Short research article.

The Population Question in Nigeria: Models and Reliable Projections.

by

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ABSTRACT
The question of Nigeria's actual population has remained a very sensitive one within the country because there are very obvious constitutional advantages of large populations conferred on constituent parts of the Nation. This sensitivity has made it increasingly difficult to conduct a credible census exercise, and this means having to depend on projections and estimates. Projections may not take into exact account so many salient social, demographic and environmental factors that could influence or alter the nature of population abundance and structure. Along this line of reasoning, this paper examines two well-known population growth models, namely, the exponential growth model (EGM) and the logistic growth model (LGM). A simple blend of the two models through the use of an arithmetic average is proposed in an Average Projection (AP). This Average Projection is more stable than both EGM and LGM in the long run. It has the capacity to moderate the explosive tendency of the exponential model and the inexplicably converging effects of the carrying capacity in the logistic model. Projections of the population of Nigeria were made for the range of years 1991 to 2050 using the EGM, LGM and AP, and they were graphically illustrated. When compared with actual (official) projection, the average projection shows closer approximation and better stability in the long run.

Keywords
Average projection, Carrying capacity, Exponential growth model, Logistic growth model, Population Dynamics, Population projections.

1.0 Introduction

Population growth models are types of mathematical models that are used in the study of population dynamics[1]. They allow for a better understanding of how complex interactions and processes work, and also provide a quantitative way of explaining how numbers change over time and in relation to one another[2, 3]. Models are useful for prediction, forecasting and simulations of future outcomes. In modelling, present and past data are used to extrapolate the future. Besides Time series and classical regression studies, the theory of models of population growth has proved to be very vital in the economics of planning and projections. Models are however laced with complex systems of equations, but their reliability, consistency and precision are most times higher.

A number of terms are used in the technical sense when we undertake the study of population dynamics. Population growth rate is the term used to describe the per capita growth of population, and it is the summary of parameters of observable trends in population density or abundance. According to [4], formulating an appropriate model for the dynamics of any population is equivalent to specifying a recipe for population change. Basically, a population model should provide answer to the question of how a population is going to change in the near future, given its current status and the prevailing environmental conditions of the individuals that make up the present population. These changes in the population may be in the total number of individuals present, and may also pertain to the composition of the population[4].

The population size and the composition of the population are two important characteristics that determine future development of any population. More formally, the first concept is referred to as the population abundance, while the second is called the population structure. Together both concepts what is generally known as the population state (or p-state for short). The p-state is the simple characterization of a given population in terms of how many individuals, and of which type, are present in the population. The individual is the fundamental entity in the dynamics of a population, since changes in the population can only come about because of events that happen with individual organisms.

The individual state or i-state is the collection of physiological traits that are used to characterize individual organisms within a population, and that will influence the life history. The environmental conditions to which a population is exposed are of importance in defining the direction of its dynamics, and usually contributes in setting the limits for its development. Environmental condition (or e-condition for short) is the collection of biotic and abiotic factors, external to the individual itself, that can influence its life history. While the i-state of an individual could be viewed as determining its potential for reproduction, the e-condition modulates this potential towards realizing this potential[4].

The rest of this paper has been organized as follows: Sections 2, 3, and 4 are reviews of relevant literature on general concepts of population dynamics, the exponential growth model and the logistic growth model, respectively. Section 5 provides a preview of the nature of the population question in Nigeria. The proposed model is presented in section 6 alongside comparative illustrations with the EGM and the LGM. This was followed by discussion and conclusion in sections 7 and 8, respectively.

2.0 Population Growth Models

If we consider a population to be the collection of individuals of a particular species that live within a well - defined area, any change in the number of individuals within this population comes about either by birth, or death, or immigration, or emigration[5].

More formally, and in its simplest form, a population dynamics model can be expressed as:

Population change = Births + Immigration – (Deaths + Emigration) i.e. B.I.D.E.

Building a model to describe the dynamics of a particular population will involve having to choose the mathematical representation of the population in our model, and choosing a particular representation for the population[6]. Populations of organisms can even be studied by counting the number of adults at a reproduction time, to get a sequence of numbers, say P_1, P_2, . . . , P_n, where P_r is the number of adults at the rth reproduction time. This sequence can be studied in various ways to predict the changes such a population will experience over some generations to come. One might expect to find a functional relationship between successive generations, and expressed in the form:

$$P_{n+1} = f(P_n), \quad .. \quad \dots \quad \dots \quad \dots \qquad (1)$$

Here, f is called the reproductive function. A simple analysis of this is to plot the successive pairs of points (P_r , P_{r+1}), r = 1, 2, …..,n on a graph. It is often found that these data actually lie on a smooth curve, known as the population's reproduction curve[2].

3.0 The Exponential Growth Model

This model was proposed by a demographer in the person of Thomas Malthus, and it was based on the notion that population dynamics is at a rate which is directly proportional to the size of the population[4]. The model is embedded in a differential equation whose solution is

$$P(t) = p_o e^{rt} \qquad \dots \qquad \dots \qquad \dots \qquad (2)$$

Where p_o is the initial population at time t_o, r is the growth rate, and is sometimes called the Malthusian parameter, and $P(t)$ is the population size at time t.

In the branch of population ecology, this model is known as the first principle of population dynamics[4]. It is an example of a model with one independent variable (time), one dependent variable (population size) and one parameter (growth rate).

If X(k) is used to denote the population size during the time period k, and with r as the population growth rate per unit time, the Malthusian population model can be expressed recursively as:

$$X(k+1) = (1+r).X(k) \qquad \dots \qquad \dots \qquad \dots \qquad (3)$$

This is a difference equation model, and we can use the model to determine population sizes at any point in the future by applying the equation repeatedly.

The population balance equation, as used then by Malthus[2], can be written more succinctly as:

$$\frac{dN(t)}{dt} = rP \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (4)$$

where r = β – δ was assumed to be the difference between birth β and death δ.

Equation (4) is the famous exponential growth equation or Malthus' growth law, which can be solved when the initial state of the population (the population size N at time t = 0) has been properly specified. As an explicit function of time t, the solution is given by (2). According to [3], exponential growth equations are useful models for simple computation and for developing intuitive ideas about a population. One of the weaknesses of this model is the assumption that the relative population growth rate is constant, and that population would grow without bound. But in reality, this cannot happen indefinitely. The exponential growth model is inadvertently modified by fact that it ignores the idea of density dependence and intra-specific competition. This, according to [3], is a complicated issue that continue to inspire debate and cannot just be wished away.

4.0 The Logistic Growth Model

In 1840, a Belgian Mathematician by name Verhulst, thought of population growth as not only depending on the population size, but also on how far this size is from an upper limit[3]. He thus proposed a new model in the form:

$$\frac{dP}{dt} = rP(1 - \frac{P}{K}) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

where $r > 0$, is the population growth rate, and $K > 0$ is the carrying capacity or the maximum supportable population. This equation is also known as a *logistic difference equation*.

The curve resulting from the plot of the logistic equation resembles an S, and for that, it is sometimes called an S-shaped curve or a Sigmoid[3]. In this model, the population initially experiences an exponential growth phase, but as the size approaches the carrying capacity, the growth slows down and then reaches a stable level. This gradual convergence to the carrying capacity is perhaps the result of some environmental conditions.

In Solving the Logistic difference equation, the constant solutions are $P=0$ and $P=K$, while the non-constant solutions may be obtained by solving the appropriate difference equation under the initial conditions $P(0)=P_0$ (assuming that P_0 is not equal to both 0 or K), to get

$$P(t)=\frac{KP_0}{P_0+(K-P_0)e^{-rt}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (6)$$

From this result, it is clearly seen that if $P(t)$ is the logistic formula and K is the carrying capacity, then $\lim_{t\to\infty} P(t) = K$.

This follows because

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{PoK}{Po+(K-Po)e^{-rt}} = \frac{PoK}{Po} = K \qquad \ldots \qquad \ldots \qquad (7)$$

Several researchers have also worked on the appropriateness and feasibility of the common growth models. A novel idea known as the mechanistic model was conceived by [1]. In it the mechanistic model was presented as a growth model that correlates food availability, agricultural production, fertility rates and the size of a population. The concept of hyperbolic modeling was introduced by [6], and it was intended to moderate the excesses of exponential growth models. A transformation of the exponential and logistic models was proposed by [7]. This involved the application of least squares principle and the use of parametric linearization so as to obtain better and more reliable estimates. Worthy also of mention is the work of [8] which computes and compares the population projections of Nigeria between 2006 and 2014, using linear, geometric and exponential models of population growth.

Comparatively, the prediction for the Logistic growth model appears more realistic than that of the exponential model by providing for limitations in the system. However, the relevance and utility of the logistic growth equation in real populations are limited because the dynamics of populations are complex and in practical context, it is difficult to come up with the real value for the carrying capacity in the study of human population[4]. In a long term prediction, the exponential model explodes unrealistically, while the logistic model freezes abnormally to an imaginary carrying capacity. The carrying capacity may not be a fixed number over time. It is subject to change depending on many conditions. The current level of technology which changes quite regularly, will definitely affect this number. Moreover, species can sometimes alter and expand their niche. If the carrying capacity of a system happens to experience a change during a period of logistic growth, a second period of logistic growth with a different carrying capacity will be needed to superimpose on the first growth pulse.

5.0 The Nigerian Population Question.

As at the year 1960, the population of Nigeria[9] was put at 45.2 million by a census exercise which was more of a projection just as in earlier pre-independence attempts at getting a reliable census. This is because only the major cities and a few other accessible or known areas were captured in the enumeration exercise. Furthermore, in addition to cultural beliefs in some areas of the nation which, supposedly, was not in favour of counting of humans, the motive of the exercise was highly misunderstood and suspected by the general public to be solely for taxation purposes. So a lot of communities and individuals did much to evade the exercise or under - represent their population. Moreover, as recent as 1985, more communities were still being discovered in Nigeria, whose existence were hitherto unknown to Government or her agencies. A celebrated case of this was the discovery of the Koma People on the mountains of the then Gongola State by an adventure of a Military Administrator in the middle 1980s. According to [9], the credibility question of the early population figures of Nigeria therefore, bothers on under – estimation. However, the general attitude towards national census issues was soon to assume another dangerous and frightening dimension. With the discovery and exploitation of crude oil in commercial quantity in Nigeria, and the resultant sharing of centrally pooled revenue, and with the advent of representative democracy and the adoption of the policies of federal character and quota system, competitive manipulation of population figures between Regions, States, Local Government Areas and Communities became the order of the day. Inflation and bloating of figures from preferred areas by the coordinating agencies was always alluded to each time there was census in Nigeria. It is on record that till date there is no reliable and non-controversial census exercise successfully carried out in Nigeria.

With a population estimated at about 182 million in 2015, Nigeria is unarguably, the most populous country in Africa, and perhaps, the seventh most populous in the world. According to [10], Nigeria is considered a high fertility country, with obvious evidence that her large population has contributed to the inability of Government to meet the basic needs of her citizens. The population of the country was projected to grow at about 3% annually. According to [11], the global population growth rate is about 1.1%, which translates to roughly 75 million additions every year. This high population growth rate of Nigeria can be attributed to several factors. The most prominent of them are the huge investment in the health sector by various levels of Government and Non Governmental Agencies, which has resulted in safer delivery, reduced miscarriages, reduced infant mortality, improvement in life expectancy, and higher fertility rates. The global fight against killer diseases, and migration into Nigeria from neighbouring countries especially in the North, also feature in the list of factors, as well as the prevalent conservative, religious attitude of most communities.

At the facial level, the Exponential Model of population growth appears to have found much credibility in the Nigerian context. This is because in a short period of time, the population of the country has greatly increased and much pressure has been brought to bear on the scarce resources and available social amenities, Being a purely subsistent and agrarian economy, with high dependence on oil resources and massive importation of food, the challenges of a high population in a country like Nigeria are very daunting [12]. From the economic angle, there is prevalence of under-employment and unemployment, low per capital income, poor standard of living, excess pressure on and over subscription of the available basic amenities and huge external debt. On the social front there is food shortage, hunger and disease, internal and external migration, communal clashes and boundary disputes, poverty and quite dangerously, the provision of recruitment base for terror groups.

6.0 The Average Projection Model

In this section, a simple population model called the Average projection (AP) is proposed. This proposed model is obtained by taking an arithmetic average of the Exponential growth model and the Logistic growth model, using equations (2) and (6). Errors associated with the average projection drastically reduced, and the model provides projections that are reasonably stable over a fairly long interval of time. This is unlike the exponential model which increases without bound in the long run, or the Logistic model which converges to the carrying capacity eventually. Illustratively, the Actual projection of the population of Nigeria was obtained from the published data of the Nigeria Population Commission, available at [13] and [14]. The Exponential Growth Model (EGM) and the Logistic Growth Model were then used to provide alternative projections of the population of Nigeria, with 1991 as the initial time period t_0. The population growth rate used in the official projection was computed using the figures of 1991 and 2006. This is because both years had actual and official census figures for Nigeria[8]. Furthermore the growth rate of 2.8% agrees with the official growth rate adopted after the 2006 census.

Applying the Exponential growth model (EGM), and using the information in Table 1 with $t = 0$ corresponding to the year 1991, we have $P_0 = 92.296$. We then solve for r, using the fact that $P =$ 140.432 when $t = 15$, and by using the equation (2), we solve to obtain

$$r = 0.028 \qquad \ldots \qquad \ldots \qquad \ldots \qquad (8)$$

This leads us to the general solution $P(t) = 92.296e^{0.028t}$ ... ... (9)

Substitute for values of t from 1 through 59 or as appropriate to obtain the EGM projections of Table 1.

Similarly, applying the Logistic growth model (LGM), and using the information of Table 1, with $t=0$ corresponding to the year 1991, we have $P_0 = 92.296$. We can solve for r, using the fact that $P =$ 140.432, when $t = 15$.

Now let us assume an economical, but arbitrary carrying capacity of 500million persons for the Nigerian State. This figure is quite arbitrary, but logical, considering the prevailing economic conditions, habitable landmass of Nigeria and other indices of development. It is reasonable to assume that Nigeria will not be able to peacefully sustain a population above 500 million.

Hence, using (6), we have the general logistic equation as

$$P(t) \ = \frac{500(92.296)}{92.296+(500-92.296)e^{-rt}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (10)$$

$$\text{i.e. } \ 140.432 \ = \frac{500(92.296)}{92.296+(500-92.296)e^{-r15}}$$

Which is solved to obtain $r = 0.036$ ... ... (11)

Substituting (11) in (10), we have the general solution for as:

$$P(t) \ = \frac{500(92.296)}{92.296+(500-92.296)e^{-0.036t}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (12)$$

Evaluating $P(t)$, at time t = 0, 4, 9, 14, ... , 59, we obtain the LGM projection in Table 1.

Consider another model which is simply an arithmetic average of the EGM and LGM projections given by (9) and (12) respectively. This is the Average Projection (AP), presented in the last column of Table 1.

Table 1: Actual, Exponential, Logistic and Average projections for population of Nigeria between 1991 and 2050.

Count t	**Date**	**Actual Projection**	**EGM projection**	**LGM projection**	**Average Projection (AP)**
1	1991	92.296	92.296	92.296	92.296
2	1995	108.425	100.384	103.628	102.006
3	2000	122.877	115.469	119.194	117.332
4	2005	139.611	136.592	136.293	136.443
5	2010	159.425	157.119	154.848	155.984
6	2015	182.202	180.729	174.716	177.723
7	2020	206.830	207.888	195.681	201.785

8	2025	233.558	239.128	217.493	228.311
9	2030	262.599	275.063	239.813	257.438
10	2035	293.965	316.398	262.297	289.348
11	2040	327.406	363.944	284.584	324.264
12	2045	362.356	418.636	306.324	362.480
13	2050	398.508	481.546	327.203	404.375

The average projection in Table 1 is simply an arithmetic average of the exponential and the logistic projections. For a better understanding, the data of Table 1 is represented, first as a time series multiple bar chart (Fig. 1) and as a line graph (Fig. 2). It can be seen that the average projection closely approximates the actual projection.

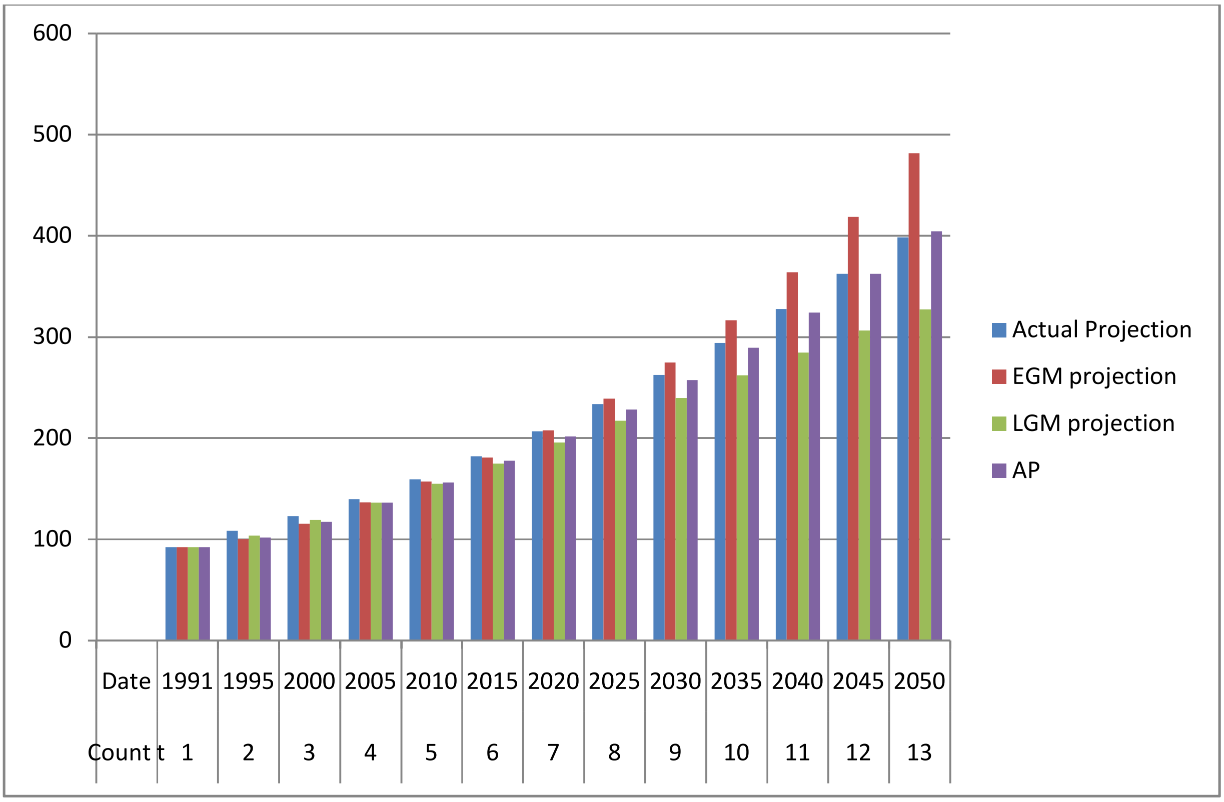

Fig.1 Time series multiple bar chart of projections of the population of Nigeria.

The deviations of the EGM, LGM and the AP from the actual projections are presented in Table 2 and plotted as line graphs in figure 2.

Table 2: Deviations of the models from the actual projection

Count	Actual Proj.	EGM dev.	LGM dev.	APM dev.
1	0.000	0.000	0.000	0.000
2	0.000	-8.041	-4.797	-6.419
3	0.000	-7.408	-3.683	-5.545
4	0.000	-3.019	-3.318	-3.168
5	0.000	-2.306	-4.577	-3.442
6	0.000	-1.473	-7.486	-4.479
7	0.000	1.058	-11.149	-5.046
8	0.000	5.570	-16.065	-5.248
9	0.000	12.464	-22.786	-5.161
10	0.000	22.433	-31.668	-4.617
11	0.000	36.538	-42.822	-3.142
12	0.000	56.280	-56.032	0.124
13	0.000	83.038	-71.305	5.867

A plot of the deviations of Table 2 as a line graph is presented in Fig. 2 for each class of projection.

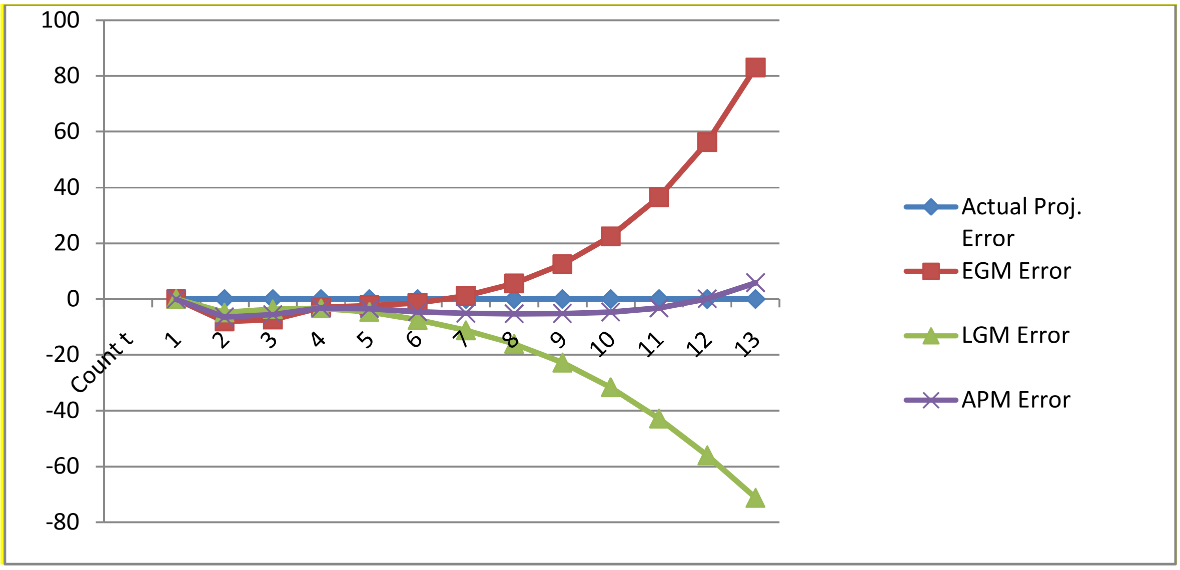

Fig.2 Deviations of the EGM, LGM and the AP from the actual projection

7.0 Discussion

From Table 2 and Fig.2, all four projections cluster around same values for the first six observations. The predicted population sizes were quite close to those of the actual projections. Major deviations start to manifest as from the seventh observation, with the magnitude of differences increasing steadily. In the long run, we see the EGM and the LGM going in opposite sides of the actual projections and diverging rapidly too. We observe some level of stability with the AP over the long run, and a reasonable approximation between the actual and the average projection. This close approximation is sustained over a long period of time.

The average projection of Table 1, which refers to the arithmetic average of the exponential (EGM) and logistic (LGM) projections, can be viewed as a model for the reliable estimation of a population at time t. It is a simple, conservative, middle-of-the-road approach that has the capacity to moderate the extreme effects of both EGM and LGM projections. The explosive tendency of the EGM and the inherent converging effects of the carrying capacity in the logistic model are balanced out to obtain a stable predictor in the long run.

8.0 Conclusion

This work has proposed a model that is a blend of the exponential and the logistic population growth models, unique especially to the Nigerian context. This model, which is obtained by an arithmetic average of both, comparatively shows better level of stability in the long run. With it, population estimates that are closer to the actual were obtained. It is hoped that this proposition will contribute positively to the search for better ways of obtaining reliable estimates of population figures, especially in Nigeria and other similar climes where so many factors are contending against this ideal.

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