

1

2 **Original Research Article**

3

4 DYNAMIC PROGRAMMING METHOD FOR OPTIMIZING STOCK
 5 ALLOCATION USING CHEBYSHEV POLYNOMIAL
 6 APPROXIMATION

7 **ABSTRACT**

8 Stock allocation is a system used to ensure that goods and services reach the ultimate users
 9 through efficient stocking in warehouses close to the consumers. The dire need for optimum
 10 distribution of goods to both retailers and consumers has cost a reasonable drift from ordinary
 11 allocation to developing a mathematical model that ensures efficient allocation of goods and services.
 12 This paper presents a method of optimizing stock allocation using the Chebyshev polynomial
 13 approximation of an n- warehouse inventory model. The features of Chebyshev polynomial are
 14 enumerated and used on stock allocation environment to obtain good approximation that would
 15 ensure high yield to the firms. It has been shown that this method is efficient, stable and provides
 16 quick access to obtaining optimal allocation. The model of the method is given, relevant algorithm and
 17 theorems are included while illustrative examples are provided.

18 Keywords: Dynamic Programming, Stock Control, Stock allocation, Optimal decision. Chebyshev
 19 polynomial, polynomial approximation.

20 **1.0 MOTIVATION**

21 The quest for optimal distribution of goods to both retailers and consumers has given rise to
 22 reasonable drift from ordinary allocation to developing a mathematical model that enhances steady
 23 and efficient allocation. Stock is the supply of goods for sale kept in the store for business. The
 24 growing global economy has caused a dramatic shift in stock management in the twentieth century
 25 from a mere approach to scientific approach. "One of the related problems is that as the complexity
 26 and specialization in an organization increases, it becomes more difficult to allocate rationally and
 27 reasonably the available resources to various sections of the organization" [1]. The proper allocation
 28 of resources in both manufacturing and distribution industries is of paramount significance to the
 29 society since the chain of distribution is complete only when the goods get to consumers. The
 30 allocation process that minimizes cost and maximizes profit is always the desire of every organization.
 31 The need to obtain such a process is the aim of this work. In this regard, a lot of methods in
 32 Operations Research are available. Methods such as linear programming model; integer programming,
 33 goal programming, dynamic programming models can be used to ascertain optimum allocation of

goods. This work will use the dynamic programming model to obtain an optimum allocation of resources that would provide the solution to the desire of any organization.

The dynamic programming is a linear optimization method that obtains optimum solution of a multivariable problem by decomposition of the problem into sub problems [2]. Dynamic programming is an approach of optimizing multistage decision processes with a recursive equation. DP is based on the observation by Bellman (1957) that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.” This reduces multi-period sequential decision problems to a sequence of one-period optimal control problems. We examine only finite horizon problems in this paper but the numerical ideas also apply to infinite-horizon problems. The recursive principle of DP implies that the solution can be characterized by value functions, $V_t(x)$, defined by the Bellman equation

$$V_t(x) = \max_{a \in D_t(x)} \{ U_t(x, a) + \beta E\{V_{t+1}(x+) / x, a\} \},$$

for $t = 0, 1, \dots, T - 1$, where T is finite, x is the state and a is the action choice (x and a can be vectors), $U_t(x, a)$ is the utility function at time $t < T$, and $U_T(x) = V_T(x)$ is the terminal utility function, β is the discount factor, $D_t(x)$ is a feasible set of a at time t , $x+$

is the (possibly random) state at time $t + 1$ if the state is x at time t and action a is chosen at time t , and $E\{\cdot\}$ is the expectation operator [14]. The different stages of the problem are linked such that the optimum feasible solution of each stage is guaranteed to be the optimum feasible solution for the entire problem. The dynamism of demand makes it necessary to keep goods in stock and the act of maintaining stock has its associated costs. The act of stocking goods to satisfy future demand gives rise to the problem of designing a very efficient allocation technique that minimizes cost and maximizes profit. This involves minimizing an appropriate cost function that balances the total cost resulting from overstocking or under-stocking. [4]. Dynamic Programming was first used by Richard Bellman in 1940 to describe a process of solving problems where one needs to find the best decisions one after the other [7]. This work uses the dynamic programming method to develop a stock allocation model that ensures optimum allocation of goods and services for maximum returns. If $r_i(Q_i)$ is the total return from the i th activity with the resource, we seek to maximize.

$$R(Q_1, Q_2, \dots, Q_n) = r_1(Q_1) + r_2(Q_2) + \dots + r_n(Q_n),$$

Given that $Q = Q_i \geq 0, i = 1, 2, \dots,$

2 INTRODUCTION

66 A polynomial function is a function that can be defined by evaluating a polynomial. A function f of
67 one argument is called a polynomial function if it satisfies $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} +$
68 $\dots + a_1 x + a_0$ for all argument x where n is a non-negative integer and $a_i / i = 0, 1, 2, \dots, n$ are
69 constant coefficients. [2]. Polynomial functions are those functions whose values can be calculated by
70 putting the value of the independent variable in a polynomial, if f is continuous on $[a, b]$, then a best
71 approximation to f from P_n exists [9]. Approximation is the act of estimating a number or an
72 amount. The approximate is almost correct and may not be exact [1]. It is a number which is taken as
73 a close estimate of another number or the method of finding such approximate number [4].
74 Approximation arises due to the difficulty in obtaining the exact area/volume or dimension of some
75 objects. In addition, the inability of man to foretell the future accurately as a result of his/her fallibility
76 gives birth to estimation of quantities.

77 The general approximation problem states that if f is an element and S , a subset of a
78 normed linear space X , then approximation theory seeks to find an element of S which is as close as
79 possible to f ; i.e. to find an element S^* of S such that $\|f - S^*\| \leq \|f - S\| \quad \forall S \in S$, S^* is called
80 the best approximation of f from S relative to the given norm. Weierstrass approximation
81 theorem states that if f is a continuous function on an interval $[a, b]$ and $\varepsilon > 0$ is given,
82 then there exists a polynomial $p(x)$ such that

$$\begin{aligned} 83 \quad & \sup_{x \in [a, b]} |f(x) - p(x)| \leq \varepsilon \\ 84 \quad & x \in [a, b] \end{aligned}$$

85 Determination of polynomial coefficients requires solution of complicated system of equations. It is
86 possible to avoid such problems by using orthogonal Chebyshev polynomials. This is a method of
87 approximation where the maximum difference between value of function and value calculated from
88 polynomials is minimized [6].

89 The Chebyshev polynomial $T_n(x)$ is a basis function and one of the spectral general class of
90 polynomials referred to as orthogonal polynomial. According to [15], the approximation of the
91 function f requires the value of f at the zeros of the $(n+1)^{\text{st}}$ Chebyshev polynomial. The Chebyshev
92 polynomial T_n and U_n are polynomials of degree n and the sequence of Chebyshev polynomials of
93 either kind composes a polynomial sequence. They are polynomials with the largest possible leading
94 coefficient but subject to the condition that their absolute value is bounded on the interval by 1. One
95 of the unique properties of the Chebyshev polynomial is that on the interval $-1 \leq x \leq 1$ all the
96 extrema have values that are either -1 or 1 . Thus these polynomials have only two finite critical
97 values. The Chebyshev approximation was chosen because of its superior convergence
98 properties)

99 .

100 3 The Chebyshev's Polynomial Approximation

101 3.1 Chebyshev's polynomial is a sequence of orthogonal polynomials which are related to De-
102 Moivre's formula and which can be defined recursively. The Chebyshev's polynomials T_n are

polynomials of degree n . The Chebyshev's polynomial of the first kind are defined by the recursive relation.

$$T_0(x) = 1$$

$$T_1(x) = x$$

$$\vdots \quad \vdots$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x). \quad [4]$$

The Chebyshev's approximation in particular states that:

If P_n is the collection of all polynomials whose degree is at most n and f be a continuous function on the interval $[a, b]$. The polynomial P is said to be the best approximation to f from P_n if $P \in P_n$ and

$$\max_{x \in [a, b]} |f(x) - P(x)| \leq \max_{x \in [a, b]} |f(x) - q(x)| \quad \forall q \in P_n \quad [5].$$

3.2 Properties of Chebyshev's Polynomial

i. Recursion formula; $T_0(x) = 1$,

ii. $T_1(x) = x = xT_0(x)$,

.

.

iii. $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \geq 1$

iii. Symmetric property; $T_n(-x) = (-1)^n T_n(x)$

iv. $T_n(x)$ has n - zeros in $[-1, 1]$ given by $x_k = \cos\left(\frac{2k+1}{n} \pi\right)$,

$$k = 0, 1, \dots, n-1.$$

v. Orthogonality property; Set $(f, g) = \int_{-1}^1 f(x) g(x) (1-x^2)^{-\frac{1}{2}} dx$, then

$$(T_i, T_j) = \begin{cases} 0 & \text{if } i \neq j \\ \frac{1}{2} \pi & \text{if } i = j \neq 0 \\ \pi & \text{if } i = j = 0 \end{cases} \quad \text{i.e continuous case}$$

$(f, g) = \sum_{k=0}^n f(x_k) g(x_k)$ where $\{x_k\}$ are the zeros of $T_{m+1}(x)$

$$\text{Then for } 0 \leq i \leq m, 0 \leq j \leq m, (T_i, T_j) = \begin{cases} 0 & \text{if } i \neq j \\ \frac{1}{2} m + 1 & \text{if } i = j \neq 0 \\ m + 1 & \text{if } i = j = 0 \end{cases}$$

(Discrete case)

vi. Minimax property; Of all n th degree polynomials with leading coefficient $1, 2^{1-n} T_n$ has the smallest maximum norm in $[-1, 1]$. The value of its maximum norm is 2^{1-n} .

The strength of the Chebyshev polynomial approximation is on the fact that it has recursive properties which makes it adapt to the dynamic programming method and stock allocation processes. It is an orthogonal polynomial whose extrema lie in the interval $[-1, 1]$

4 REVIEW OF RELATED LITERATURE

Stock allocation is an important part of any manufacturing organization. The availability of goods to customers when needed is a sign of preparedness and efficient stock management [11]. Many researchers have worked on stock allocation and inventory models and some of them are cited in this section.. Optimal stock allocation policy generally requires comprehensive knowledge of the nature of demand of goods in an environment [4].

Dynamic Programming is one of the numerous linear optimization methods. It is a method for solving complex problems by breaking it into simpler sub-problems [3]. It determines the optimum solution of a multivariable problem by decomposing it into stages with each stage comprising a single variable sub-problem. It is a recursive equation that links the different stages of the problem in a manner that guarantees that the optimal feasible solution of each stage is also optimal and feasible for the entire problem [5]. According to [6], the work of Bellman implies that the process refers to the act of supplying a decision through breaking down the problem into a sequence of decision steps which is done by defining a sequence of value function $v_1, v_2, \dots, v_n$ with an argument y representing the state of the system at times i from 1 to n . Dynamic programming is guaranteed to give a mathematically optimal solution and the equations for the stages are written as follows:

Let $f_i(x)$ be the shortest distance to node x_i at stage i ;

Define $d(x_{i-1}, x_i)$ as the distance from node x_{i-1} to x_i .

The f_i is computed from f_{i-1} , by the following recursive equation

$$f_i(x_i) = \min_{\text{All feasible } ((x_{i-1}, x_i) \text{ nodes})} \{d(x_{i-1}, x_i) + f_{i-1}(x_{i-1})\} \quad i=1,2,3,\dots \quad [8]$$

The computation in dynamic programming is done recursively so that the optimum solution of one sub-problem is used as an input to the next sub-program and by the time the last sub-program is solved, the optimum solution for the entire problem is ascertained [10].

The Chebyshev polynomial of degree n is denoted $T_n(x)$, and is given by the explicit formula $T_n(x) = \cos(\arccos x)$

This may look trigonometric at first glance (and there is in fact a close relation between the Chebyshev polynomials and the discrete Fourier transform); however can be combined with trigonometric identities to yield explicit expressions for $T_n(x)$ and $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$.

The Chebyshev polynomials are orthogonal in the interval $[-1; 1]$ over a weight $(1 - x^2)^{-1/2}$. In particular, the polynomial $T_n(x)$ has n - zeros in the interval $[-1; 1]$ and they are located at the points

$$x = \cos(\pi(k - 1/2)), \quad k = 1, 2, \dots, n$$

In this same interval there are $n + 1$ extrema (maxima and minima), located at

$$x = \cos(\pi k) \quad k = 0; 1; \dots; n$$

171 n
 172 At all of the maxima $T_n(x) = 1$, while at all of the minima $T_n(x) = -1$;
 173 it is precisely this property that makes the Chebyshev polynomials so useful in orthogonal relation.
 174 The Chebyshev basis polynomial yields an extremely well conditioned polynomial
 175 approximation of functions. The Chebyshev polynomial satisfies the discrete and continuous equation
 176 that can be accurately and efficiently solved even with high degree approximation. The chebyshev
 177 polynomial is orthogonal with Euclidean norm condition number 2^5 . This implies that the basis
 178 coefficient θ_j can be computed quickly and accurately

179 5 ALGORITHM/THEOREM

180 5.1 ALGORITHM [19]

181 The Remez algorithm is a successive approximation technique for computing the
 182 coefficients of the polynomial in P_n which provides the best Chebyshev approximation to a
 183 given continuous function f . The general approach is to find the best approximation to f on
 184 a set of $n+2$ points in such a manner that the successive best approximation on the finite
 185 sets approaches the best approximation over the whole interval.

186 Before stating the algorithm the following assertions /theorems will be stated as they
 187 preclude it.

188 i). Let $x_0 < x_1 < \dots < x_{n+1}$ be $n+2$ distinct numbers. Then the system of equations.
 189 $f(x_k) - \sum_{j=0}^n a_j x_k^j = (-1)_\lambda^k$ $k = 0, 1, 2, \dots, n+1$ has a unique solution in $a_0, a_1, a_2,$
 190 $\dots, a_n, \lambda$. [19]

191 ii). Suppose $f \in C[a, b]$, f^1 exists on (c, b) and $a_1, a_2 < \dots < a_m$ are points of $[a, b]$.
 192 If $(-1)^k f(a_k) \geq 0$, $1 \leq k \leq m$, then f has at least $m-1$ zeros on $[a_1, a_m]$. [19]

193 iii). Suppose $T = \{x_0, x_1, x_2, \dots, x_{n+1}\}$ is a set of $n+2$ points indexed, so that $\{x_0 < x_1 < \dots <$
 194 x_{n+1} and f is a function defined on T . [22]

195 Let $a_0, a_1, a_2, \dots, a_n, \lambda$ be the unique solution to $f(x_k) - \sum_{j=0}^n a_j x_k^j = (-1)_n^k \lambda$, and define
 196 p by $p(x) = \sum_{j=0}^n a_j x^j$

197 Then P is unique best approximation to f on the set T .

198 iv). Let f be continuous on $[a, b]$, and let $T = \{x_0, \dots, x_{n+1}\}$ be a set of $n+2$ distinct points in
 199 $[a, b]$ indexed so that $x_0 < x_1 < \dots < x_{n+1}$.

200 Suppose $p \in P_n$ and $f - p$ alternates sign at the points $x_0, x_1, \dots, x_{n+1}$.

201 Then

$$202 E_n(f) \geq E_{n,T}^{(f)} \geq \min_{0 \leq j \leq n+1} |f(x_j) - p(x_j)|. \quad [19]$$

203 The algorithm now proceeds as follows ; [19]

204 i) Choose a set $T_0 = \{x_0^0, x_1^0, \dots, x_{n+1}^0\}$ of $n+1$ points in $[a, b]$

205 ii) Solve the system of equations

$$f(x_k^0) - \sum_{j=0}^n a_j^0 (x_k^0)^j = (-1)^k \lambda_0, \quad k = 0, 1, \dots, n+1$$

iii) For $a_0^0, a_1^0, \dots, a_n^0$ and λ_0 (if $\lambda_0 = 0$ choose a new set of points for T_0),
Now proceed inductively.

Having chosen a set of points

$$T_m = \{x_0^m, x_1^m, \dots, x_{n+1}^m\},$$

$$\text{Let } \lambda_m = f(x_0^m) - P_m(x_0^m),$$

Let P_m be the best approximation of f on T_m and Let $\lambda_m = f(x_0^m) - P_m(x_0^m)$,

Then Choose a new set

$$T_{m+1} = \{x_0^{m+1}, x_1^{m+1}, \dots, x_{n+1}^{m+1}\} \text{ so that the following conditions are satisfied.}$$

$$i) \quad x_0^{m+1} < x_1^{m+1} < \dots < x_{n+1}^{m+1}$$

$$ii) \quad |f(x_k^{m+1}) - p_m(x_k^{m+1})| = \|f - P_m\| \text{ for some } k,$$

iii) $f - P_m$ alternates sign at the points of T_{n+1} and

$$iv) \quad |f(x_k^{m+1}) - p_m(x_k^{m+1})| \geq |\lambda_m|, \quad k = 0, 1, \dots, n+1$$

if $|\lambda_m| = \|f - p_m\|$, then P_m is the best approximation.

5.2 THEOREMS

In problems of approximation, the convergence of function play important role. This is because the approximate value must be very close (almost exactly) to the actual solution. The uniqueness of the solution is also important. This section will consider theorems that assert the quality, unbiasedness and consistency of the approximates if it exists.

5.2.1 Let $f \in [-1, 1]$ and let S_n be the best approximate to f from P_n relative to

$$\|f\| = \left[\int_{-1}^1 \frac{f^2(x) dx}{\sqrt{1-x^2}} \right]^{\frac{1}{2}}, \text{ then}$$

$$\lim_{n \rightarrow \infty} \|f - S_n\| = 0 \quad [6]$$

Proof

Let $\| \cdot \|$ and $\| \cdot \|_0$ be the norms defined by $\|f\|$ above and

$$\|f\| = \left[\int_{-\pi}^{\pi} \frac{f^2(x) dx}{\sqrt{1-x^2}} \right]^{\frac{1}{2}} \text{ Respectively.}$$

$$\text{Let } S_n(\theta) = \frac{a_0}{2} + \sum_{j=1}^n a_j \cos j\theta$$

Making, the change of variable $x = \cos\theta$ and using $f(x) - \frac{a_0 T_0(x)}{2} - \sum_{j=1}^n a_j T_j(x) = g(\theta) - \frac{a_0}{2} -$

$$\sum_{j=1}^n a_j \cos j\theta$$

gives

$$\|f - S_n\|^2 = \int_{-1}^1 \frac{|f(x) - S_n(x)|^2 dx}{\sqrt{1-x^2}} = \int_0^\pi [g(\theta) - S_n(\theta)]^2 d\theta$$

$$= \frac{1}{2} \|g - S_n\|_0^2$$

237 Recall that if $a_j = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos jx dx$ and

238 $b_j = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin jx dx$, $j = 1, 2, \dots, n$

239 and $S_n(x) = T^{(x)} = \frac{a_0}{2} + \sum_{j=1}^n (a_j \cos jx + b_j \sin jx)$, then

$$\lim_{n \rightarrow \infty} \|f - S_n\| = 0$$

240 Hence $\frac{1}{2} \|g - S_n\|_0^2 = 0$

241 Then the polynomial P^* whose coefficient is c^* is a best approximation of f .

242 5.2.2. If $f \in [a, b]$, then the best chebyshev polynomial approximation to f from P_n is unique [3]

243 Proof Let p_1 and p_2 be best approximations to f from P_n , $P_1, P_2 \in P_n$

244 Now define $P_0 = \frac{(p_1 + p_2)}{2}$ and

245 Let $E = \|f - P_1\| = \|f - p_2\|$

246 Then $E \leq \|f - P_0\| = \left\| \frac{f - p_1}{2} + \frac{(f - p_2)}{2} \right\| \leq \left\| \frac{f - p_1}{2} \right\| + \left\| \frac{f - p_2}{2} \right\| = E$

247 Thus P_0 is also a best approximation to f .

248 Let $x_0 < x_1 < \dots < x_{n+1}$ be points satisfying the equation.

$$f(x_j) - P_0(x_j) = (-1)^j \delta E, j = 0, 1, \dots, n+1$$

249 Then for $j = 0, 1, \dots, n+1$

$$E = |f(x_j) - P_0(x_j)| = \left| \frac{f(x_j) - P_1(x_j)}{2} + \frac{f(x_j) - P_1(x_j)}{2} \right|$$

250 But $|f(x_j) - p(x_j)|$ and $|f(x_j) - p(x_j)|$ are both less than or equal to E

251 $\Rightarrow f(x_j) - P(x_j) = f(x_j) - P_2(x_j) = (-1)^j \sqrt{E}$

252 Thus

253 $P_1(x_j) = P_2(x_j), j = 0, 1, \dots, n+1$

254 $\Rightarrow P_1 - P_2$ has $n+2$ zeros.

255 Hence

$$P_1 = P_2$$

256 5.2.3 If f is continuous on $[a, b]$, then a best approximation to f from P_n exists [9]

257 6 MATHEMATICAL MODEL

258 Let Q be a certain quantity of resource that will be distributed among n -number of depots. Let

259 R be the return which depends on the quantity of resource allotted to the depots.

260 The objective is to optimize total return

261 If $r_i(Q_i)$ denotes the return from the i th activity with resource Q_i , then the total return is given as

262 $R(Q_1, Q_2, \dots, Q_n) = r_1(Q_1) + r_2(Q_2) + \dots + r_n(Q_n)$ (1)

263 The quantity of resource Q is limited. Hence this gives rise to the constant.

264 $Q = (Q_1 + Q_2 + \dots + Q_n, Q_i \geq 0, i = 1, 2, \dots, n)$ (2)

265 Hence the problem is stated as:

266 Optimize $R(Q_1, Q_2, \dots, Q_n) = R(Q_1) + R(Q_2) + \dots + R(Q_n)$
 267 Given that $Q = Q_1 + Q_2 + \dots + Q_n$, $Q_i \geq 0$, $i = 1, 2, \dots, n$
 268 If $f_n(Q) = \max_{0 \leq Q_1 \leq Q_n} [R(Q_1, Q_2, \dots, Q_n)]$
 269 $= \max_{0 \leq Q_1 \leq Q_n} [r_1(Q_1) + r_2(Q_2) + \dots + r_n(Q_n)]$ (3)
 270 $f_n(Q)$ is the maximum return from the distribution of the resource Q to the n activities/depots.
 271 To approximate $f_n(Q)$, use a corresponding polynomial function p_n of $f_n(Q)$ from the allocation
 272 table, i.e

$$273 \quad P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0, \quad (4)$$

274 Choose another polynomial function $P_i(x)$, $i = 1, 2, \dots, n-1$ in reference to the
 275 Chebyshev polynomial $T_n(x)$ to approximate $f_n(x)$
 276 where $p_i(x) \in P_n(x)$ and $\deg p_i \leq \deg P_n(x)$ and
 277 where $f(x) - T(x) = p(x)$

$$278 \quad \text{Find } F(x) = f(x) - p_i(x) \text{ where } |f(x) - p_i(x)| < \varepsilon \quad (5)$$

279 Obtain $F'(x)$ if it exists and then

280 Calculate k_i , the zeros of $F'(x)$ where $k_i \in [-1, 1]$

281 Proceed to find $\|F\| = \max [F(a = k_1), F(k_2), \dots, F(k_{n-1}), F(k_n = b)]$

$$282 \quad k \in [a, b]$$

$$283 \quad = \max [F(-1), F(k_2), \dots, F(k_{n-1}), F(1)] = 1 \quad (6)$$

$$284 \quad k \in [-1, 1]$$

285 Then test if k , the zeros of p forms an oscillation set in F .

286 If it does, then p_i approximates f .

$$287 \quad \text{For the best approximation, find } \|f - p_m\| \text{ where } f - p_m = \gamma_m \quad (7)$$

288 and where $p_m \in P_n$

289 If $|\gamma_m| \leq \|f - p_m\|$, then p_m is the best approximation of f if

$$290 \quad \text{Max } |f(x) - p(x)| < \text{max } |f(x) - q(x)| \text{ where } q(x) \in P_n(x)$$

$$291 \quad x \in [-1, 1] \quad x \in [-1, 1]$$

292 7 ILLUSTRATION

293 7.1 The table below is the stock allocation of a company to their six warehouses. Obtain the best
 294 approximation of allocation to ensure profitability of the company.

Warehouse (x)	1	2	3	4	5	6
Allocation (in thousand units) $f(x)$	130	293	8	556	943	43

295

296 The corresponding function to the allocation table above from regression is $f(x) =$
 297 $4x^3 + 2x^2 + x + 1$.

298 Use $p(x) = 2x^2 + 4x + 1$ as an approximate of $f(x)$ i.e $f(x) - T_3(x) = p(x)$

$$F(x) = f(x) - p(x)$$

$$F(x) = 4x^3 - 3x$$

$$F'(x) = 12x^2 - 3 = 0$$

$$4x^2 - 1 = 0$$

$$\Rightarrow x^2 = \pm \frac{1}{4}$$

299 $x = \pm 0.5$ i.e. $\lambda = \pm 0.5 \in [-1, 1]$

300 i.e. F' is zero at ± 0.5

301 $\|f\| = \max [|f(-1)|, |f(-0.5)|, |f(0.5)|, |f(1)|] = 1$

302 We note that $-1, -0.5, 0.5$ and 1 form an oscillation, set of four points

303 P is the best polynomial approximate for f from P_2 .

304 To show that p is the best approximation, we try two other approximations in P_n i.e.

305 $P_a(x) = 4x^2 + 2x + 1$ and $P_b(x) = x^2 + 2x + 1$

306 Now

$$f(x) - P_a(x) = F_1(x)$$

$$F_a(x) = 4x^3 - 2x^2 - x$$

$$F'_a(x) = 0$$

$$\Rightarrow \lambda_1 = -0.1667 \text{ and } 0.5 \in [-1, 1]$$

$$F_b(x) = f - P_b(x) = 4x^3 + x^2 - x$$

307 $F'_b(x)$ is zero at -0.342 and 0.217

$$\lambda_2 = -0.343 \text{ and } 0.217 \in [-1, 1]$$

308 The zeros of F'_a and F'_b do not form oscillation sets though they are all approximations of f .

309 Hence $\lambda = \pm 0.5$ i.e. p is the best approximation. Furthermore $|\lambda| = \pm 0.5$ applied in $|f(x) - p(x)|$

310 shows that

$$\max_{x \in [-1, 1]} |f(x) - P(x)| < \max_{x \in (-1, 1)} |f(x) - P_a(x)| < \max_{x \in (-1, 1)} |f(x) - P_b(x)|$$

311 For $P \in P_n$, $P_a \in P_n$ and $P_b \in P_n$

312 P is the best approximation to f from P_n relative to the uniform (Chebyshev) norm. It is unique.

313 7.2. The supply of a firm to her eight depots are given below. Use the record to obtain an

314 optimal approximate for the firm.

315

X	1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---	---

316

Y	8	128	648	2048	5000	10368	19208	37768
---	---	-----	-----	------	------	-------	-------	-------

317

318 Solution

319 The corresponding functional equation for the table above is $8x^4$

320 The equation is quartic hence use the Chebyshev polynomial $T_4(x)$. i.e. $T_4(x) = 4x^4 - 4x^2 +$

321 $1 = p_1(x)$

322 $8x^4 - (4x^4 + 4x^2 - 1) = 4x^4 - 4x^2 + 1$

323 $F(x) = 4x^4 - 4x^2 + 1$

324 $F'(x) = 16x^3 - 8x = 8x(2x^2 - 1)$

325 $X = 0, x = \pm \sqrt{1/2}$

326 The zeros of F' is the set $K = (-1, -\sqrt{1/2}, 0, \sqrt{1/2}, 1)$ and

327 $(-\sqrt{1/2}, 0, \sqrt{1/2},) \in [-1, 1]$

328 $\|F\| = \max \left[|f(-1)|, |f(-\sqrt{\frac{1}{2}})|, |f(0)|, \left| f\left(\sqrt{\frac{1}{2}}\right) \right|, |f(1)| \right] = 1$

329 Also $-1, -\sqrt{1/2}, 0, \sqrt{1/2}$ and 1 form an oscillation set.

330 P is therefore the best approximation of f

331 7.3 DISCUSSION OF RESULT

332 The result above were compared with results got using the normal DP method

333 And the step function method. It was discovered that less computation time and steps were
334 used in the current method (Chebyshev). It is clear that the zeros of the best approximating
335 polynomials in $[-1,1]$. The stability of the method is ensured since the region of stability $[-$
336 $1,1]$ contains zero. [22]

337 8 SUMMARY/CONCLUSION

338 Therefore the proper allocation of resources in manufacturing and distribution firm is of
339 paramount importance to the company and the society. There is therefore the need to obtain an
340 allocation that optimizes the returns of a company that puts the firm on a good footing. The act of
341 stocking to satisfy future demand has its associated costs. There is therefore the need to design a
342 model that minimizes the appropriate cost function that balances the total cost resulting from
343 overstocking or under-stocking. The Dynamic Programming approach has been explained and
344 applied to stock allocation environment. The Chebyshev polynomial approximation has been
345 employed on the resultant polynomial function from the allocations. An algorithm has been
346 adopted while a hybrid model is developed. The model is applied to allocation of stock to
347 obtain an optimum allocation.

348 REFERENCES

- 349 [1] Chikwendu, C. R. (2012) Elementary Operation Research with Applications. Nnewi. God's Time
350 publishing concept. Pp 9-11
- 351 [2] Berovic D. P and Vinters, R.B (2004): The Application of Dynamic Programming to Optimal
352 Inventory Control. IEEE Transactions on automatic control. Vol. 49, No. 45
- 353 [3] Bates J. J. (2007): A Dynamic Inventory/Maintenance Model. University of Florida Scholars
354 Common, Graduate School Thesis and dissertation. <http://scholarcommon.uf.edu/etd/619>. Pp
355 4-6

- 356 [4] Kutaglu, E. (2007) An Inventory Sharing and Allocation Method for a multi-location Service
357 parts logistic network with time based service levels. Research in OR and Industrial
358 Engineering University of Texas. Austin. Pp 85-94
- 359 [5] Aldasoro, R. et al (2011) A hybrid model based on Dynamic Programming neural networks and
360 surrogate value for inventory optimization application. Journal of Operations Research
361 Society. 50, pp 85-94
- 362 [6] Mpwanya, M.F. (2005) Inventory management as a determinant for improvement of
363 customer service. A dissertation to the Department of Business Administration, University
364 of Pretoria, South Africa. Pp 7-9
- 365 [7] Song and Zipkin (1993). Inventory control in a Fluctuating Demand Environment, Operation
366 Research 41(2), Pp. 351 – 370.
- 367 [8] Taha, H. A (2007). Operations Research. An Introduction 8th Edition London, Pearson
368 Education International prentice Hall. Pp 399-410
- 369 [9] Mersereau J.A. (2011) Information Sensitive Replenishment when Inventory Records are
370 Inaccurate Kenan-Flagler Business School, University of North Carolina. Pp 4-5
- 371 [10] Chan and Plambeck (2008). Dynamic Inventory Management with Learning about the
372 Demand Distribution and Substitution Probability. manufacturing Service Operations
373 Management 10(2), 236 – 256.
- 374 [11] Eddy R. E. (2004). What is Dynamic Programming. Nature publishing group. [http:](http://www.nature.com/nature)
375 [//www.nature.com/nature](http://www.nature.com/nature) biotechnology. Pp 2-3
- 376 [12] Chikwudu C.R. and Emenonye, C. E (2013). Chebyshev's Polynomial Approximation of an N
377 - Warehouse Stock Allocation Model (Dynamic Programming Approach). International
378 Journal of Engineering Innovation Research (IJEIIF). 41-45
- 379 [13] Righmic G (2004). Dynamic Algorithm for Elementary Shortest Path Problem with Resource
380 Consternates. Discrete Mathematics University of Inlaw 12247 – 12249.
- 381 [14] Guglieno, M.C (2008) Chebyshev polynomial approximation to approximate Differential
382 Equations. Brunel University pp 1-3
- 383 [15] Milovanic G and Mastroianni G (2008) Basic theory of interpolation (orthogonal polynomials and
384 weighted applications) <http://www.springer.com> pp 75-77

385 [16] Makajuma, M(2006)Evaluation of functions. Numerical Recipe in the art of scientific
386 computing, Cambridge University press. Pp 190-192

387 [17] Makajuma,M(2006) Notes on Neoclassical Growth Model: Value function.Heration
388 +Chebyshev Regression. Pp 1-2

389 [18] Cai, Y and Judd, L.K (2010) Stable and Efficient Computational methods for Dynamic
390 Programming.Journal of the European Economic Association. 8(2-3) 626 – 634

391 [19] Rowland H.J and Jenson, J.A(1975) Method of computation. The linear space approach) Illinois,
392 Scott,Foreman and company. pp 254-278

393 [20]Santaaulalia-Llopis, R(2014) Function Approximation, Quantitative Macoeconomics(Lecture
394 notes)Washington University,St Louis. Pp 2-5

395 [21] Caporale, G.M(2008)Chebyshev polynomial approximation of Partial Differential
396 Equations.Brunnel University and London Metropolitan University,Mario Cerrato.
397 University of Glasgow. Pp 4-6

398 [22] Germund, D(1974)Numerical methods.New Jersey.Prentice-Hall Inc, pp 375-377

399

400

401

402

403

404

405