

CORDIAL AND MEAN LABELINGS ON EXTENDED DUPLICATE GRAPH OF COMB GRAPH.

Abstract

A graph labeling is a mapping that carries a set of graph elements onto a set of numbers called labels(usually the set of integers). In this paper we prove the existence of graph labeling such as cordial, total cordial, product cordial, total product cordial, prime cordial,odd mean labeling and even mean labeling for extended duplicate graph of Comb graph by presenting algorithms .

AMS Subject Classification : 05C78

Keywords: Graph labeling, Comb , Duplicate graph.

1 . Introduction

Graph labeling has the origin from a seminal paper by Rosa in 1967 . It is defined as an assignment of integers to the vertices or edges or both subject to certain conditions. Andar Boxwala and Limaya[1] have shown that a cordial labeling of G can be extended to a cordial labeling of the graph obtained from G by attaching $2m$ pendent edges at each vertex of G .

The concept of product, total product and prime cordial labeling was introduced by M Sundaram, R Ponraj and S Somasundaram [8] . They proved that the following graphs are prime cordial: C_n if and only if $n \geq 6$; P_n if and only if $n \neq 3$ or 5 ; bistars; dragons; crowns ;ladders. Entringer proved that the disjoint union of C_{2k} and C_n is prime;the complete graph K_n does not have a prime labeling.

Somasundaram and Ponraj [9] have introduced the notion of mean labeling of graphs.The notion of odd and even mean labeling was introduced by Nirmala.

K.Thirusangu, et al have introduced the concept of extended duplicate graph and they proved that $EDG(P_m)$ is cordial and also proved that the extended duplicate graph of a path P_m admits cordial ,total cordial, product cordial and total product cordial labeling [10].

In this paper, we prove that, the extended duplicate graph of Comb graph is cordial, total cordial, product cordial, total product cordial, prime cordial, odd mean and even mean labeling by presenting algorithms.

2 Preliminaries

In this section, we give basic notions relevant to this paper.

Definition 2.1:

Let P_{m+1} be a path graph. Comb graph is defined as $P_{m+1} \odot (m+1)K_1$. It has $2m+2$ vertices and $2m+1$ edges.

Definition 2.2:

Let $G(V, E)$ be a Comb graph. A Duplicate graph of G is $DG=(V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \emptyset$ and $f: V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as follows: The edge $v_i v_j$ is in E if and only if both $v_i v_j'$ and $v_i' v_j$ are edges in E_1 . Clearly duplicate graph is disconnected.

Definition 2.3:

A function $f: V \rightarrow \{0, 1\}$ is said to be cordial labeling if each edge uv receives the label $|f(u) - f(v)|$ such that the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one, and the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one. Moreover, it admits total cordial labeling if the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by at most one.

Definition 2.4 :

A function $f: V \rightarrow \{0, 1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one, and the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one. Moreover, it is said to be total product cordial labeling if the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by at most one.

Definition 2.5:

A prime cordial labeling of a graph G with vertex set V is a function $f : V \rightarrow \{1, 2, 3, \dots, p\}$ such that the induced function $f^* : E \rightarrow \mathbb{N}$ defined $f^*(v_i v_j)$ is assigned the label 1, if $\gcd(f(v_i), f(v_j)) = 1$ and 0, if $\gcd(f(v_i), f(v_j)) > 1$, then the number of edges labeled with '0' and '1' differ by at most 1.

Definition 2.6:

A function f is called an odd mean labeling of a graph G if there exists an injective function $f : V(G) \rightarrow \{1, 3, 5, \dots, 2q-1\}$ such that when each edge $v_i v_j$ receives a label $(f(v_i) + f(v_j))/2$, then the resulting edge labels are distinct. A graph which admits an odd mean labeling is said to be an odd mean graph.

Definition 2.7:

A function f is called an even mean labeling of a graph G if there exists an injective function $f : V(G) \rightarrow \{2, 4, \dots, 2q\}$ such that when each edge $v_i v_j$ receives a label $(f(v_i) + f(v_j))/2$ then the resulting edge labels are distinct. A graph which admits an even labeling is said to be an even mean graph.

3. Main Result

In this section we prove the existence of cordial, total cordial, product cordial, total product cordial, prime cordial, odd and even mean labeling for the extended duplicate graph of Comb graph by presenting algorithms.

Definition:

Let $DG(\text{comb}) = (V, E)$ be the duplicate graph of comb graph. The extended duplicate graph of comb graph is obtained as follows: (1) Add the edges $v_1 v_{m+1}$ and $v_{m+2} v_{2m+2}'$ if $m \equiv 1 \pmod{2}$. (2) Add $v_1 v_{m+1}'$ and $v_{m+2}' v_{2m+2}$ to $DG(\text{comb})$ if $m \equiv 0 \pmod{2}$.

$$V = \{v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}'\}.$$

$$E = E_1 \cup E_2.$$

Where $E_1 = \{v_i v_{i+1}' \cup v_i' v_{i+1} \text{ for } 1 \leq i \leq m\} \cup \{v_i' v_{m+i+1} \cup v_i v_{m+i+1}' \text{ for } 1 \leq i \leq m+1\}$.

$$E_2 = \{v_1 v_{m+1}, v_{m+2}' v_{2m+2}'\} \text{ if } m \equiv 1 \pmod{2}.$$

$$= \{v_1 v_{m+1}', v_{m+2}' v_{2m+2}\} \text{ if } m \equiv 0 \pmod{2}.$$

Thus the extended duplicate graph of comb graph has $4m+4$ vertices and $4m+4$ edges.

91 It is denoted by $EDG(\text{comb})$.

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94 **Algorithm : 1**

95 **Procedure :** (cordial labeling for extended duplicate graph of comb graph).

96 $V = \{ v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}' \}$

97 // assignment of labels to vertices

98 **If $m=2n$**

99 for $i = 1$ to $m+1$ do

100 $\{ v_i, v_{m+i+1}' \leftarrow 0$

101 $v_i', v_{m+i+1} \leftarrow 1$

102 $\}$

103 end for

104 **if $m=2n+1$**

105 for $i = 1$ to $2m+2$ do

106 $\{ v_i, v_i' \leftarrow \begin{cases} 0 & \text{if } i \equiv 0 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$

108 $\}$

109 end for

110 end procedure

111 **output :** Vertex labeled extended duplicate graph of comb graph.

112 **Theorem 3.1:**

113 Extended duplicate graph of comb graph is cordial

114 **Proof :**

115 From the construction of extended duplicate graph of comb graph , it is clear that

116 $EDG(\text{comb})$ graph has $4m+4$ vertices and $4m+4$ edges

117 In order to label the vertices, define a function $f: V \rightarrow \{0, 1\}$ as given in algorithm 1.

118 Clearly

119 The number of vertices labeled with '0' = $m+1 + m+1 = 2m+2$

120 The number of vertices labeled with '1' = $m+1 + m+1 = 2m+2$

121 Thus the number of vertices labeled with '0' and '1' is differ by atmost '1'.

122 In order to get the edge labels, define the induced map $f^* : E \rightarrow \mathbb{N}$ such that
 123 $f^*(v_i v_j) = [f(v_i) + f(v_j)] \pmod{2}$.

124 The edge labels are obtained as follows:

125 For $1 \leq i \leq m$

126 $f^*(v_i v_{i+1}) = f^*(v_i' v_{i+1}') = 1$

127 $f^*(v_i v_{m+i+1}) = f^*(v_i' v_{m+i+1}') = 0$

128 **If $m = 2n$**

129 $f^*(v_1 v_{m+1}) = f^*(v_{m+2}' v_{2m+2}') = 1$

130 $f^*(v_1 v_{m+2}) = f^*(v_{m+1}' v_{2m+2}') = 0$

131 **If $m = 2n + 1$**

132 $f^*(v_1 v_{m+1}) = f^*(v_{m+2}' v_{2m+2}') = 1$

133 $f^*(v_{m+1} v_{2m+2}') = f^*(v_{m+1}' v_{2m+2}') = 0$

134 Thus the number of edges labeled with '0' = $m+m+2 = 2m+2$

135 The number of edges labeled with '1' = $m+m+2 = 2m+2$

136 Hence the number of edges labeled with '0' and '1' differ by atmost one .

137 Therefore **EDG (comb)** graph is cordial.

138 **Theorem 3.2 :**

139 Extended duplicate graph of comb graph EDG (comb), $m \geq 2$ is total cordial.

140 **Proof :**

141 By theorem 3.1, we have that the number of vertices labeled with '1' and '0' are same as
 142 $2m + 2$. The number of edges labeled with '1' and '0' are same as $2m + 2$. Thus the total number
 143 of vertices and edges labled with '1' is $4m+4$ and the total number of vertices and edges labled
 144 with '0' is $4m+4$. Clearly the total number of vertices and edges labeled with '1' and the total
 145 number of vertices and edges labeled with '0' differ by atmost one.

146 Hence the extended duplicate graph of comb graph is total cordial.

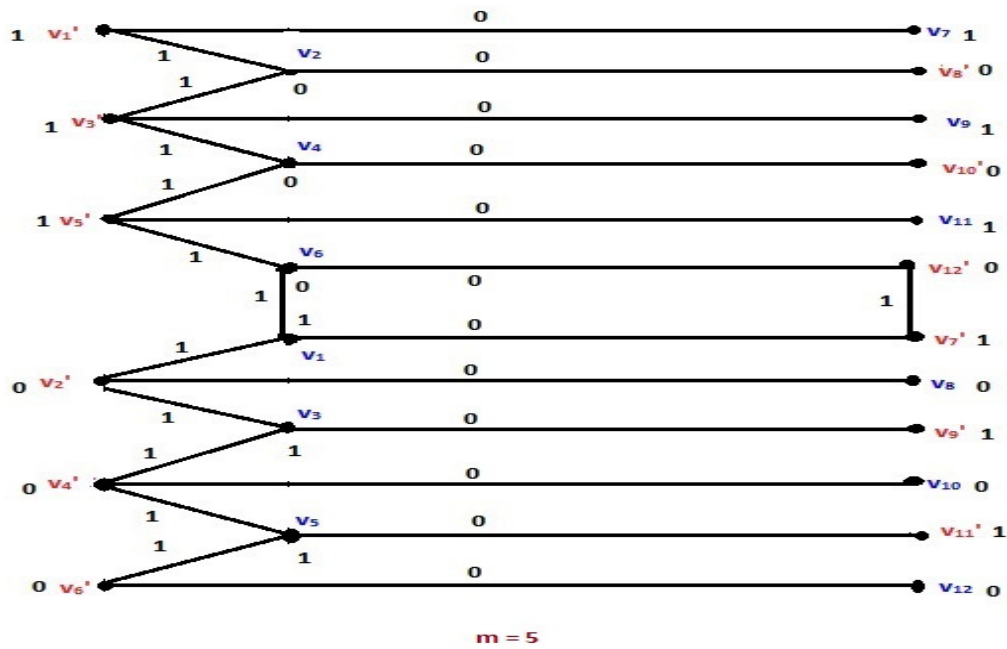
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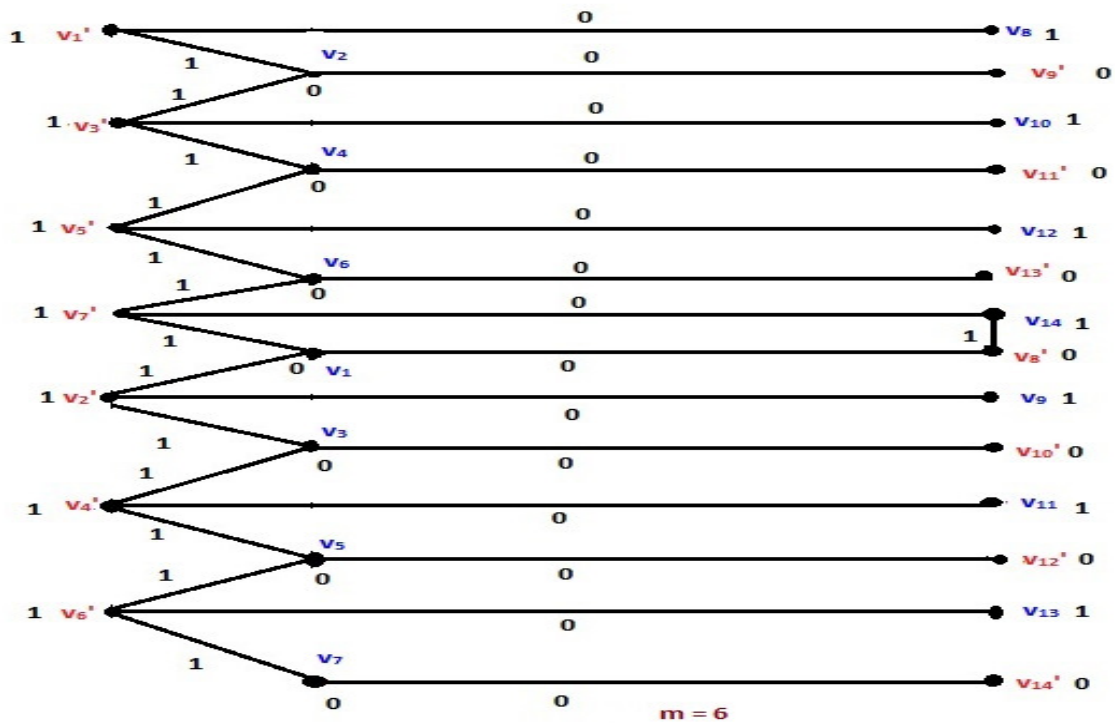
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150 **Example 3.1:**

151 Cordial labeling of EDG (comb) for $m=5$ and $m=6$ are given below.



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156 **Algorithm : 2**

157 **Procedure :** (product cordial labeling for extended duplicate graph of comb graph).

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158           $V = \{ v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}' \}$ 
159  // assignment of labels to vertices
160           $v_1', v_2 \leftarrow 0$  ;  $v_1, v_{m+2}' \leftarrow 1$ 
161  for i = 3 to m+1 do
162      {  $v_i \leftarrow 1$ 
163      }
164  end for
165  for i = 2 to m+1 do
166      {  $v_i' \leftarrow 1$ 
167      }
168  end for
169  for i = 1 to m do
170      {  $v_{m+i+1} \leftarrow 0$ 
171      }
172  end for
173  for i = 2 to m do
174      {  $v_{m+i+1}' \leftarrow 0$ 
175      }
176  end for
177  if  $m = 2n$ 
178       $v_{2m+2} \leftarrow 1$  :  $v_{2m+2}' \leftarrow 0$ 
179  if  $m = 2n+1$ 
180       $v_{2m+2} \leftarrow 0$  :  $v_{2m+2}' \leftarrow 1$ 
181  end for
182  end procedure
183  output :Vertex labeled extended duplicate graph of comb graph.
184  Theorem 3.3 :
185      Extended duplicate graph of comb graph EDG (comb),  $m \geq 2$  is product cordial.
186
187  Proof :

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188 Clearly, EDG (comb) has $4m + 4$ vertices and $4m + 4$ edges. In order to label the vertices,
189 define a function $f : V \rightarrow \{0, 1\}$, using algorithm 2.

190 Clearly the number of vertices labeled with '0' $= 2+m-1+m+1 = 2m+2$

191 Number of vertices labeled with '1' $= 2+m-1+m+1 = 2m+2$

192 Hence the number of vertices labeled with '0' and '1' is differ by atmost one. In order to
193 get the edge labels, define the induced map $f^* : E \rightarrow \mathbb{N}$ such that $f^*[v_i v_j] = [f(v_i) \times f(v_j)]$

194 Thus the edge labels are obtained as follows :

$$195 \quad f^*(v_1 v_2') = f^*(v_1 v_{m+2}') = 1 : f^*(v_1' v_2) = f^*(v_2 v_3') = 0$$

196 For $1 \leq i \leq m$

$$197 \quad f^*(v_i' v_{m+i+1}) = 0$$

198 For $2 \leq i \leq m$

$$199 \quad f^*(v_i' v_{i+1}) = 1 : f^*(v_i v_{m+i+1}') = 0$$

200 For $3 \leq i \leq m$

$$201 \quad f^*(v_i v_{i+1}') = 1$$

202 **If $m = 2n$**

$$203 \quad f^*(v_{m+1} v_{2m+2}') = 0$$

$$204 \quad f^*(v_1 v_{m+1}') = f^*(v_{m+1}' v_{2m+2}) = f^*(v_{m+2}' v_{2m+2}) = 1$$

205 **If $m = 2n + 1$**

$$206 \quad f^*(v_{m+1}' v_{2m+2}) = 0$$

$$207 \quad f^*(v_1 v_{m+1}) = f^*(v_{m+1} v_{2m+2}') = f^*(v_{m+2}' v_{2m+2}') = 1$$

208 Thus the number of edges labeled with '0' $= 2+m+m-1+1 = 2m+2$

209 Number of edges labeled with '1' $= 2+m-1+m-2 + 3 = 2m+2$

210 Hence the number of edges labeled with '0' and '1' differ by atmost one.

211 Therefore **EDG (comb)** graph is product cordial.

212 **Theorem 3.4 :**

213 Extended duplicate graph of comb graph , EDG (comb) $m \geq 2$ is total product cordial.

214

215 **Proof :**

216 By theorem 3.3, we have the number of vertices labeled with '0' and '1' are same as

217 $2m+2$. The number of edges labeled with '0' and '1' are same as $2m+2$.

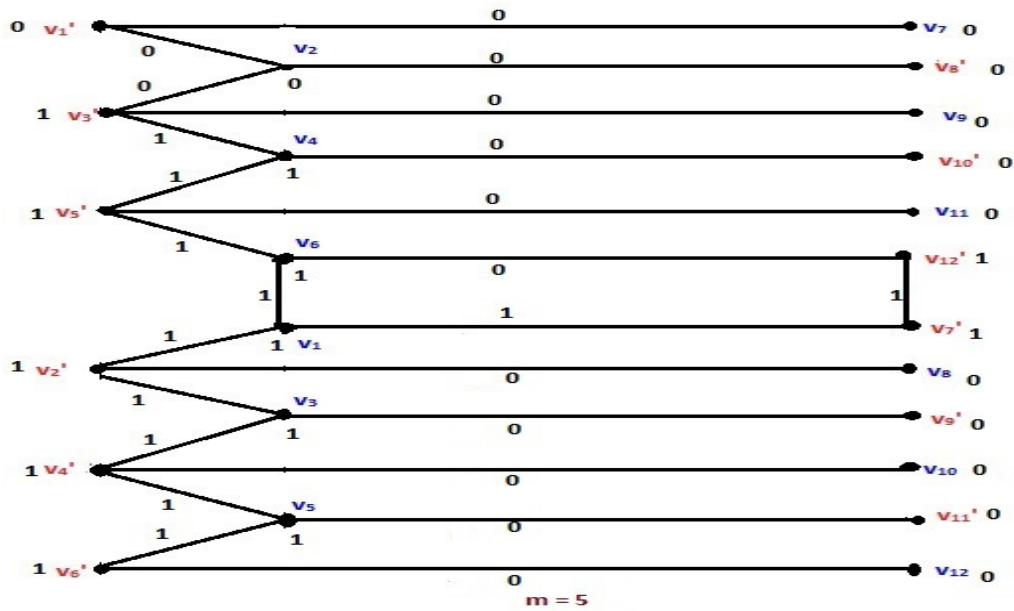
Thus the total number of vertices and edges labeled with '0' is $4m + 4$ and the total number of vertices and edges labeled with '1' is $4m + 4$.

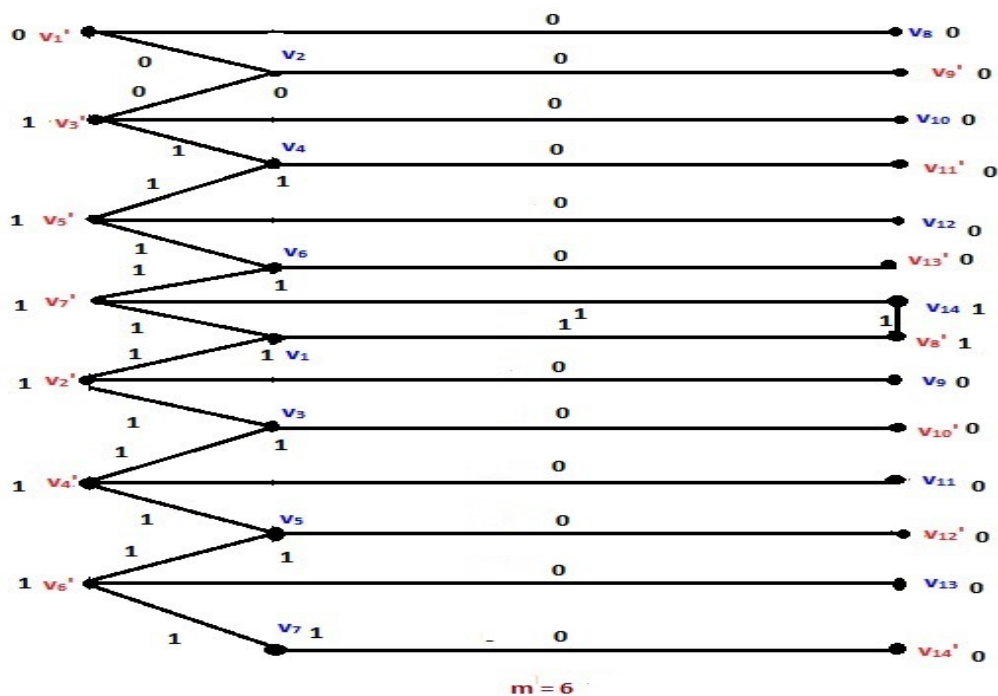
Hence the total number of vertices and edges labeled with '0' and '1' differ by at most one.

Therefore **EDG (comb)** graph is total product cordial.

Example 3.2 :

Product cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given below.





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230 **Algorithm : 3**

231 **Procedure :** (prime cordial labeling for extended duplicate graph of comb).

232 $V = \{ v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}' \}$

233 // assignment of labels to vertices

234 $v_1 \leftarrow 4m-2$; $v_{m+2}' \leftarrow 4m+2$

235 for $i = 2$ to $m-1$ do

236 $\{ v_i \leftarrow \begin{cases} 2i & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i-2 & \text{otherwise} \end{cases}$

237 $\}$

238 end for

240 for $i = 1$ to $m-1$ do

241 $\{ v_i' \leftarrow \begin{cases} 2i & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i-2 & \text{otherwise} \end{cases}$

242 $\}$

243 end for

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247   for i = 2 to m do
248       {
249            $v_{m+i+1} \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i-3 & \text{otherwise} \end{cases}$ 
250       }
251   }
252   end for
253   for i = 2 to m do
254       {  $v_{m+i+1}' \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i-3 & \text{otherwise} \end{cases}$ 
255       }
256   }
257   end for
258   if m = 2n
259        $v_m \leftarrow 2m$  ,  $v_m' \leftarrow 1$  ,  $v_{m+2} \leftarrow 4m+3$ ,  $v_{m+1} \leftarrow 4m+1$ 
260        $v_{m+1}' \leftarrow 4m$  ,  $v_{2m+2} \leftarrow 4m+4$  ,  $v_{2m+2}' \leftarrow 4m-1$ 
261   if m = 2n+1
262        $v_m \leftarrow 4m+3$  ,  $v_m' \leftarrow 2m$  ,  $v_{m+1} \leftarrow 4m$ 
263        $v_{m+1}' \leftarrow 4m+1$  ,  $v_{2m+2}' \leftarrow 4m+4$  ,  $v_{2m+2} \leftarrow 4m-1$ ,  $v_{m+2} \leftarrow 1$ .
264   end procedure

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output : Vertex labeled extended duplicate graph of comb graph.

Theorem : 3.5

Extended duplicate graph of comb graph, $m \geq 2$ admits prime Cordial labeling.

Proof :

The EDG (comb) graph has $4m+4$ vertices and $4m+4$ edges. Label the vertices of EDG(comb) graph by defining a function $f : V \rightarrow \{1, 2, 3, 4, \dots, 4m+4\}$, as given in algorithm 3. Thus all the $4m+4$ vertices are labeled.

Define the induced map $f^* : E \rightarrow N$ such that

$$f^*(v_i v_j) = \begin{cases} 1 & \text{if g. c. d. } (f(v_i), f(v_j)) = 1 \\ 0 & \text{if g. c. d. } (f(v_i), f(v_j)) > 1 \end{cases}$$

276 By the above induced function, the edge labels are obtained as follows :

277 For $1 \leq i \leq m-2$

$$278 \quad f^*(v_i v_{i+1}') = f^*(v_i' v_{i+1}) = 0$$

279 For $1 \leq i \leq m$

$$280 \quad f^*(v_i' v_{m+i+1}) = 1$$

281 For $2 \leq i \leq m$

$$282 \quad f^*(v_i v_{m+i+1}') = 1$$

283 **If $m = 2n$**

$$284 \quad f^*(v_1 v_{m+1}') = f^*(v_1 v_{m+2}') = f^*(v_{m+1}' v_{2m+2}) = f^*(v_{m+2}' v_{2m+2}) = f^*(v_{m+1}' v_m) = f^*(v_m v_{m+1}') = 0$$

$$285 \quad f^*(v_{m-1} v_m') = f^*(v_m' v_{m+1}) = f^*(v_{m+1} v_{2m+2}') = 1$$

286 **If $m = 2n + 1$**

$$287 \quad f^*(v_1 v_{m+1}) = f^*(v_1 v_{m+2}') = f^*(v_{m+1} v_{2m+2}') = f^*(v_{m+2}' v_{2m+2}') = f^*(v_{m-1} v_m') =$$

$$288 \quad f^*(v_m' v_{m+1}) = 0$$

$$289 \quad f^*(v_{m-1}' v_m) = f^*(v_m v_{m+1}') = f^*(v_{m+1}' v_{2m+2}) = 1$$

290 Thus in both the above cases,

$$291 \quad \text{The number of edges labeled with '1' } = m + m - 1 + 3 = 2m + 2.$$

$$292 \quad \text{Number of edges labeled with '0' } = m - 2 + m - 2 + 6 = 2m + 2.$$

293 Hence then number of edges labeled with '1' and '0' differ by atmost '1'

294 Therefore **EDG (comb)** graph is prime cordial .

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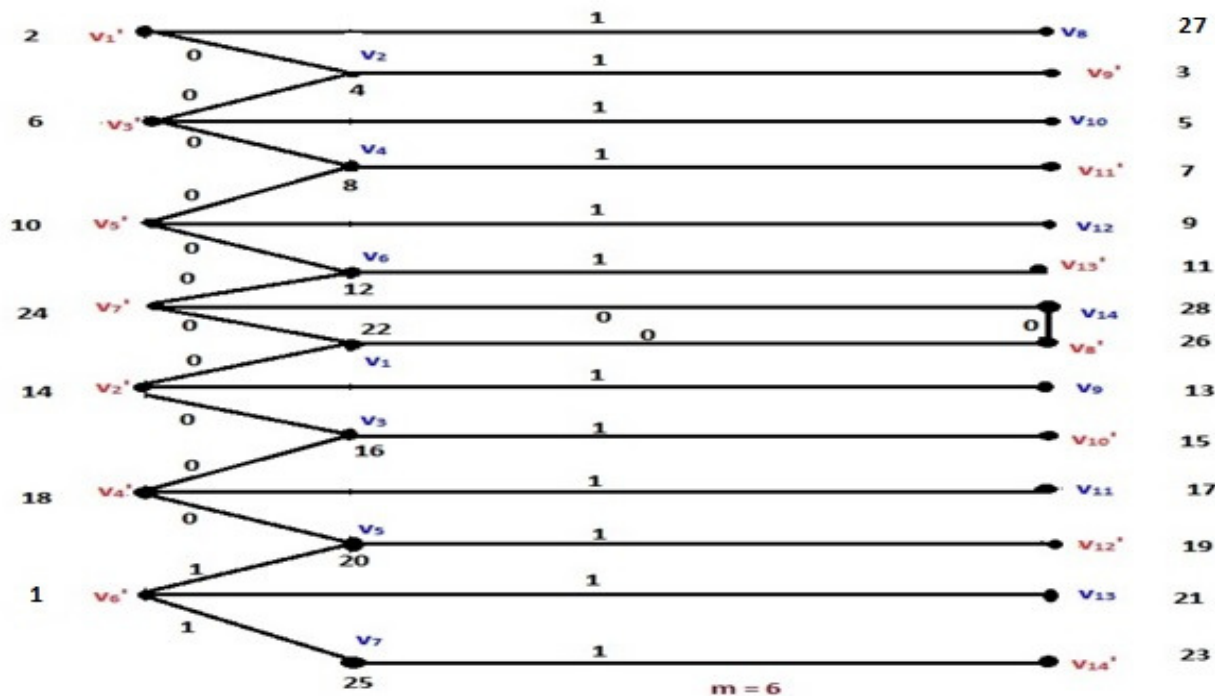
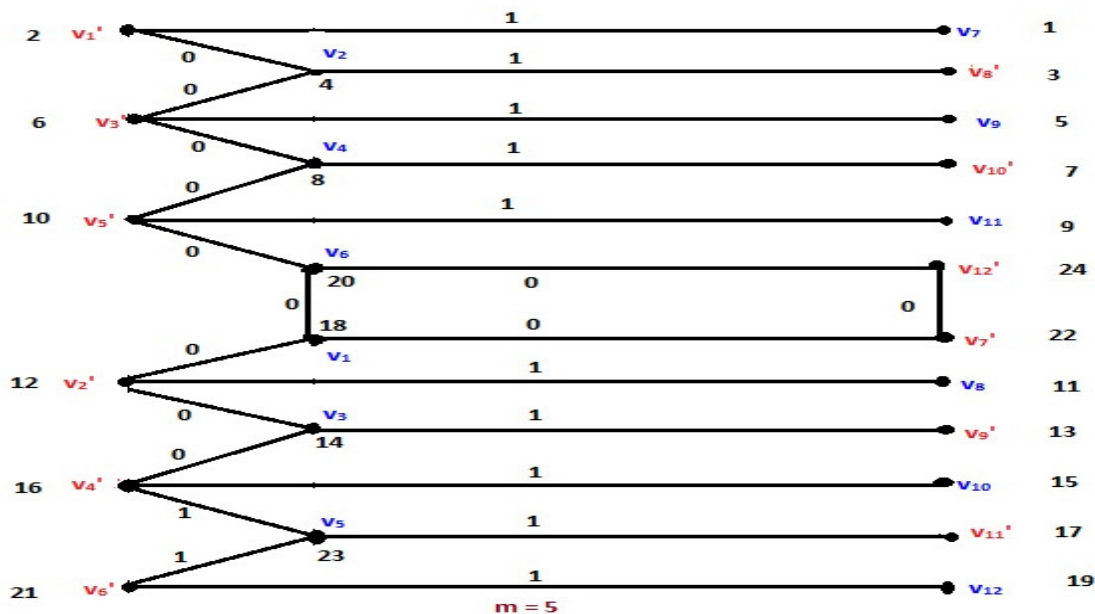
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Example 3.3:

Prime cordial labeling of EDG(comb) graphs for $m=5$ and $m=6$ are given below.



314 **Algorithm : 4**

315 **Procedure :** (odd mean labeling for extended duplicate graph of comb graph).

$$316 \quad V = \{ v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}' \}$$

317 // assignment of labels to vertices

318 for $i = 1$ to $m+1$ do

$$319 \quad \{ \quad v_i \quad \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i+1 & \text{otherwise} \end{cases}$$

321

$$322 \quad \quad \quad v_i' \quad \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i+1 & \text{otherwise} \end{cases}$$

324 }

325 end for

326 for $i = 2$ to $m+2$ do

$$327 \quad \{ \quad v_{m+i} \quad \leftarrow \begin{cases} 4m+2i+1 & \text{if } i \equiv 0(\text{mod } 2) \\ 6m+2i+3 & \text{otherwise} \end{cases}$$

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$$331 \quad \quad \quad v_{m+i}' \quad \leftarrow \begin{cases} 4m+2i+1 & \text{if } i \equiv 1(\text{mod } 2) \\ 6m+2i+3 & \text{otherwise} \end{cases}$$

333 }

334 end for

335 end procedure

336 **output :** Vertex labeled extended duplicate graph of comb graph

337 **Theorem : 3.6**

338 Extended duplicate graph of EDG (comb) graph admits odd mean labeling.

339 **Proof:**

340 Clearly EDG (comb) graph has $p = 4m+4$ vertices and $q = 4m+4$ edges.

341 To label the vertices , define a function

342 $f : V \rightarrow \{ 1,3,5,\dots, 2q-1 \}$ as given in algorithm 4.

343 Thus all the $4m+4$ vertices are labeled.

344 To get the edge labels, define, the induced map

345 $f^* : E \rightarrow \{1,2,3,\dots,2q-1\}$ such that

346 $f^*(v_i v_j) = f(v_i) + f(v_j)/2$ for all $v_i v_j \in E$

347 By the induced function , the edge labels are obtained as follows:

348 For $1 \leq i \leq m$

$$349 \quad f^*(v_i v_{i+1}') = \begin{cases} 2i & \text{if } i \equiv 0 \pmod{2} \\ 2m+2i+2 & \text{otherwise} \end{cases}$$

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$$352 \quad f^*(v_i' v_{i+1}) = \begin{cases} 2i & \text{if } i \equiv 1 \pmod{2} \\ 2m+2i+2 & \text{otherwise} \end{cases}$$

353

354 For $1 \leq i \leq m+1$

$$355 \quad f^*(v_i v_{m+i+1}') = \begin{cases} 2m+2i+1 & \text{if } i \equiv 0 \pmod{2} \\ 4m+2i+3 & \text{otherwise} \end{cases}$$

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$$358 \quad f^*(v_i' v_{m+i+1}) = \begin{cases} 2m+2i+1 & \text{if } i \equiv 1 \pmod{2} \\ 4m+2i+3 & \text{otherwise} \end{cases}$$

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360 **If $m = 2n$**

$$361 \quad f^*(v_1 v_{m+1}') = 2m+2, \quad f^*(v_{m+2}' v_{2m+2}) = 6m+6$$

362

363 **If $m = 2n + 1$**

$$364 \quad f^*(v_1 v_{m+1}) = 2m+2, \quad f^*(v_{m+2}' v_{2m+2}') = 6m+6$$

365 Thus all the labeled edges are distinct .

366 Hence **EDG (comb)** graph admits odd mean labeling

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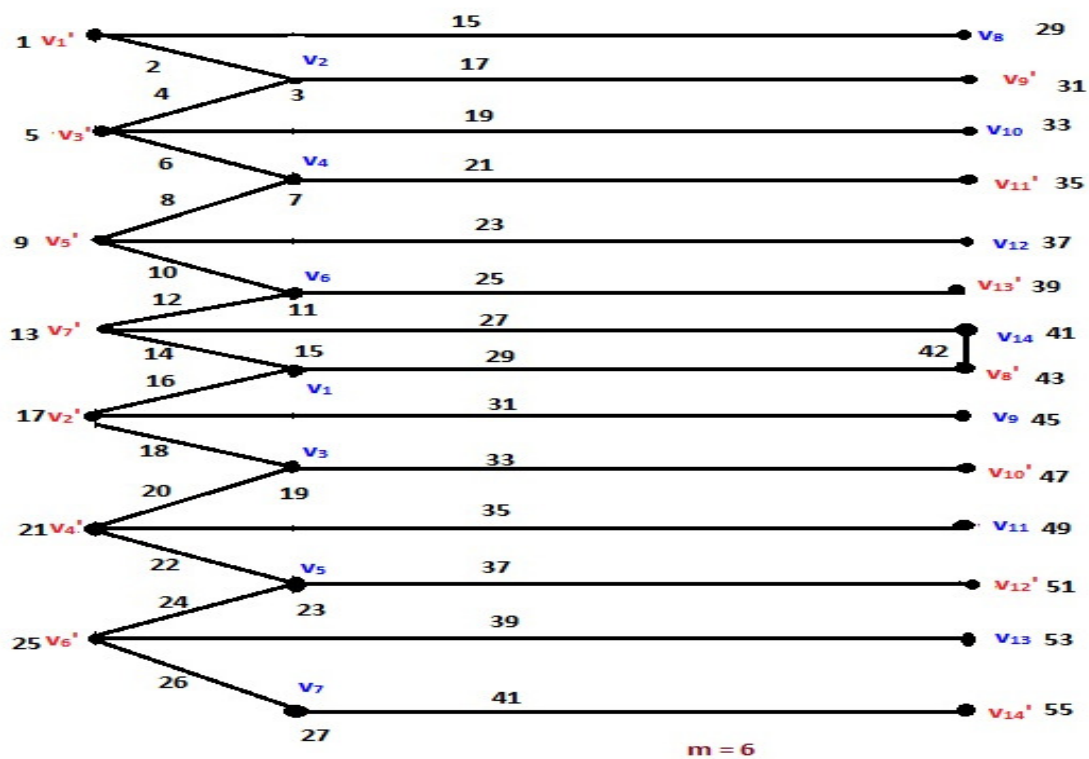
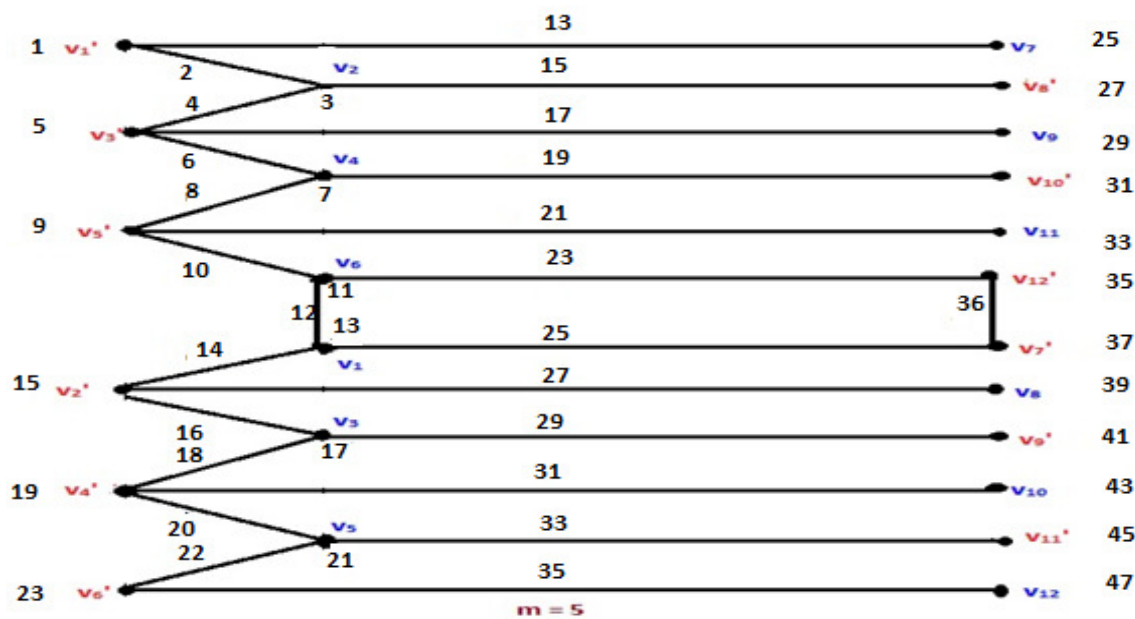
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Example 3.4:

Odd mean labeling of EDG(comb) graphs for $m=5$ and $m=6$ are given below.



382 **Algorithm : 5**

383 **Procedure :** (even mean labeling for extended duplicate graph of comb graph).

$$384 \quad V = \{ v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}' \}$$

385 // assignment of labels to vertices

386 for $i = 1$ to $m+1$ do

$$387 \quad \{ \quad v_i \quad \leftarrow \quad \begin{cases} 2i & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i+2 & \text{otherwise} \end{cases}$$

$$389 \quad v_i' \quad \leftarrow \quad \begin{cases} 2i & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i+2 & \text{otherwise} \end{cases}$$

391 }

392 end for

393 for $i = 2$ to $m+2$ do

$$394 \quad \{ \quad v_{m+i} \quad \leftarrow \quad \begin{cases} 4m+2i+2 & \text{if } i \equiv 0(\text{mod } 2) \\ 6m+2i+4 & \text{otherwise} \end{cases}$$

$$396 \quad v_{m+i}' \quad \leftarrow \quad \begin{cases} 4m+2i+2 & \text{if } i \equiv 1(\text{mod } 2) \\ 6m+2i+4 & \text{otherwise} \end{cases}$$

398 }

399 end for

400 end procedure

401 **output :** Vertex labeled extended duplicate graph of comb graph.

402 **Theorem : 3.7**

403 Extended duplicate graph of comb graph, EDG (comb)graph admits even mean labeling.

404 **Proof:**

405 Clearly EDG (comb) graph has $p = 4m+4$ vertices and $q = 6m+4$ edges.

406 To label the vertices , define a function

407 $f : V \rightarrow \{ 2,4,6,\dots, 2q \}$ as given in algorithm 5.

408 Thus $4m+4$ vertices are labeled.

409 To get the edge labels, define, the induced map

410 $f^* : E \rightarrow \{ 1,2,3,\dots,2q \}$ such that

411 $f^*(v_i v_j) = [f(v_i)+f(v_j)] / 2$ for all $v_i v_j \in E$

412 By the induced function , the edge labels are obtained as follows:

413 For $1 \leq i \leq m$

$$414 \quad f^*(v_i v_{i+1}') = \begin{cases} 2i+1 & \text{if } i \equiv 0 \pmod{2} \\ 2m+2i+3 & \text{otherwise} \end{cases}$$

$$416 \quad f^*(v_i' v_{i+1}) = \begin{cases} 2i+1 & \text{if } i \equiv 1 \pmod{2} \\ 2m+2i+3 & \text{otherwise} \end{cases}$$

418 For $1 \leq i \leq m+1$

$$419 \quad f^*(v_i v_{m+i+1}') = \begin{cases} 2m+2i+2 & \text{if } i \equiv 0 \pmod{2} \\ 4m+2i+4 & \text{otherwise} \end{cases}$$

$$421 \quad f^*(v_i' v_{m+i+1}) = \begin{cases} 2m+2i+2 & \text{if } i \equiv 1 \pmod{2} \\ 4m+2i+4 & \text{otherwise} \end{cases}$$

423 **If $m = 2n$**

$$424 \quad f^*(v_1 v_{m+1}') = 2m+3 \quad ; \quad f^*(v_{m+2}' v_{2m+2}) = 6m+7$$

425 **If $m = 2n+1$**

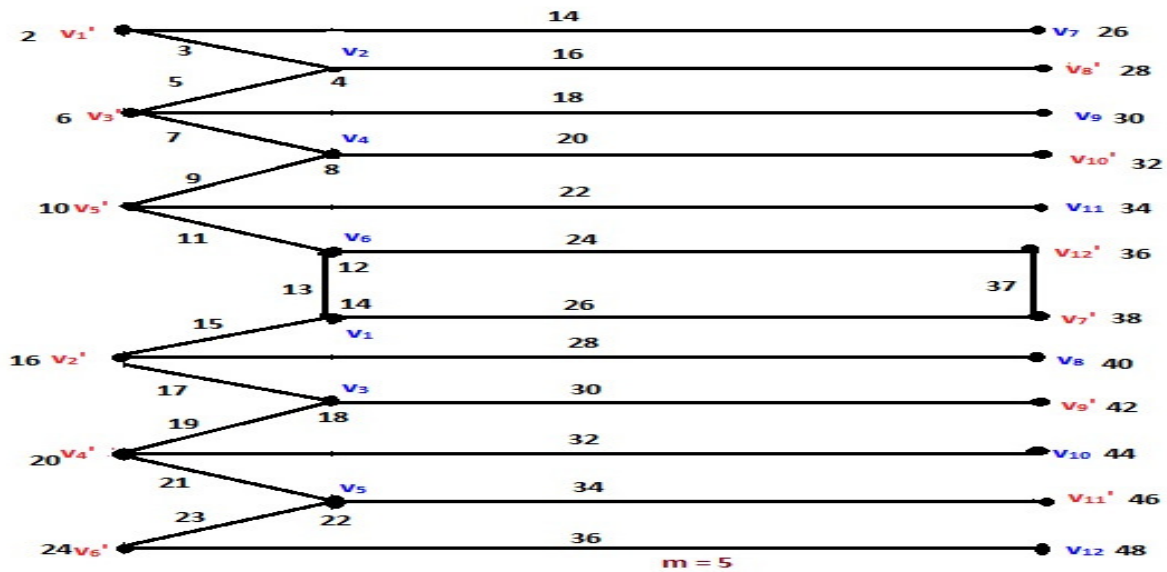
$$426 \quad f^*(v_1 v_{m+1}) = 2m+3 \quad ; \quad f^*(v_{m+2}' v_{2m+2}') = 6m+7$$

427 Thus all the labeled edges are distinct .

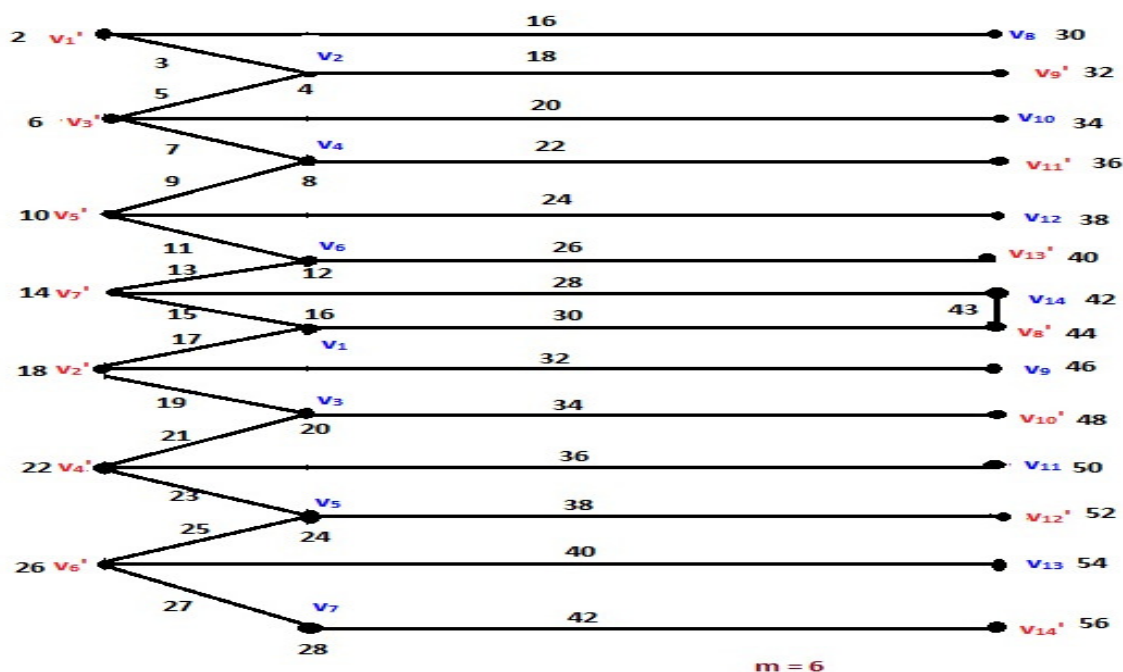
428 Hence **EDG (comb)** graph admits even mean labeling

429 **Example 3.5:**

430 Even mean labeling of EDG(comb) graph for $m = 5$ and $m = 6$ are given below.



431



Conclusion:

In this paper we have presented algorithms and proved that the extended duplicate graph of Comb graph admits cordial, total cordial, product cordial, total product cordial, prime cordial, odd mean labeling and even mean labeling.

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