

DYNAMIC PROGRAMMING METHOD FOR OPTIMIZING STOCK
ALLOCATION USING CHEBYSHEV POLYNOMIAL
APPROXIMATION

ABSTRACT

Stock allocation is a system used to ensure that goods and services reach the ultimate users through efficient stocking in warehouses close to the consumers . The dire need for optimum distribution of goods to both retailers and consumers has cost a reasonable drift from ordinary allocation to developing a mathematical model that ensures efficient allocation of goods and services. This paper presents a method of optimizing stock allocation using the Chebyshev polynomial approximation of an n- warehouse inventory model .The features of Chebyshev polynomial are enumerated and used on stock allocation environment to obtain good approximation that would ensure high yield to the firms. It has been shown that this method is efficient, stable and provides quick access to obtaining optimal allocation. The model of the method is given, relevant algorithm and theorems are included while illustrative examples are provided.

.Keywords: Dynamic Programming, Stock Control, Stock allocation,Optimal decision. Chebyshev polynomial, polynomial approximation.

1.0 MOTIVATION

The quest for optimal distribution of goods to both retailers and consumers has given rise to reasonable drift from ordinary allocation to developing a mathematical model that enhances steady and efficient allocation.

30 Stock is the supply of goods for sale kept in the store for business. The
 31 growing global economy has caused a dramatic shift in stock management in
 32 the twentieth century from a mere approach to scientific approach. "One of the
 33 related problems is that as the complexity and specialization in an organization
 34 increases, it becomes more difficult to allocate rationally and reasonably the
 35 available resources to various sections of the organization" [1]. The proper
 36 allocation of resources in both manufacturing and distribution industries is of
 37 paramount significance to the society since the chain of distribution is complete
 38 only when the goods get to consumers. The allocation process that minimizes
 39 cost and maximizes profit is always the desire of every organization. The need
 40 to obtain such a process is the aim of this work. In this regard, a lot of methods
 41 in Operations Research are available. Methods such as linear programming
 42 model; integer programming, goal programming, dynamic programming models
 43 etc can be used to ascertain optimum allocation of goods. This work will use the
 44 dynamic programming model to obtain an optimum allocation of resources that
 45 would provide the solution to the desire of any organization.

46 The dynamic programming is a linear optimization method that obtains
 47 optimum solution of a multivariable problem by decomposition of the problem
 48 into sub problems [2]. Dynamic programming is an approach of optimizing
 49 multistage decision processes with a recursive equation. DP is based on the observation
 50 by Bellman (1957) that an optimal policy has the property that whatever the
 51 initial state and initial decision are, the remaining decisions must constitute an
 52 optimal policy with regard to the state resulting from the first decision." This
 53 reduces multi-period sequential decision problems to a sequence of one-period
 54 optimal control problems. We examine only finite horizon problems in this
 55 paper but the numerical ideas also apply to infinite-horizon problems. The
 56 recursive principle of DP implies that the solution can be characterized by value
 57 functions, $V_t(x)$, defined by the Bellman equation

$$58 \quad V_t(x) = \max_{a \in D_t(x)} U_t(x, a) + \beta E\{V_{t+1}(x+) \mid x, a\},$$

59

60
 61 for $t = 0, 1, \dots, T - 1$, where T is finite, x is the state and a is the
 62 action choice (x and a can be vectors), $U_t(x, a)$ is the utility function at
 63 time $t < T$, and $U_T(x) = V_T(x)$ is the terminal utility function,

β is the discount factor, $D_t(x)$ is a feasible set of a at time t, x is the (possibly random) state at time $t + 1$ if the state is x at time t and action a is chosen at time t , and $E\{\cdot\}$ is the expectation operator [14]. The different stages of the problem are linked such that the optimum feasible solution of each stage is guaranteed to be the optimum feasible solution for the entire problem. The dynamism of demand makes it necessary to keep goods in stock and the act of maintaining stock has its associated costs. The act of stocking goods to satisfy future demand gives rise to the problem of designing a very efficient allocation technique that minimizes cost and maximizes profit. This involves minimizing an appropriate cost function that balances the total cost resulting from overstocking or under-stocking. [4]. Dynamic Programming was first used by Richard Bellman in 1940 to describe a process of solving problems where one needs to find the best decisions one after the other [7]. This work uses the dynamic programming method to develop a stock allocation model that ensures optimum allocation of goods and services for maximum returns. If $r_i(Q_i)$ is the total return from the i th activity with the resource we seek to maximize.

$$R(Q_1, Q_2, \dots, Q_n) = r_1(Q_1) + r_2(Q_2) + \dots + r_n(Q_n)$$

Given that $Q = Q_i \geq 0, i = 1, 2, \dots,$

2 INTRODUCTION

A polynomial is an expression constructed from variables and constants using the operations of addition, multiplication, subtraction and division. It is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_n x + a_o$ where $a_n \neq o$. A polynomial function is a function that can be defined by evaluating a polynomial. A function f of one argument is called a polynomial function if it satisfies $f(x) = a_n x^n + a_n x^{n-1} + a_{n-2} x^{n-2} + \dots + ax + a_o$ for all argument x where n is a non-negative integer and $a_i / i = 0, 1, 2 \dots, n$ are constant coefficients. [2]. Polynomial functions are those functions whose values can be calculated by putting the value of the independent variable in a polynomial. If f is continuous on $[a, b]$, then a best approximation to f from P_n exists [9] Approximation is the act of estimating a number or an amount. The approximate is almost correct and may not be exact [1]. It is a number which is taken as a close estimate of another number or the method of finding such approximate number [4]. Approximation arises due to the difficulty in obtaining the exact area/volume or dimension of some objects. In addition, the inability of man to foretell the future accurately as a result of his fallibility gives birth to estimation of quantities.

The general approximation problem states that if f is an element and S , a subset of a normed linear space X , then approximation theory seeks to find

104 an elements of S which is as close as possible to f ; i.e. to find an element S^* of
 105 S such that $\|f - S^*\| \leq \|f - S\| \quad \forall s \in S$ S^* is called the best
 106 approximation of f from S relative to the given norm. Weierstrass
 107 approximation theorem states that if f is a continuous function on an
 108 interval $[a; b]$ and $\varepsilon > 0$ is given, then there exists a polynomial $p(x)$
 109 such that $\sup_{x \in [a, b]} |f(x) - p(x)| \leq \varepsilon$

110 $x \in [a, b]$

111 Determination of polynomial coefficients requires solution of complicated
 112 system of equations. It is possible to avoid such problems by using orthogonal
 113 Chebyshev polynomials. This is a method of approximation where the
 114 maximum difference between value of function and value calculated from
 115 polynomials is minimized [6].

116 The Chebyshev polynomial $T_n(x)$ is a basis function and one of the spectral
 117 general class of polynomials referred to as orthogonal polynomial. According to
 118 [15], the approximation of the function f requires the value of f at the zeros of
 119 the $(n+1)^{\text{st}}$ Chebyshev polynomial. The Chebyshev polynomial T_n and U_n are
 120 polynomials of degree n and the sequence of Chebyshev polynomials of either
 121 kind composes a polynomial sequence. They are polynomials with the largest
 122 possible leading coefficient but subject to the condition that their absolute value
 123 is bounded on the interval by 1. One of the unique properties of the Chebyshev
 124 polynomial is that on the interval $-1 \leq x \leq 1$ all the extrema have values
 125 that are either -1 or 1. Thus these polynomials have only two finite critical
 126 values. The Chebyshev approximation was chosen because of its
 127 superior convergence properties).

128

129 3 The Chebyshev's Polynomial Approximation

130 3.1 Chebyshev's polynomial is a sequence of orthogonal polynomials which
 131 are related to De-moivre's formula and which can be defined recursively. The
 132 Chebyshev's polynomials T_n are polynomials of degree n . The Chebyshev's
 133 polynomial of the first kind are defined by the recursive relation.

$$134 \quad T_0(x) = 1$$

$$135 \quad T_1(x) = x$$

$$136 \quad \vdots \quad \vdots$$

$$137 \quad T_{n+1}(x) = 2x T_n(x) - T_{n-1}(x). \quad [4]$$

138 The Chebyshev's approximation in particular states that:

139 If P_n is the collection of all polynomials whose degree is at most n and f
 140 be a continuous function on the interval $[a, b]$. The polynomial P is said to be
 141 the best approximation to f from p_n if $P \in P_n$ and

$$142 \quad \max_{x \in [a, b]} |f(x) - P(x)| \leq \max_{x \in [a, b]} |f(x) - q(x)| \quad \forall q \in P_n \quad [5].$$

143 3.2 Properties of Chebyshev's Polynomial

144 i. Recursion formula; $T_0(x) = 1$, $T_1(x) = x = xT_0(x)$,

$$145 \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \geq I$$

146 ii. The leading coefficient is 2^{n-1} for $n \geq 1$ and 1 for $n = 0$

147 iii. Symmetric property; $T_n(-x) = (-1)^n T_n(x)$

148 iv. $T_n(x)$ has n – zeros in $[-1, 1]$ given by $x_k = \cos\left(\frac{2k+1}{n} \pi\right)$,

$$149 \quad k = 0, 1, \dots, n-1.$$

150 v. Orthogonality property; Set $(f, g) = \int_{-1}^1 f(x) g(x) (1-x^2)^{-\frac{1}{2}} dx$, then

$$151 \quad (T_i, T_j) = \begin{cases} 0 & \text{if } i \neq j \\ \frac{1}{2} \pi & \text{if } i = j \neq 0 \\ \pi & \text{if } i = j = 0 \end{cases} \quad \text{i.e continuous case}$$

152 $(f, g) = \sum_{k=0}^n f(x_k) g(x_k)$ where $\{x_k\}$ are the zeros of $T_{m+1}^{(x)}$

153 Then for $0 \leq i \leq m, 0 \leq j \leq m, (T_i, T_j) =$

$$154 \quad \begin{cases} 0 & \text{if } i \neq j \\ \frac{1}{2} m + 1 & \text{if } i = j \neq 0 \\ m + 1 & \text{if } i = j = 0 \end{cases}$$

155 (Discrete case)

156 vi. Minimax property; Of all n th degree polynomials with leading
 157 coefficient 1, $2^{1-n} T_n$ has the smallest maximum norm in $[-1, 1]$. The
 158 value of its maximum norm is 2^{1-n} .

159 The strength of the Chebyshev polynomial approximation lies on the
 160 fact that it has a recursive properties which makes it adapt to the
 161 dynamic programming method and stock allocation processes. It is
 162 an orthogonal polynomial whose extrema lie in the interval $[-1, 1]$

163

164 4 REVIEW OF RELATED LITERATURE

165 Lots of research have been done in this area and a few are stated here.
 166 Stock allocation is an important part of any manufacturing organization and the
 167 availability of goods as at and when due is a sign of preparedness and efficient
 168 stock management which retain customers [11]. Optimal stock allocation
 169 policy generally require comprehensive knowledge of the nature of
 170 demand of goods in an environment[4]. Dynamic Programming is one of the
 171 numerous linear optimization methods. It is a method for solving complex
 172 problems by breaking it down into simpler sub-problems [3]. It determines the
 173 optimum solution of a multivariable problem by decomposing it into stages
 174 with each stage comprising a single variable sub-problem. It is a recursive
 175 equation that links the different stages of the problem in a manner that
 176 guarantees that the optimal feasible solution of each stage is also optimal
 177 and feasible for the entire problem [5]. According to [6], the work of Bellman
 178 implies that the process refers to the act of supplying a decision through
 179 breaking down the problem into a sequence of decision steps which is done by
 180 defining a sequence of value function $v_1, v_2, \dots, v_n$ with an argument y
 181 representing the state of the system at times i from 1 to n . Dynamic
 182 programming is guaranteed to give a mathematically optimal solution and the
 183 equations for the stages are written as follows:

184 Let $f_i(x)$ be the shortest distance to node x_i at stage i ;

185 Define $d(x_{i-1}, x_i)$ as the distance from node x_{i-1} to x_i .

186 The f_i is computed from f_{i-1} , by the following recursive equation

$$187 \quad f_i(x_i) = \min_{\text{All feasible } ((x_{i-1}, x_i) \text{ nodes})} \{ d(x_{i-1}, x_i) + f_{i-1}(x_{i-1}) \}, i = 1, 2, 3 \dots \quad [8]$$

188 The computation in dynamic programming is done recursively so that the
 189 optimum solution of one sub-problem is used as an input to the next sub-
 190 program and by the time the last sub-program is solved, the optimum solution
 191 for the entire problem is ascertained [10].

192 The Chebyshev polynomial of degree n is denoted $T_n(x)$, and is given by
 193 the explicit formula $T_n(x) = \cos(n \arccos x)$

This may look trigonometric at first glance (and there is in fact a close relation between the Chebyshev polynomials and the discrete Fourier transform); however can be combined with trigonometric identities to yield explicit expressions for

$$T_n(x)$$

$$T_0(x) = 1$$

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

.....

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

The Chebyshev polynomials are orthogonal in the interval $[-1; 1]$ over a weight $(1 - x^2)^{-1/2}$. In particular, the polynomial $T_n(x)$ has n -zeros in the interval $[-1; 1]$ and they are located at the points

$$x = \cos(\pi(k - 1/2)/n) \quad k = 1, 2, \dots, n$$

In this same interval there are $n + 1$ extrema (maxima and minima), located at

$$x = \cos(\pi k/n) \quad k = 0, 1, \dots, n$$

At all of the maxima $T_n(x) = 1$, while at all of the minima $T_n(x) = -1$; it is precisely this property that makes the Chebyshev polynomials so useful in polynomial approximation of functions. The Chebyshev polynomial satisfies the discrete and continuous orthogonal relations.

5 ALGORITHM/THEOREM

5.1 ALGORITHM [9]

The Remez algorithm is a successive approximation technique for computing the coefficients of the polynomial in P_n which provides the best Chebyshev approximation to a given continuous function f . The general approach is to find the best approximation to f on a set of $n+2$ points in such a manner that the successive best approximation on the finite sets approaches the best approximation over the whole interval.

Before stating the algorithm the following assertions /theorems will be stated as they preclude it.

- i). Let $x_0 < x_1 < \dots < x_{n+1}$ be $n + 2$ distinct numbers. Then the system of equations.

229 $f(x_k) - \sum_{j=0}^n a_j x_k^j = (-1)^k \lambda$ $k = 0, 1, 2, \dots, n+1$ has a unique
 230 solution in $a_0, a_1, a_2, \dots, a_n, \lambda$.

231 ii). Suppose $f \in C[a, b]$, f' exists on (c, b) and $a_1, < a_2 < \dots < a_m$ are
 232 points of $[a, b]$.

233 If $(-1)^k f(a_k) \geq 0, 1 \leq k \leq m$, then f has at least $m-1$ zeros on
 234 $[a_1, a_m]$

235 iii). Suppose $T = \{x_0, x_1, x_2, \dots, x_{n+1}\}$ is a set of $n+2$ points indexed, so that
 236 $\{x_0 < x_1 < \dots < x_{n+1}$ and f is a function defined on T .

237 Let $a_0, a_1, a_2, \dots, a_n, \lambda$ be the unique solution to $f(x_k) - \sum_{j=0}^n a_j x_k^j =$
 238 $(-1)^k \lambda$, and define p by $p(x) = \sum_{j=0}^n a_j x^j$

239 Then P is unique best approximation to f on the set T .

240 iv). Let f be continuous on $[a, b]$, and let $T = \{x_0, \dots, x_{n+1}\}$ be a set of $n+2$
 241 distinct points in $[a, b]$ indexed so that $x_0 < x_1 < \dots < x_{n+1}$.

242 Suppose $p \in P_n$ and $f - p$ alternates sign at the points $x_0, x_1, \dots, x_{n+1}$.

243 Then

$$E_n(f) \geq E_{n,T}^{(f)} \geq \min_{0 \leq j \leq n+1} |f(x_j) - p(x_j)|$$

244 The algorithm now proceeds as follows

245 i) Choose a set $T_0 = \{x_0^0, x_1^0, \dots, x_{n+1}^0\}$ of $n+1$ points in $[a, b]$

246 ii) Solve the system of equations

$$f(x_k^0) - \sum_{j=0}^n a_j^0 (x_k^0)^j = (-1)^k \lambda_0, \quad k = 0, 1, \dots, n+1$$

247 iii) For $a_0^0, a_1^0, \dots, a_n^0$ and λ_0 (if $\lambda_0 \neq 0$ choose a new set of points for T_0),

248 Now proceed inductively having chosen a set of points

249 $T_m = \{x_0^m, x_1^m, \dots, x_{n+1}^m\},$

250 Let $\lambda_m = f(x_0^m) - P_m(x_0^m),$

251 Let P_m be the best approximation of f on T_m and

252 Then

253 Choose a new set

254 $T_{m+1} = \{x_0^{m+1}, x_1^{m+1} \dots x_{n+1}^m\}$ so that the following conditions are
 255 satisfied.

- 256 i) $x_0^{m+1} < x_1^{m+1} < \dots < x_{n+1}^{m+1}$
 257 ii) $|f(x_k^{m+1}) - p_m(x_k^{m+1})| = \|f - P_m\|$ for some K ,
 258 iii) $f - P_m$ alternates sign at the points of T_{n+1} and
 259 iv) $|f(x_k^{m+1}) - p_m(x_k^{m+1})| \geq |\lambda_m|$, $k = 0, 1, \dots, n+1$
 260 if $|\lambda_m| = \|f - p_n\|$, then P_m is the best approximation.

261

262 5.2 THEOREMS

263 In problems of approximation, the convergence of function play important
 264 role. This is because the approximate value must be very close (almost exactly) to
 265 the actual solution. The uniqueness of the solution is also important. This section
 266 will consider theorems that assert the quality, unbiasedness and consistency of the
 267 approximates if it exists.

268 5.2.1 . Let $f \in [-1, 1]$ and let S_n be the best approximate to f from P_n

269 relative to $\|f\| = \left[\int_{-1}^1 \frac{f^2(x) dx}{\sqrt{1-x^2}} \right]^{\frac{1}{2}}$, then

$$270 \lim_{n \rightarrow \infty} \|f - S_n\| = 0 \quad [6]$$

271 Proof

272 Let $\| \cdot \|$ and $\| \cdot \|_0$ be the norms defined by $\|f\|$ above and

$$273 \|f\| = \left[\int_{-\pi}^{\pi} \frac{f^2(x) dx}{\sqrt{1-x^2}} \right]^{\frac{1}{2}} \text{ Respectively.}$$

$$274 \text{ Let } S_n(\theta) = \frac{a_0}{2} + \sum_{j=1}^n a_j \cos j\theta$$

275 Making, the change of variable $x = \cos\theta$ and using $f(x) = \frac{a_0 T_0(x)}{2} -$

$$276 \sum_{j=1}^n a_j T_j(x) = g(\theta) - \frac{a_0}{2} - \sum_{j=1}^n a_j \cos j\theta$$

277 gives

$$278 \|f - S_n\|^2 = \int_{-1}^1 \frac{|f(x) - S_n(x)|^2 dx}{\sqrt{1-x^2}} = \int_0^\pi [g(\theta) - S_n(\theta)]^2 d\theta$$

$$279 = \frac{1}{2} \|g - S_n\|_0^2$$

280 Recall that if $a_j = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos jx dx$ and

$$281 \quad b_j = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin jx dx \quad j = 1, 2, \dots, n$$

282 and $S_n(x) = T^{(x)} = \frac{a_0}{2} + \sum_{j=1}^n (a_j \cos jx + b_j \sin jx)$, then

$$\lim_{n \rightarrow \infty} \|f - S_n\| = 0$$

283 Hence $\frac{1}{2} \|g - S_n\|_0^2 = o$

284 Then the polynomial P^* whose coefficient is c^* is a best approximation of f .

285 5.2.2. If $f \in [a, b]$, then the best chebyshev approximation to f from

286 P_n is unique.[3]

287

288 Proof Let p_1 and p_2 be best approximations to f from P_n , $P_1, P_2 \in P_n$

289 Now define $P_0 = \frac{(p_1 + p_2)}{2}$ and

290 Let $E = \|f - P_1\| = \|f - p_2\|$

$$291 \quad \text{Then } E \leq \|f - P_0\| = \left\| \frac{f - p_1}{2} + \frac{(f - p_2)}{2} \right\| \leq \left\| \frac{f - p_1}{2} \right\| + \left\| \frac{f - p_2}{2} \right\| = E$$

292 Thus P_0 is also a best approximation to f .

293 Let $x_0 < x_1 < \dots < x_{n+1}$ be points satisfying the equation.

$$f(x_j) - P_0(x_j) = (-1)^j \delta E, j = 0, 1, \dots, n+1$$

294 Then for $j = 0, 1, \dots, n+1$

$$E = |f(x_j) - P_0(x_j)| = \left| \frac{f(x_j) - P_1(x_j)}{2} + \frac{f(x_j) - P_2(x_j)}{2} \right|$$

295 But $|f(x_j) - p_1(x_j)|$ and $|f(x_j) - p_2(x_j)|$ are both less than or equal to E

$$296 \quad \Rightarrow f(x_j) - P(x_j) = f(x_j) - P_2(x_j) = (-1)^j \sqrt{E}$$

297 Thus

$$298 \quad P_1(x_j) = P_2(x_j), j = 0, 1, \dots, n+1$$

299 $\Rightarrow P_1 - P_2$ has $n+2$ zeros hence

$$P_1 = P_2$$

300 5.2.3 5.2.3. If f is continuous on $[a, b]$, then a best approximation to f

301 from P_n exists [9]

302 6 MATHEMATICAL MODEL

303 Let Q be a certain quantity of resource that will be distributed among n -
 304 number of depots. Let R be the return which depends on the quantity of resource
 305 allotted to the depots.

306 The objective is to optimize total return

307 If $r_i(Q_i)$ denotes the return from the i th activity with resource Q_i , then the total
 308 return is given as

$$309 \quad R(Q_1, Q_2, \dots, Q_n) = r_1(Q_1) + r_2(Q_2) + \dots + r_n(Q_n) \quad (1)$$

310 The quantity of resource Q is limited hence this gives rise to the constant.

$$311 \quad Q = (Q_1 + Q_2 + \dots + Q_n, Q_i \geq 0, i = 1, 2, \dots, n) \quad (2)$$

312 Hence the problem is therefore stated as:

$$313 \quad \text{Optimize } R(Q_1, Q_2, \dots, Q_n) = R(Q_1) + R(Q_2) + \dots + R(Q_n)$$

$$314 \quad \text{Given that } Q = Q_1 + Q_2 + \dots + Q_n, Q_i \geq 0, i = 1, 2, \dots, n$$

$$315 \quad \text{If } f_n(Q) = \max_{0 \leq Q_1 \leq Q_n} [R(Q_1, Q_2, \dots, Q_n)]$$

$$316 \quad = \max_{0 \leq Q_1 \leq Q_n} [r_1(Q_1) + r_2(Q_2) + \dots + r_n(Q_n)] \quad (3)$$

317 $f_n(Q)$ is the maximum return from the distribution of the resource Q to the n
 318 activities/depots.

319 To approximate $f_n(Q)$, use a corresponding polynomial function p_n of
 320 $f_n(Q)$ from the allocation table. i.e

$$321 \quad P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0, \quad (4)$$

322 Choose another polynomial function $P_i(x)$, $i = 1, 2, \dots, n-1$ in reference to the
 323 Chebyshev polynomial $T_n(x)$ to approximate $f_n(x)$

324 where $p_i(x) \leq P_n(x)$ and $\deg p_i \leq \deg P_n(x)$ and

325 where $f(x) - T(x) = p(x)$

$$326 \quad \text{Find } F(x) = f(x) - p_i(x) \text{ where } |f(x) - p_i(x)| < \varepsilon \quad (5)$$

327 Obtain $F'(x)$ if it exists and then

328 Calculate k_i , the zeros of $F'(x)$ where $k_i \in [-1, 1]$

329 Proceed to find $\|F\| = \max [F(a = k_1), F(k_2), \dots, F(k_{n-1}), F(k_n = b)]$

$$330 \quad k \in [a, b]$$

$$331 \quad = \max [F(-1), F(k_2), \dots, F(k_{n-1}), F(1)] = 1 \quad (6)$$

$$332 \quad k \in [-1, 1]$$

333 Then test if k form an oscillation set in F .

334 If it does, then p_i approximates f .

335 For the best approximation, find $\|f - p_m\|$ where $f - p_m = \gamma_m$ (7)

336 and where $p_m \in P_n$

337 If $|\gamma_m| \leq \|f - p_m\|$, then p_m is the best approximation of f if

338 $\max_{x \in [-1,1]} |f(x) - p(x)| < \max_{x \in [-1,1]} |f(x) - q(x)|$ where $q(x) \in P_n(x)$

339 $x \in [-1,1]$ $x \in [-1,1]$

340 7 ILLUSTRATION

341 7.1 The table below is the stock allocation of a company to their six warehouses. Obtain the
342 best approximation of allocation to ensure profitability of the company.

Warehouse (x)	1	2	3	4	5	6
Allocation (000 units) $f(x)$	130	293	8	556	943	43

343

344 The corresponding function to the allocation table above from geometry or cubic
345 regression is $f(x) = 4x^3 + 2x^2 + x + 1$.

346 Now, $f(x) = 4x^3 + 2x^2 + x + 1$,

347 Use $p(x) = 2x^2 + 4x + 1$ as an approximate of $f(x)$ i.e $f(x) - T_3(x) = p(x)$

$$\therefore F(x) = f(x) - p(x)$$

$$F(x) = 4x^3 + 2x^2 + x + 1 - (2x^2 + 4x + 1 +$$

$$= 4x^3 - 3x$$

$$F'(x) = 12x^2 - 3 = 0$$

$$= 3(4x^2 - 1)$$

$$4x^2 - 1 = 0$$

$$\Rightarrow x^2 = \pm \frac{1}{4}$$

348 $x = \pm 0.5$ i.e. $\lambda = \pm 0.5 \in [-1,1]$

349 i.e F' is zero at ± 0.5

350 $\|f\| = \max [|f(-1)|, |F(-0.5)|, |F(0.5)|, |F(1)|] = 1$

351 We note that $-1, -0.5, 0.5$ and 1 form an oscillation, set of four points

352 $\|p\|$ is the best polynomial approximate for f from P_2 .

353 To show that p is the best approximation, we try two other approximations in P_n i.e.

354 $P_a(x) = 4x^2 + 2x + 1$ and $P_b(x) = x^2 + 2x + 1$

355 Now

$$f(x) - P_a(x) = F_1(x)$$

$$F_a(x) = 4x^3 - 2x^2 - x$$

$$F'_a(x) = 12x^2 - 4x - 1$$

$$F'_a(x) = 0$$

$$\Rightarrow \lambda_1 = -0.1667 \text{ and } 0.5 \in [-1, 1]$$

$$F_b(x) = f - P_b(x) = 4x^3 + x^2 - x$$

$$= F'_b(x) = 12x^2 + 2x - 1$$

$$F'_b(x) \text{ is zero at } -0.342 \text{ and } 0.217$$

$$\lambda_2 = -0.343 \text{ and } 0.217 \in [-1, 1]$$

358 The zeros of F'_a and F'_b do not form oscillation sets though they are all approximations of

359 f . Hence $\lambda \pm 0.5$ i.e p is the best approximation. Furthermore $|\lambda| = \pm 0.5$ applied in

360 $\|f(x) - p(x)\|$ shows that

$$\max_{x \in [-1, 1]} |f(x) - P(x)| < \max_{x \in (-1, 1)} |f(x) - P_a(x)| < \max_{x \in (-1, 1)} |f(x) - P_b(x)|$$

361 For $P \in P_n$, $P_a \in P_n$ and $P_b \in P_n$

362 $\square$ P is the best approximation to f from P_n relative to the uniform (or chebyshev) norm. It is
363 unique.

364 7.2. The supply of a firm to her eight depots are given below. Use the
365 record to obtain an optimal approximate for the firm.

366	X	1	2	3	4	5	6	7	8
367	Y	8	128	648	2048	5000	10368	19208	37768

368

369 Solution

370 The corresponding functional equation for the table above is $8x^4$

371 The equation is quartic hence use the Chebyshev polynomial $T_4(x)$. i.e

$$372 T_4(x) = 4x^4 - 4x^2 + 1 = p_1(x)$$

$$373 8x^4 - (4x^4 + 4x^2 - 1) = 4x^4 - 4x^2 + 1$$

$$374 F(x) = 4x^4 - 4x^2 + 1$$

$$375 F'(x) = 16x^3 - 8x = 8x(2x^2 - 1)$$

$$376 X = 0, x = \pm \sqrt{1/2}$$

377 The zeros of F' is the set $K = (-1, -\sqrt{1/2}, 0, \sqrt{1/2}, 1)$ and

$$378 (-\sqrt{1/2}, 0, \sqrt{1/2},) \subseteq [-1, 1]$$

$$\|F\| = \max \left[|f(-1)|, |f(-\sqrt{\frac{1}{2}})|, |f(0)|, \left|f\left(\sqrt{\frac{1}{2}}\right)\right|, |f(1)| \right] = 1$$

Also $-1, -\sqrt{1/2}, 0, \sqrt{1/2}$ and 1 form an oscillation set.

P is therefore the best approximation of f .

5 SUMMARY/CONCLUSION

The proper allocation of resources in manufacturing and distribution firm is of paramount importance to the company and the society. The ability to obtain an allocation that optimizes the returns of a company puts the firm on a good footing. The act of stocking to satisfy future demand has its associated costs. There is therefore the need to design a model that minimizes the appropriate cost function that balances the total cost resulting from overstocking or under-stocking. The Dynamic Programming approach has been explained and applied to stock allocation environment. The Chebyshev polynomial approximation has been employed on the resultant polynomial function from the allocations. An algorithm has been adopted while a hybrid model is developed. The model is applied to allocation of stock to obtain an optimum allocation.

REFERENCES

- [1] Chikwendu, C. R. (2012) Elementary Operation Research with Applications. Nnewi. God's Time publishing concept.
- [2] Berovic D. P and Vinters, R.B (2004): The Application of Dynamic Programming to Optimal Inventory Control. IEEE Transactions on automatic control. Vol. 49, No. 45
- [3] Bates J. J. (2007): A Dynamic Inventory/Maintenance Model. University of Florida Scholars Common, Graduate School Thesis and dissertation. <http://scholarcommon.uf.edu/etd/619>.
- [4] Kutaglu, E. (2007) An Inventory Sharing and Allocation Method for a multi-location Service parts logistic network with time based service levels. Research in OR and Industrial Engineering University of Texas. Austin.

- 408 [5] Pressman, E. and Setlu S. P. (2004) Stochastic Inventory Models with
409 Continuous and Poisson Demand (Working Page). The University of
410 Texas at Dallas RichardsonT. X.
- 411 [6] Ayan, G. W. (1998) Purchasing handbook. Fourth Edition New York.
412 McGraw Hill Book Company.
- 413 [7] Song andZipkin (1993). Inventory control in a Fluctuating Demand
414 Environment, Operation Research 41(2), Pp. 351 – 370.
- 415 [8] Taha, H. A (2007).Operations Research.An Introduction 8th Edition
416 London, Pearson Education International prentice Hall.
- 417 [9] Mersereau J.A. (2011) Information Sensitive Replenishment when
418 Inventory Records are Inaccurate Kenan-Flagler Business School,
419 University of North Carolina. ajm@unc.edu.
- 420 [10] Chan and Plambeck (2008). Dynamic Inventory Management with
421 Learning about the Demand Distribution and Substitution
422 Probability.Manufacturing Service Operations Management 10(2), 236 –
423 256.
- 424 [11] Eddy R. E. (2004). What is Dynamic Programming.Nature publishing
425 group.[http: //www.nature.com/naturebiotechnology](http://www.nature.com/naturebiotechnology).
- 426 [12] Chikwudu C.R. and Emenonye, C. E (2013). Chebyshev’s Polynomial
427 Approximation of an N - Warehouse Stock Allocation Model (Dynamic
428 Programming Approach). International Journal of Engineering Innovation
429 Research (IJEIIF).
- 430 [13] Righmic G (2004). Dynamic Algorithm for Elementary Shortest Path
431 Problem with Resource Consternates. Discrete Mathematics University of
432 Inlaw 17247 – 12249.
- 433 [14] Guglieno, M.C (2008) Chebyshev polynomial approximation to
434 approximate Differential Equations. Brunnel University
- 435 [15] Milovanic G and Mastrion G (2008) Basic theory of
436 interpolation(orthogonal polynomials and weighted applications)
437 <http://www.springer.com>
- 438 [16] Makajuma, M(2006)Evaluation of functions. Numerical Recipe in the
439 art of scientific computing. Cambridge University press

440 [17] Makajuma,M(2006) Notes on Neoclassical Growth Model: Value
441 function.Heration + Chebyshev Regression.

442 [18] Cai, Y and Judd, L.K 92010) Stable and Efficient Computational
443 methods for Dynamic Programming.Journal of the European
444 Economic Association. 8(2-3)626 - 634

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