificación con cal en  
420 suelos expansivos. M. Sc. Thesis, Universidad Autónoma de Querétaro. 2016.  
421 Querétaro, México.