

**CORDIAL AND MEAN LABELINGS
ON EXTENDED DUPLICATE GRAPH OF COMB GRAPH.**

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Abstract

A graph labeling is a mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper we prove the existence of graph labeling such as cordial, total cordial, product cordial, total product cordial, primecordial, odd mean labeling and even mean labeling for extended duplicate graph of Comb graph by presenting algorithms.

AMS Subject Classification : 05C78

Keywords: Graph labeling, Comb, Duplicate graph.

1 . Introduction

Graph labeling has the origin from a seminal paper by Rosa in 1967 [7]. It is defined as an assignment of integers to the vertices or edges or both subject to certain conditions. The notion of cordial labeling was introduced by Cahit [5]. A function $f : V \rightarrow \{0,1\}$ is said to be cordial labeling if each edge uv receives the label $|f(u) - f(v)|$ such that the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one, and the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one. Moreover, it admits total cordial labeling if the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by at most one.

Andar et. al., [1] proved that a cordial labeling of G can be extended to a cordial labeling of the graph obtained from G by attaching $2m$ pendent edges at each vertex of G .

The concept of product, total product and prime cordial labeling was introduced by

Sundaram et. al,[8]. A function $f: V \rightarrow \{0,1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be product cordial labeling if the number of vertices labeled with '0' and the number of vertices labeled with '1' differ by at most one, and the number of edges labeled with '0' and the number of edges labeled with '1' differ by at most one. Moreover, it is said to be total product cordial labeling if the number of vertices and edges labeled with '0' and the number of vertices and edges labeled with '1' differ by at most one.

A prime cordial labeling of a graph G with vertex set V is a function $f: V \rightarrow \{1, 2, 3, \dots, p\}$ such that the induced function $f^*: E \rightarrow N$ defined $f^*(v_i v_j)$ is assigned the label 1, if $\gcd(f(v_i), f(v_j)) = 1$ and 0, if $\gcd(f(v_i), f(v_j)) > 1$, then the number of edges labeled with '0' and '1' differ by at most 1. It was proved that the graphs C_n if and only if $n \geq 6$; P_n if and only if $n \neq 3$ or 5; bistars; dragons; crowns ; ladders are prime cordial.

Nirmala investigated the concept of odd and even mean labeling. A function f is called an odd mean labeling of a graph G if there exists an injective function $f: V(G) \rightarrow \{1, 3, 5, \dots, 2q-1\}$ such that when each edge $v_i v_j$ receives a label $(f(v_i) + f(v_j))/2$, then the resulting edge labels are distinct. A graph which admits an odd mean labeling is called an odd mean graph. A function f is called an even mean labeling of a graph G if there exists an injective function $f: V \rightarrow \{2, 4, \dots, 2q\}$ such that when each edge $v_i v_j$ receives a label $(f(v_i) + f(v_j))/2$ then the resulting edge labels are distinct. A graph which admits an even mean labeling is called an even mean graph.

Thirusangu et.al,[10] have introduced the concept of extended duplicate graph.

A Duplicate graph of a path graph $G(V, E)$ is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \Phi$ and $f: V \rightarrow V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as follows: The edge $v_i v_j$ is in E if and only if both $v_i v_j'$ and $v_i' v_j$ are edges in E_1 . Clearly duplicate graph is disconnected. In order to make it as connected, add an edge $v_i v_i'$ for any i . This graph is called the Extended Duplicate Graph of a path graph G .

It was shown that the extended duplicate graph of a path P_m admits cordial, total cordial, product cordial and total product cordial labeling.

Singh et.al, [9] proved some results on Comb graph. Let P_{m+1} be a path graph. Comb graph is defined as $P_{m+1} \odot (m+1)K_1$. It has $2m+2$ vertices and $2m+1$ edges.

In this paper, we prove that, the extended duplicate graph of Comb graph is cordial, total cordial, product cordial, total product cordial, prime cordial, odd mean and even mean labeling by presenting algorithms.

2. Main Result

In this section we present the structure of the extended duplicate graph of a comb graph and establish the existence of cordial, total cordial, product cordial, total product cordial, prime cordial, odd and even mean labeling for the extended duplicate graph of Comb graph by presenting algorithms.

Definition: (Structure of the extended duplicate graph of a comb graph)

Let $G(V,E)$ be a comb graph. $DG(comb) = (V_1, E_1)$ is the duplicate graph of comb graph with $4m+4$ vertices and $4m+2$ edges.

Denote the vertex set as $V_1 = \{v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}'\}$ and the edge set as $E_1 = \{v_i v_{i+1}' \cup v_i' v_{i+1} \text{ for } 1 \leq i \leq m\} \cup \{v_i' v_{m+i+1} \cup v_i v_{m+i+1}' \text{ for } 1 \leq i \leq m+1\}$. Clearly $DG(comb)$ is disconnected. The extended duplicate graph of a comb $EDG(Comb)$ is obtained from $DG(comb)$ by adding the edges (i) $v_1 v_{m+1}$ and $v_{m+2}' v_{2m+2}'$ if $m \equiv 1 \pmod{2}$. (ii) $v_1 v_{m+1}'$ and $v_{m+2}' v_{2m+2}$ if $m \equiv 0 \pmod{2}$.

Thus the extended duplicate graph of comb graph has $4m+4$ vertices and $4m+4$ edges.

Algorithm2.1

Procedure (cordial labeling for extended duplicate graph of comb graph)

$V = \{v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}'\}$

// assignment of labels to vertices

If $m = 2n$

for $i = 1$ to $m+1$ do

{ $v_i, v_{m+i+1}' \leftarrow 0$

$v_i', v_{m+i+1} \leftarrow 1$ }

}

end for

If $m = 2n+1$

for $i = 1$ to $2m+2$ do

{ $v_i, v_i' \leftarrow \begin{cases} 0 & \text{if } i \equiv 0 \pmod{2} \\ 1 & \text{otherwise} \end{cases}$

}

end for

end procedure

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output : vertex labeled extended duplicate graph of comb graph.

Theorem 2.1

The extended duplicate graph of comb graph is cordial.

Proof :

From the construction of extended duplicate graph of comb graph , it is clear that $EDG(comb) (V,E)$ graph has $4m+4$ vertices and $4m+4$ edges.

In order to label the vertices, define a function $f : V \rightarrow \{0, 1\}$ as given in algorithm 2.1. Clearly,

The number of vertices labeled with '0' = $m+1 + m+1 = 2m+2$

The number of vertices labeled with '1' = $m+1 + m+1 = 2m+2$

Thus the number of vertices labeled with '0' and '1' is differ by atmost '1'.

In order to get the edge labels, define the induced map $f^* : E \rightarrow \mathbb{N}$ such that $f^*(v_i v_j) = [f(v_i) + f(v_j)] \pmod{2}$. The edge labels are obtained as follows:

For $1 \leq i \leq m$

$$f^*(v_i v_{i+1}') = f^*(v_i' v_{i+1}) = 1, \quad f^*(v_i v_{m+i+1}') = f^*(v_i' v_{m+i+1}) = 0$$

If $m = 2n$

$$f^*(v_1 v_{m+1}') = f^*(v_{m+2}' v_{2m+2}) = 1, \quad f^*(v_1 v_{m+2}') = f^*(v_{m+1}' v_{2m+2}) = 0$$

If $m = 2n + 1$

$$f^*(v_1 v_{m+1}') = f^*(v_{m+2}' v_{2m+2}') = 1, \quad f^*(v_{m+1} v_{2m+2}') = f^*(v_{m+1}' v_{2m+2}) = 0$$

Thus the number of edges labeled with '0' = $m+m+2 = 2m+2$

The number of edges labeled with '1' = $m+m+2 = 2m+2$

Hence the number of edges labeled with '0' and '1' differ by atmost one.

Therefore $EDG(comb)$ graph is cordial.

Theorem 2.2

The extended duplicate graph of comb graph $EDG(comb)$, $m \geq 2$ is total cordial.

Proof :

By theorem 2.1, the number of vertices labeled with '1' and '0' are same as $2m + 2$. The number of edges labeled with '1' and '0' are same as $2m + 2$. Thus the total number of vertices and edges labeled with '1' is $4m+4$ and the total number of vertices and edges labeled with '0' is $4m+4$. Clearly the total number of vertices and edges labeled with '1' and the total number of vertices and edges labeled with '0' differ by atmost one.

Hence the extended duplicate graph of comb graph is total cordial.

Example 2.1

Cordial labeling of EDG (comb) for $m=5$ and $m=6$ are given in figure 1 and figure 2 respectively.

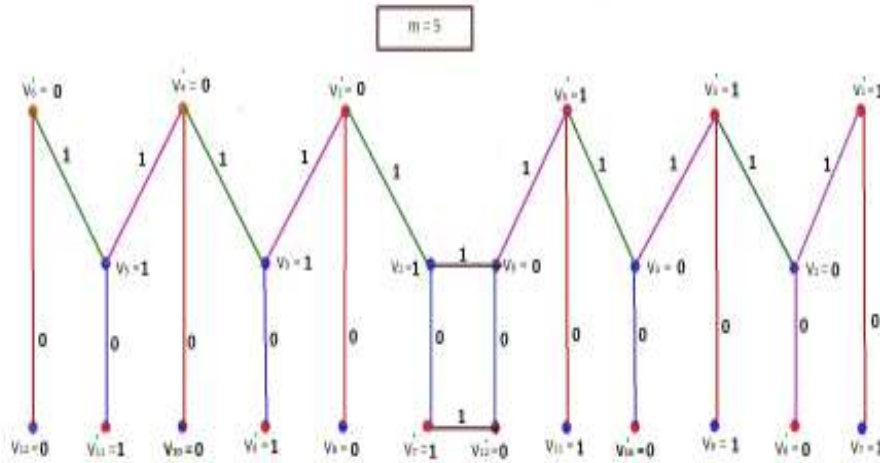


Figure:1 cordial labeling of EDG(comb) graph for $m = 5$

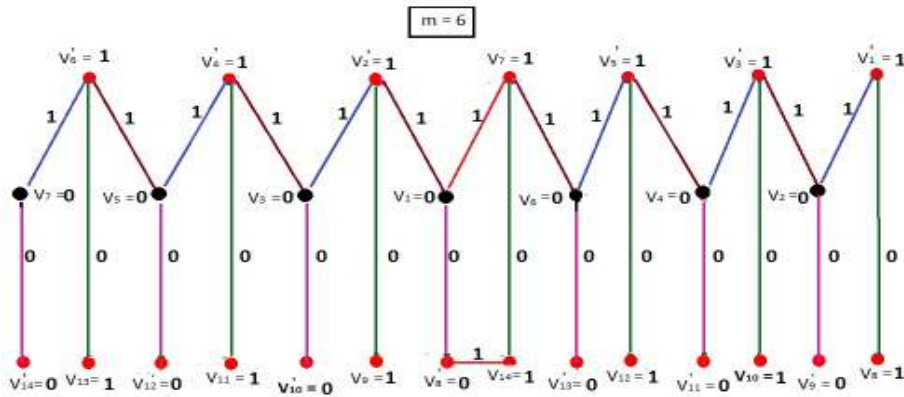


Figure : 2 cordial labeling of EDG(comb) graph for $m = 6$

Algorithm 2.2

Procedure :(product cordial labeling for extended duplicate graph of comb graph)

$$V = \{ v_1, v_2, \dots, v_{2m+2}, v_1', v_2', \dots, v_{2m+2}' \}$$

```

// assignment of labels to vertices
 $v_1', v_2 \leftarrow 0$  ;  $v_1, v_{m+2}' \leftarrow 1$ 
for i = 3 to m+1 do
    {  $v_i \leftarrow 1$  }
end for
for i = 2 to m+1 do
    {  $v_i' \leftarrow 1$  }
end for
for i = 1 to m do
    {  $v_{m+i+1} \leftarrow 0$  }
end for
for i = 2 to m do
    {  $v_{m+i+1}' \leftarrow 0$  }
end for
if m = 2n
     $v_{2m+2} \leftarrow 1$  :  $v_{2m+2}' \leftarrow 0$ 
if m = 2n+1
     $v_{2m+2} \leftarrow 0$  :  $v_{2m+2}' \leftarrow 1$ 
end for
end procedure
output : vertex labeled extended duplicate graph of comb graph.

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Theorem 2.3

The extended duplicate graph of comb graph EDG (comb), $m \geq 2$ is product cordial.

Proof :

Clearly, $\text{EDG}(\text{comb})(V, E)$ has $4m + 4$ vertices and $4m + 4$ edges. In order to label the vertices, define a function $f : V \rightarrow \{0, 1\}$, using algorithm 2.2.

It is evident that the number of vertices labeled with '0' = $2+m-1+m+1 = 2m+2$.

Number of vertices labeled with '1' = $2+m-1+m+1 = 2m+2$.

Hence the number of vertices labeled with '0' and '1' differ by atmost one. In order to get the edge labels, define the induced map $f^* : E \rightarrow N$ such that $f^*[v_i v_j] = [f(v_i) \times f(v_j)]$.

Thus the edge labels are obtained as follows :

$$f^*(v_1 v_2') = f^*(v_1 v_{m+2}') = 1 : f^*(v_1' v_2) = f^*(v_2 v_3') = 0$$

For $1 \leq i \leq m$

$$f^*(v_i' v_{m+i+1}) = 0$$

For $2 \leq i \leq m$

$$f^*(v_i' v_{i+1}) = 1 : f^*(v_i v_{m+i+1}') = 0$$

For $3 \leq i \leq m$; $f^*(v_i v_{i+1}') = 1$

If $m = 2n$

$$f^*(v_{m+1} v_{2m+2}') = 0, f^*(v_1 v_{m+1}') = f^*(v_{m+1}' v_{2m+2}) = f^*(v_{m+2}' v_{2m+2}) = 1$$

If $m = 2n + 1$

$$f^*(v_{m+1}' v_{2m+2}) = 0, f^*(v_1 v_{m+1}) = f^*(v_{m+1} v_{2m+2}') = f^*(v_{m+2}' v_{2m+2}') = 1$$

Thus the number of edges labeled with '0' = $2+m+m-1+1 = 2m+2$.

Number of edges labeled with '1' = $2+m-1+m-2 + 3 = 2m+2$.

Hence the number of edges labeled with '0' and '1' differ by atmost one.

Therefore $\text{EDG}(\text{comb})$ graph is product cordial.

Theorem 2.4 :

The extended duplicate graph of comb graph , $\text{EDG}(\text{comb})$ $m \geq 2$ is total product cordial.

Proof :

By theorem 2.3, the number of vertices labeled with '0' and '1' are same as $2m+2$. The number of edges labeled with '0' and '1' are same as $2m+2$. Thus the total number of vertices and edges labeled with '0' is $4m + 4$ and the total number of vertices and edges labeled with '1' is $4m + 4$. Hence the total number of vertices and edges labeled with '0' and '1' differ by atmost one.

Therefore $\text{EDG}(\text{comb})$ graph is total product cordial.

Example 2.2

Product cordial labeling of EDG(comb) graph for $m=5$ and $m=6$ are given in figure 3 and figure 4 respectively .

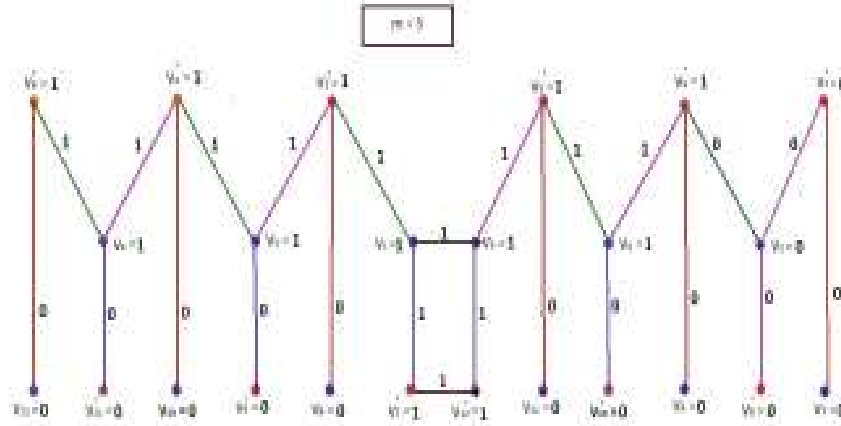


Figure:3 product cordial labeling of EDG(comb) graph for $m = 5$

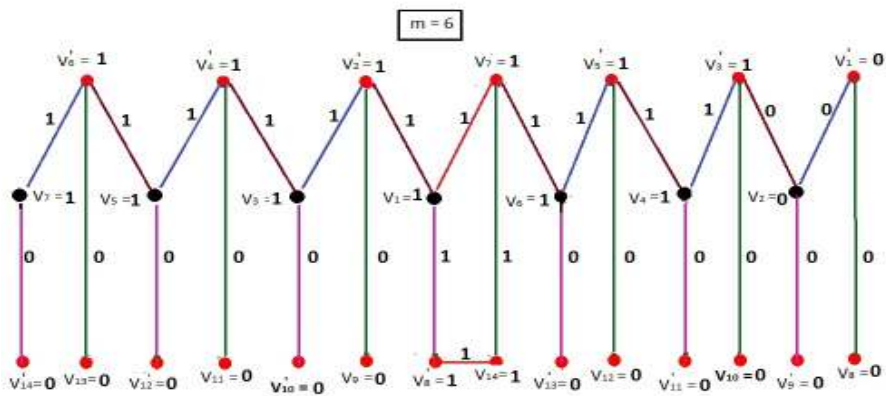


Figure:4 product cordial labeling of EDG(comb) graph for $m = 6$

Algorithm 2.3

Procedure :(prime cordial labeling for extended duplicate graph of comb graph)

$V = \{ v_1, v_2, \dots, v_{2m+2}, v'_1, v'_2, \dots, v'_{2m+2} \}$

// assignment of labels to vertices

$v_1 \leftarrow 4m-2 ; \quad v_{m+2}' \leftarrow 4m+2$

for $i = 2$ to $m-1$ do

$\{ v_i \leftarrow \begin{cases} 2i & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i-2 & \text{otherwise} \end{cases}$

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end for

for $i = 1$ to $m-1$ do

$\{ v'_i \leftarrow \begin{cases} 2i & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i-2 & \text{otherwise} \end{cases}$

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end for

for $i = 2$ to m do

$\{ v_{m+i+1} \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i-3 & \text{otherwise} \end{cases}$

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end for

for $i = 2$ to m do

$\{ v_{m+i+1}' \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i-3 & \text{otherwise} \end{cases}$

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end for

if $m = 2n$

$v_m \leftarrow 2m, v'_m \leftarrow 1, v_{m+2} \leftarrow 4m+3, v_{m+1} \leftarrow 4m+1$

$v_{m+1}' \leftarrow 4m, v_{2m+2} \leftarrow 4m+4, v_{2m+2}' \leftarrow 4m-1$

if $m = 2n+1$

$v_m \leftarrow 4m+3, v'_m \leftarrow 2m, v_{m+1} \leftarrow 4m, v_{m+1}' \leftarrow 4m+1,$

$v_{2m+2}' \leftarrow 4m+4, v_{2m+2} \leftarrow 4m-1, v_{m+2} \leftarrow 1.$

end procedure

output: vertex labeled extended duplicate graph of comb graph.

Theorem 2.5

The extended duplicate graph of comb graph, $m \geq 2$ admits prime Cordial labeling.

Proof :

The $EDG(comb)(V,E)$ graph has $4m+4$ vertices and $4m+4$ edges. Label the vertices of $EDG(comb)$ graph by defining a function $f : V \rightarrow \{1, 2, 3, 4, \dots, 4m+4\}$, as given in algorithm 2.3. Thus all the $4m+4$ vertices are labeled.

Define the induced map $f^* : E \rightarrow N$ such that

$$f^*(v_i v_j) = \begin{cases} 1 & \text{if g. c. d.}(f(v_i), f(v_j)) = 1 \\ 0 & \text{if g. c. d.}(f(v_i), f(v_j)) > 1 \end{cases}$$

By the above induced function, the edge labels are obtained as follows :

For $1 \leq i \leq m-2$

$$f^*(v_i v_{i+1}') = f^*(v_i' v_{i+1}) = 0$$

$$\text{For } 1 \leq i \leq m; f^*(v_i' v_{m+i+1}) = 1$$

$$\text{For } 2 \leq i \leq m; f^*(v_i v_{m+i+1}') = 1$$

If $m = 2n$

$$f^*(v_1 v_{m+1}') = f^*(v_1 v_{m+2}') = f^*(v_{m+1}' v_{2m+2}) = f^*(v_{m+2}' v_{2m+2}) = f^*(v_{m+1}' v_m) = f^*(v_m v_{m+1}') = 0$$

$$f^*(v_{m-1} v_m') = f^*(v_m' v_{m+1}) = f^*(v_{m+1} v_{2m+2}') = 1$$

If $m = 2n+1$

$$f^*(v_1 v_{m+1}) = f^*(v_1 v_{m+2}') = f^*(v_{m+1} v_{2m+2}') = f^*(v_{m+2}' v_{2m+2}') = f^*(v_{m-1} v_m') = f^*(v_m' v_{m+1}) = 0$$

$$f^*(v_{m-1}' v_m) = f^*(v_m v_{m+1}') = f^*(v_{m+1}' v_{2m+2}) = 1$$

Thus in both the above cases,

The number of edges labeled with '1' = $m+m-1+3 = 2m+2$.

Number of edges labeled with '0' = $m-2+m-2+6 = 2m+2$.

Hence the number of edges labeled with '1' and '0' differ by atmost '1'

Therefore $EDG(comb)$ graph is prime cordial .

Example 2.3

Prime cordial labeling of $EDG(comb)$ graphs for $m=5$ and $m=6$ are given in figure 5 and figure 6 respectively.

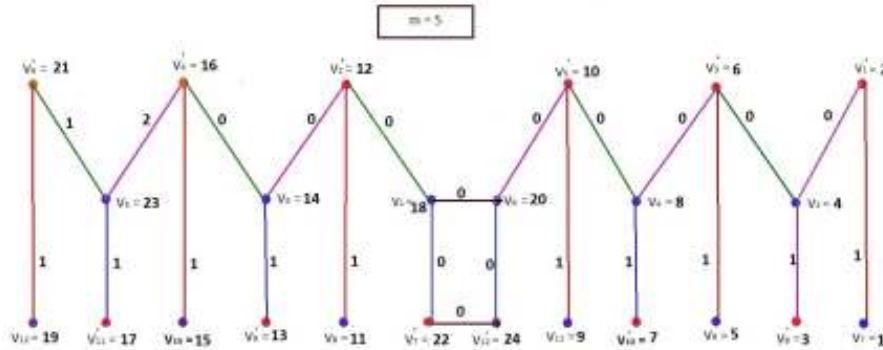


Figure: 5 prime cordial labeling of EDG(comb) graph for $m = 5$

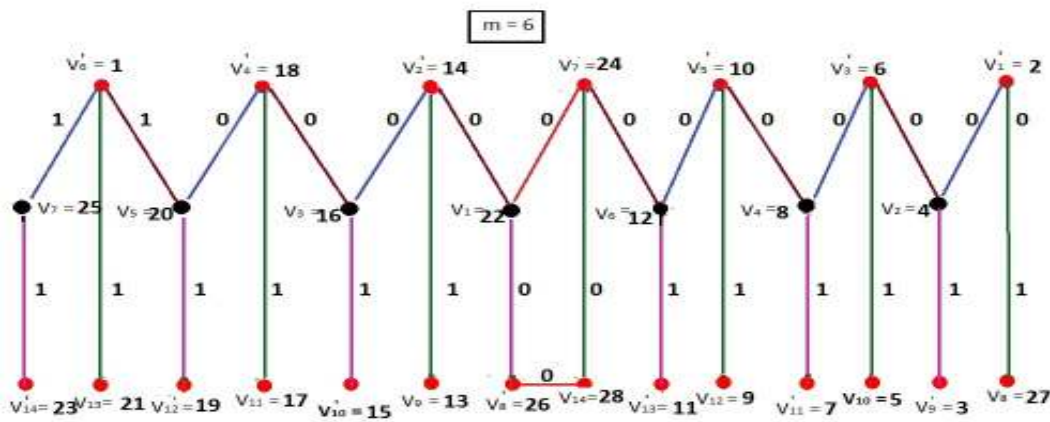


Figure: 6 prime cordial labeling of EDG(comb) graph for $m = 6$

Algorithm 2.4

Procedure : (odd mean labeling for extended duplicate graph of comb graph)

$V = \{ v_1, v_2, \dots, v_{2m+2}, v'_1, v'_2, \dots, v'_{2m+2} \}$

// assignment of labels to vertices

for $i = 1$ to $m+1$ do

$$v_i \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 0 \pmod{2} \\ 2m+2i+1 & \text{otherwise} \end{cases}$$

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$$v_i' \leftarrow \begin{cases} 2i-1 & \text{if } i \equiv 1 \pmod{2} \\ 2m+2i+1 & \text{otherwise} \end{cases}$$

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end for

for $i = 2$ to $m+2$ do

$$\{ \quad v_{m+i} \leftarrow \begin{cases} 4m+2i+1 & \text{if } i \equiv 0 \pmod{2} \\ 6m+2i+3 & \text{otherwise} \end{cases}$$

$$v_{m+i}' \leftarrow \begin{cases} 4m+2i+1 & \text{if } i \equiv 1 \pmod{2} \\ 6m+2i+3 & \text{otherwise} \end{cases}$$

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}

end for

end procedure

output : vertex labeled extended duplicate graph of comb graph.

Theorem 2.6

The extended duplicate graph of EDG (comb) graph admits odd mean labeling.

Proof:

Clearly $\text{EDG}(\text{comb})(V, E)$ graph has $p = 4m+4$ vertices and $q = 4m+4$ edges.

To label the vertices, define a function $f : V \rightarrow \{1, 3, 5, \dots, 2q-1\}$ as given in algorithm 2.4.

Thus all the $4m+4$ vertices are labeled.

To get the edge labels, define the induced map

$f^* : E \rightarrow \{1, 2, 3, \dots, 2q-1\}$ such that $f^*(v_i v_j) = f(v_i) + f(v_j) / 2$ for all $v_i v_j \in E$

By the induced function, the edge labels are obtained as follows:

For $1 \leq i \leq m$

$$f^*(v_i v_{i+1}') = \begin{cases} 2i & \text{if } i \equiv 0 \pmod{2} \\ 2m+2i+2 & \text{otherwise} \end{cases}$$

$$f^*(v_i' v_{i+1}) = \begin{cases} 2i & \text{if } i \equiv 1 \pmod{2} \\ 2m+2i+2 & \text{otherwise} \end{cases}$$

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For $1 \leq i \leq m+1$

$$f^*(v_i v_{m+i+1}') = \begin{cases} 2m+2i+1 & \text{if } i \equiv 0 \pmod{2} \\ 4m+2i+3 & \text{otherwise} \end{cases}$$

$$f^*(v_i'v_{m+i+1}) = \begin{cases} 2m+2i+1 & \text{if } i \equiv 1 \pmod{2} \\ 4m+2i+3 & \text{otherwise} \end{cases}$$

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If $m = 2n$

$$f^*(v_1v_{m+1}) = 2m+2, \quad f^*(v_{m+2}'v_{2m+2}) = 6m+6.$$

If $m = 2n+1$

$$f^*(v_1v_{m+1}) = 2m+2, \quad f^*(v_{m+2}'v_{2m+2}') = 6m+6.$$

Thus all the labeled edges are distinct .

Hence EDG (comb) graph admits odd mean labeling .

Example 2.4

Odd mean labeling of EDG(comb) graphs for $m=5$ and $m=6$ are given in figure 7 and figure 8 respectively.

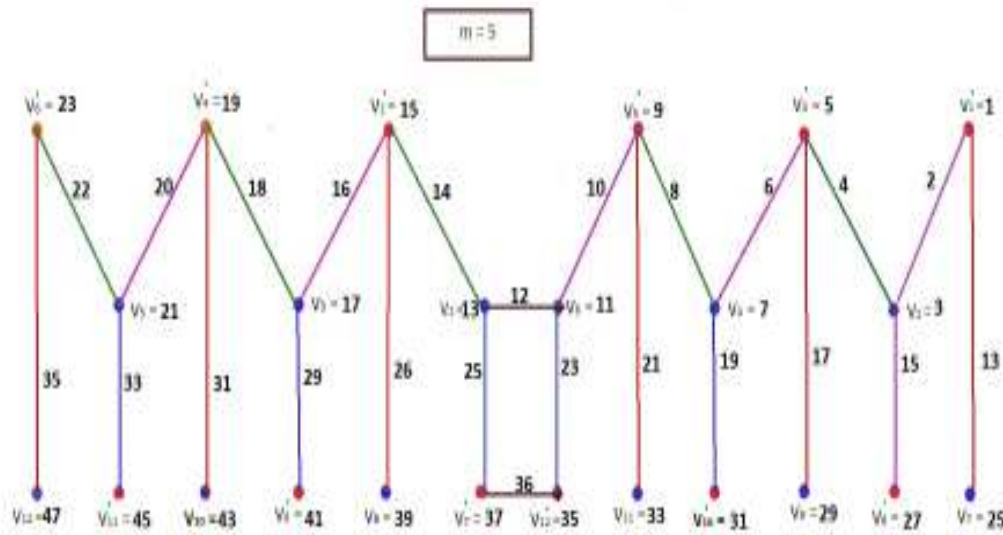


Figure : 7 odd mean labeling of EDG(comb) graph for $m = 5$

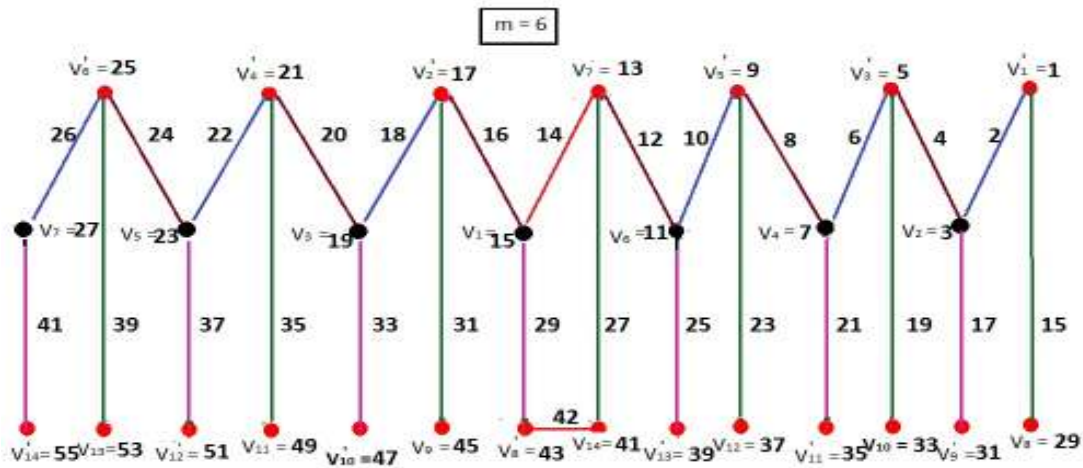


Figure : 8 odd mean labeling of EDG(comb) graph for $m = 6$

Algorithm 2.5

Procedure :(even mean labeling for extended duplicate graph of comb graph)

$V = \{ v_1, v_2, \dots, v_{2m+2}, v'_1, v'_2, \dots, v'_{2m+2} \}$

// assignment of labels to vertices

for $i = 1$ to $m+1$ do

$\{ \quad v_i \leftarrow \begin{cases} 2i & \text{if } i \equiv 0(\text{mod } 2) \\ 2m+2i+2 & \text{otherwise} \end{cases}$

$\quad v'_i \leftarrow \begin{cases} 2i & \text{if } i \equiv 1(\text{mod } 2) \\ 2m+2i+2 & \text{otherwise} \end{cases}$

$\}$

end for

for $i = 2$ to $m+2$ do

$\{ \quad v_{m+i} \leftarrow \begin{cases} 4m+2i+2 & \text{if } i \equiv 0(\text{mod } 2) \\ 6m+2i+4 & \text{otherwise} \end{cases}$

$\quad v'_{m+i} \leftarrow \begin{cases} 4m+2i+2 & \text{if } i \equiv 1(\text{mod } 2) \\ 6m+2i+4 & \text{otherwise} \end{cases}$

$\}$

end for

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end procedure

output : vertex labeled extended duplicate graph of comb graph.

Theorem 2.7

The extended duplicate graph of comb graph, $EDG(\text{comb})$ graph admits even mean labeling.

Proof:

Clearly $EDG(\text{comb})(V,E)$ graph has $p = 4m+4$ vertices and $q = 6m+4$ edges.

To label the vertices , define a function

$f : V \rightarrow \{ 2,4,6,\dots, 2q \}$ as given in algorithm 2.5.

Thus $4m+4$ vertices are labeled.

To get the edge labels, define, the induced map $f^* : E \rightarrow \{1,2,3,\dots,2q\}$ such that

$f^*(v_i v_j) = [f(v_i) + f(v_j)] / 2$ for all $v_i v_j \in E$

By the induced function , the edge labels are obtained as follows:

For $1 \leq i \leq m$

$$f^*(v_i v_{i+1}') = \begin{cases} 2i+1 & \text{if } i \equiv 0 \pmod{2} \\ 2m+2i+3 & \text{otherwise} \end{cases}$$

$$f^*(v_i' v_{i+1}) = \begin{cases} 2i+1 & \text{if } i \equiv 1 \pmod{2} \\ 2m+2i+3 & \text{otherwise} \end{cases}$$

For $1 \leq i \leq m+1$

$$f^*(v_i v_{m+i+1}') = \begin{cases} 2m+2i+2 & \text{if } i \equiv 0 \pmod{2} \\ 4m+2i+4 & \text{otherwise} \end{cases}$$

$$f^*(v_i' v_{m+i+1}) = \begin{cases} 2m+2i+2 & \text{if } i \equiv 1 \pmod{2} \\ 4m+2i+4 & \text{otherwise} \end{cases}$$

If $m = 2n$

$$f^*(v_1 v_{m+1}') = 2m+3 \quad ; \quad f^*(v_{m+2}' v_{2m+2}) = 6m+7.$$

If $m = 2n+1$

$$f^*(v_1 v_{m+1}) = 2m+3 \quad ; \quad f^*(v_{m+2}' v_{2m+2}') = 6m+7.$$

Thus all the labeled edges are distinct .

Hence $EDG(\text{comb})$ graph admits even mean labeling .

Example 2.5

Even mean labeling of $EDG(\text{comb})$ graph for $m = 5$ and $m = 6$ are given in figure 9 and figure 10 respectively.

Comment [r13]: Please generate it using mathtype

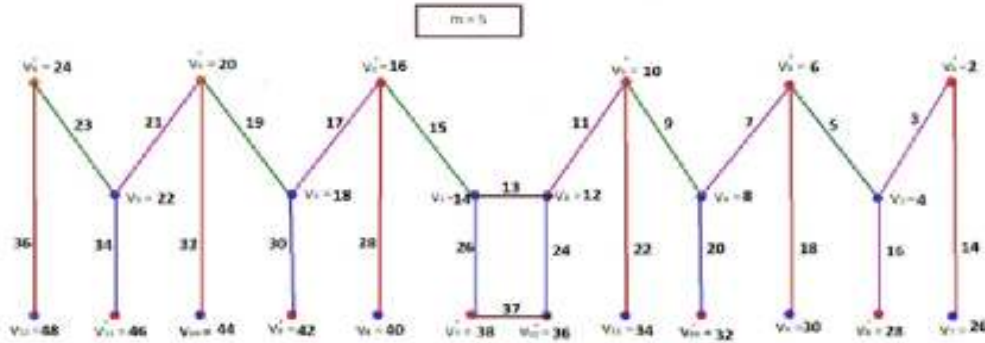


Figure : 9 even mean labeling of EDG(comb) graph for $m = 5$

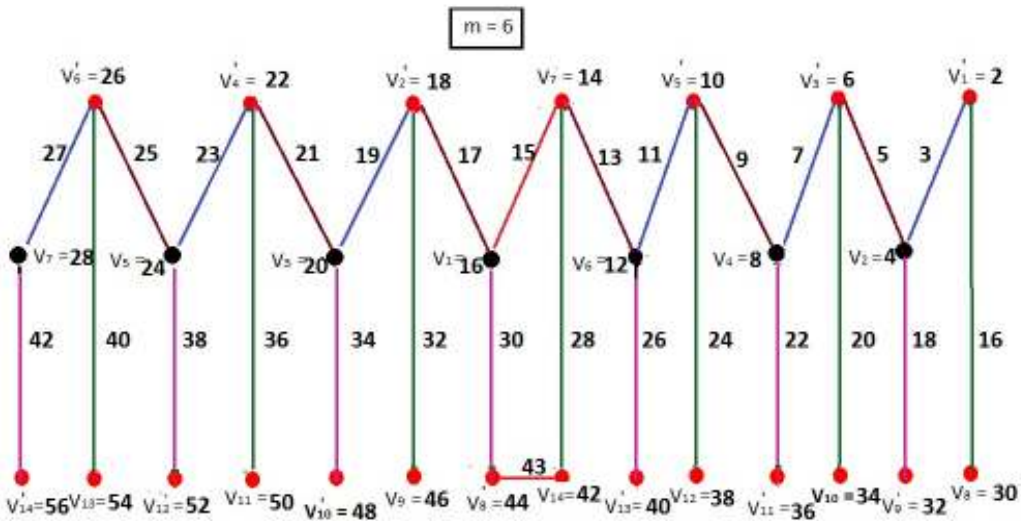


Figure : 10 even mean labeling of EDG(comb) graph for $m = 6$

Conclusion:

In this paper we have presented algorithms and proved that the extended duplicate graph of Comb graph admits cordial, total cordial, product cordial, total product cordial, prime cordial, odd mean labeling and even mean labeling.

References:

Comment [r14]: Please unify the format of all references.

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