$A \subseteq$ Sg-continuity in Topological ordered spaces

Abstract

Semi generalized closed set in a Topological space was introduced by P.Bhattacharya and B.K.Lahiri in 1987. A subset A of a topological space (X, τ) is a semi generalized closed (sg-closed) set if $scl(A) \subseteq U$ whenever $A \subseteq$ U and U is semi open in (X, τ).Some authors introduced the notion of sg-continuity in topological spaces. The same notion can be extended to topological ordered spaces. A topological ordered space is a topological space together with a partial order. In this paper, we introduce and study the notion of semi generalized increasing continuous function (sgi-continuous function), semi generalized decreasing continuous function (sgd-continuous function) and semi generalized balanced continuous function (sgb-continuous function) and study the relationships between them.

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Key words: topological ordered space, increasing set, decreasing set, balanced set and semi generalized closed set.

1. Introduction

The study of topological ordered spaces (TOS) was introduced by L.Nachbin **[2]**. It is a triple $(X, \tau, \leq)$ where τ is a topology and $\leq$ is a partial order on X. Let $(X, \tau, \leq)$ be a TOS. For any $x \in X, [x, \rightarrow] = \{ y \in X / x \leq y \}$ and $[\leftarrow, x] = \{ y \in X / y \leq x \}$ .A subset A of a TOS $(X, \tau, \leq)$ is increasing if $A = i[A]$ and decreasing if $A = d[A]$ where $i[A] = \bigcup_{a \in A} [a, \rightarrow]$ and $d[A] = \bigcup_{a \in A} [\leftarrow, a]$ .The complement of an increasing set is a decreasing set and vice versa. A subset of a TOS $(X, \tau, \leq)$ is balanced set if it is both increasing and decreasing.

2. Preliminaries

In the present paper (X, τ) represents a non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ), the closure is the intersection of all closed sets containing A and semi closure is the intersection of all semi closed sets containing A.They are denoted by $cl(A)$ and $scl(A)$.respectively.

Definition 2.1. A subset A of a topological space (X,τ) is a semi-open set **[5]** if $A \subseteq cl(\text{int}(A))$ and a *semi-closed* set if $\text{int}(cl(A)) \subseteq A$.

Definition 2.2. A subset A of a topological space (X,τ) is a sg-closed set **[1]** if $scl(A) \subseteq U$ whenever $A \subseteq$ U and U is semi-open in (X,τ).

Definition 2.3. A subset A of a topological ordered space $(X,\tau,\leq)$ is a sgi-closed (resp. sgd-closed, sgb-closed) set**[4]** if A is sg-closed and increasing (resp. decreasing, balanced).

3. Sg-continuity in Topological ordered spaces

We define new types of semi generalized continuous functions in topological ordered spaces. A function $f:(X,\tau_1)\to(Y,\tau_2)$ is sg-continuous [**1**] if $f^{-1}(V)$ is sg-closed whenever V is a closed set in Y.

The following notions are introduced in a topological ordered space.

Definition 3.1. A function $f:(X,\tau,\leq)\to(Y,\tau^1,\leq^1)$ is

(1) a semi generalized increasing continuous function (briefly sgi-continuous) if $f^{-1}(V)$ is a sgi-closed set in X whenever V is an i-closed set in Y.

(2) a semi generalized decreasing continuous function (briefly sgd-continuous) if $f^{-1}(V)$ is a sgd-closed set in X whenever V is a d-closed set in Y.

(3) a sg-balanced continuous function (briefly sgb-continuous) if $f^{-1}(V)$ is a sgb-closed set in X whenever V is a b-closed set in Y.

The following examples support the above definitions.

Example 3.2. Let $X=Y=\{a,b,c\}$, $\tau_8=\{\phi,X,\{a,b\}\}$, $\leq_3=\{(a,a),(b,b),(c,c),(a,b),(a,c)\}$ $\leq_4=\{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$.Then $(X,\tau_8,\leq_3)$ and $(Y,\tau_8,\leq_4)$ are topological ordered spaces. The i-closed sets in Y are ϕ,X and sgi-closed sets in X are $\phi,X,\{c\},\{b,c\}$. Define $f:X\to Y$ as $f(a)=b,\ f(b)=c$ and $f(c)=a$ then, f is a sgi-continuous function.

Example 3.3. Let $X=Y=\{a,b,c\}$, $\tau_7=\{\phi,X,\{a\},\{b\},\{b,c\},\{a,b\}\}$ $\tau_8=\{\phi,X,\{a,b\}\}$, $\leq_7=\{(a,a),(b,b),(c,c),(b,a)\}$, $\leq_8=\{(a,a),(b,b),(c,c)\}$, Then, $(X,\tau_7,\leq_7)$ and $(Y,\tau_8,\leq_8)$ are topological ordered spaces. The d-closed sets in Y are $\phi,X,\{c\}$ and sgd-closed sets in X are $\phi,X,\{c\},\{b,c\}$. Define $f:X\to Y$ as $f(a)=b,\ f(b)=a$ and $f(c)=c$ then, f is a sgd-continuous function.

Example 3.4. Let $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\tau_9 = \{\phi, X, \{b\}, \{c\}, \{b,c\}\}$ $\leq_1 = \{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)\}$, $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$ Then, $(X,\tau_8,\leq_5)$ and $(Y,\tau_9,\leq_1)$ are topological ordered spaces. The b-closed sets in Y are ϕ, X and sgb-closed sets in X are ϕ, X. Define $f: X \to Y$ as $f(a) = b,\ f(b) = c$ and $f(c) = a$. Then, f is a sgb-continuous function.

4. Independency of the functions

Remark 4.1: The notions sgi-continuity and sgd-continuity are independent as seen in the following example.

Example 4.2: Let $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_3 = \{(a,a),(b,b),(c,c),(a,b),(a,c)\}$ $\leq_4 = \{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$.Then, $(X,\tau_8,\leq_3)$ and $(Y,\tau_8,\leq_4)$ are topological ordered spaces. The i-closed sets in Y are ϕ, X and sgi-closed sets in X are $\phi, X, \{c\}, \{b,c\}$. Define $f: X \to Y$ as $f(a) = a,\ f(b) = c$ and $f(c) = b$ then f is a **sgi-continuous function**. The d-closed sets in Y are $\phi, X, \{c\}$ and the sgd-closed sets in X are $\phi, X, \{a,c\}$.Then f is **not a sgd-continuous function**.

If we take $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_4 = \{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$, $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$.Then, $(X,\tau_8,\leq_4)$ and $(Y,\tau_8,\leq_5)$ are topological ordered spaces. The i-closed sets in Y are $\phi, X, \{c\}$ and sgi-closed sets in X are ϕ, X. Define $f: X \to Y$ as $f(a) = a,\ f(b) = a$ and $f(c) = c$. Then, f is a **not a sgi-continuous function**. The d-closed sets in Y are ϕ, X and sgd-closed sets in X are $\phi, X, \{a,c\}, \{c\}$.Then, f is a **sgd-continuous function**.

Remark 4.3: The notions sgi-continuity and sgb-continuity are independent as seen in the following example.

Example 4.4: In the topological ordered spaces $(X,\tau_8,\leq_4)$ and $(Y,\tau_8,\leq_5)$ where $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_4 = \{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$ and $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$, the i-closed sets in Y are $\phi, X, \{c\}$ and sgi-closed sets in X are ϕ, X. Define $f: X \to Y$ as $f(a) = a,\ f(b) = a$ and $f(c) = c$. Then, f is a **not a sgi-continuous function**. The b-closed sets in Y are ϕ, X and sgb-closed sets in X are ϕ, X.Then, f is **a sgb-continuous function.**

On the other hand, in the spaces $(X,\tau_{11},\leq_7)$ and $(Y,\tau_{11},\leq_8)$ where $X=Y=\{a,b,c\}$, $\tau_{11}=\{\phi,X,\{c\},\{c,b\}\}$, $\leq_7=\{(a,a),(b,b),(c,c),(b,a)\}$ and $\leq_8=\{(a,a),(b,b),(c,c)\}$, the i-closed sets in Y are $\phi,X,\{a\},\{a,b\}$ and sgi-closed sets in X are $\phi,X,\{a\},\{a,b\}$. Define $f:X\to Y$ as $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then, f is a sgi**-continuous function**. The b-closed sets in Y are $\phi,X,\{a\},\{a,b\}$ and sgb-closed sets in X are $\phi,X,\{a,b\}$.Then, f is **not a sgb-continuous function**.

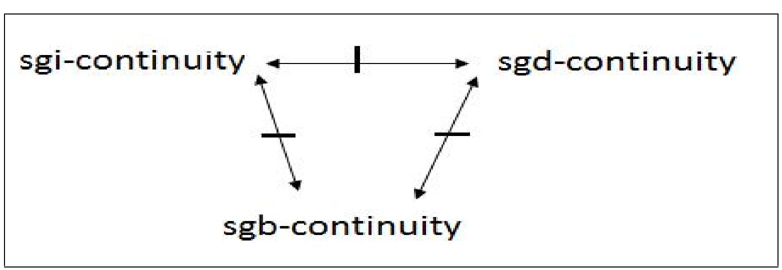
Remark 4.5 : The notions sgd-continuity and sgb-continuity are independent as seen in the following example.

Example 4.6: Consider the spaces $(X,\tau_{11},\leq_8)$ and $(Y,\tau_{11},\leq_9)$ where $X=Y=\{a,b,c\}$, $\leq_8=\{(a,a),(b,b),(c,c)\}$ and $\leq_9=\{(a,a),(b,b),(c,c),(a,c)\}$. The b-closed sets in Y are ϕ,X and sgb-closed sets in X are $\phi,X,\{a\},\{b\},\{a,b\}$. Define $f:X\to Y$ as $f(a)=c$, $f(b)=b$ and $f(c)=a$. Then, f is a sgb**-continuous function**. The d-closed sets in Y are $\phi,X,\{a\},\{a,b\}$ and sgd-closed sets in X are $\phi,X,\{a\},\{b\},\{a,b\}$.Then, f is **not a sgd-continuous function.**

For the other part, consider the topological ordered spaces $(X,\tau_{11},\leq_6)$ and $(Y,\tau_{11},\leq_7)$ where $X=Y=\{a,b,c\}$, $\leq_6=\{(a,a),(b,b),(c,c),(b,a),(a,c),(b,c)\}$ and $\leq_7=\{(a,a),(b,b),(c,c),(b,a)\}$. The d-closed sets in Y are $\phi,X,\{a,b\}$ and sgd-closed sets in X are $\phi,X,\{b\},\{a,b\}$. Define $f:X\to Y$ as $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then, f is a **sgd-continuous function.** The b-closed sets in Y are $\phi,X,\{a,b\}$ and sgb-closed sets in X are ϕ,X.Then, f is **not a sgb-continuous function.**

Conclusion:

The following results were proved in this paper.



Here the symbol A $\nleftrightarrow$ B indicates A and B are independent notions.

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[**5**]. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, **70**(1963), 36-41. a *semi-open* set [**17**] if $A \subseteq cl(int(A))$ and a *semi-closed* set if $int(cl(A)) \subseteq A$.
