Short research article.

The Population Question in Nigeria: Models and Reliable Projections.

ABSTRACT

The question of Nigeria's actual population has remained a very sensitive one within the country because there are very obvious constitutional advantages of large populations conferred on constituent parts of the Nation. This sensitivity has made it increasingly difficult to conduct a credible census exercise, and this means having to depend on projections and estimates. Projections may not take into exact account so many salient social, demographic and environmental factors that could influence or alter the nature of population abundance and structure. Along this line of reasoning, this paper examines two well-known population growth models, namely, the exponential growth model and the logistic growth model. With each model, a projection of the population of Nigeria is made up to 2050, and compared with the existing official projections. With time, these various projections manifest differences that should be of concern to those who use the figures. This study proposes a blend of the growth models through the use of simple average, which has the capacity to moderate the explosive tendency of the exponential model and the inexplicably converging effects of the carrying capacity in the logistic model.

Keywords

Carrying capacity, Exponential growth model, Logistic growth model, Population Dynamics, Population projections, Simple average.

1.0 Introduction

Population growth models are types of mathematical models that are used in the study of population dynamics. They allow for a better understanding of how complex interactions and processes work. They also provide a quantitative way of explaining how numbers change over time and in relation to one another[1]. Models are useful for prediction, forecasting and simulations of future outcomes. In modelling, present and past data are used to extrapolate the future. Besides Time series and classical regression studies, the theory of models of population growth has proved to be very vital in the economics of planning and projections. Models are however laced with complex systems of equations, but their reliability, consistency and precision are most times higher.

A number of terms are used in the technical sense when we undertake the study of population dynamics. Population growth rate is the term used to describe the per capita growth of population, and it is the summary of parameters of observable trends in population density or abundance. According to [2], formulating an appropriate model for the dynamics of any population is equivalent to specifying a recipe for population change. Basically, a population model should provide answer to the question of how a population is going to change in the near future, given its current status and the prevailing environmental conditions of the individuals that make up the present population. These changes in the population may be in the total number of individuals present, and may also pertain to the composition of the population[2].

The population size and the composition of the population are two important characteristics that determine future development of any population. More formally, the first concept is referred to as the population abundance, while the second is called the population structure. Together both concepts what is generally known as the population state (or p-state for short). The p-state is the simple characterization of a given population in terms of how many individuals, and of which type, are present in the population. The individual is the fundamental entity in the dynamics of a population, since changes in the population can only come about because of events that happen with individual organisms.

The individual state or i-state is the collection of physiological traits that are used to characterize individual organisms within a population, and that will influence the life history. The environmental conditions to which a population is exposed are of importance in defining the direction of its dynamics, and usually contributes in setting the limits for its development. Environmental condition (or e-condition for short) is the collection of biotic and abiotic factors, external to the individual itself, that can influence its life history. While the i-state of an individual could be viewed as determining its potential for reproduction, the e-condition modulates this potential towards realizing this potential[2].

2.0 Population Growth Models

If we consider a population to be the collection of individuals of a particular species that live within a well-defined area, any change in the number of individuals within this population comes about either by birth, or death, or immigration, or emigration[3].

More formally, and in its simplest form, a population dynamics model can be expressed as:

Population change = Births + Immigration – (Deaths + Emigration) i.e. B.I.D.E.

Building a model to describe the dynamics of a particular population will involve having to choose the mathematical representation of the population in our model, and choosing a particular representation for the population. Populations of organisms can even be studied by counting the number of adults at a reproduction time, to get a sequence of numbers, say $P_1, P_2, . . . , P_n$, where P_r is the number of adults at the r^{th} reproduction time. This sequence can be studied in various ways to predict the changes such a population will experience over some generations to come. One might expect to find a functional relationship between successive generations, and expressed in the form:

$$P_{n+1} = f(P_n), \quad .. \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Here, f is called the reproductive function. A simple analysis of this is to plot the successive pairs of points (P_r , P_{r+1}), r = 1, 2, …..,n on a graph. It is often found that these data actually lie on a smooth curve, known as the population's reproduction curve.

2.1 The Exponential Growth Model

This model was proposed by a demographer in the person of Thomas Malthus, and it is based on the notion that population dynamics is a rate which is directly proportional to the size of the population[2]. The model is embedded in a differential equation whose solution is

$$P(t) = p_o e^{rt} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Where p_o is the initial population at time t_o, r is the growth rate, and is sometimes called the Malthusian parameter, and $P(t)$ is the population size at time t.

In the branch of population ecology, this model is known as the first principle of population dynamics. It is an example of a model with one independent variable (time), one dependent variable (population size) and one parameter (growth rate).

If X(k) is used to denote the population size during the time period k, and with r as the population growth rate per unit time, the Malthusian population model can be expressed recursively as:

$$X(k+1) = (1+r).X(k) \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

This is a difference equation model, and we can use the model to determine population sizes at any point in the future by applying the equation repeatedly.

The population balance equation, as used then by Malthus, can be written more succinctly as:

$$\frac{dN(t)}{dt} = rP \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where r = β – δ was assumed to be the difference between birth β and death δ.

Equation (4) is the famous exponential growth equation or Malthus' growth law, which can be solved when the initial state of the population (the population size N at time t = 0) has been properly specified. As an explicit function of time t, the solution is given by (2).

2.2 The Logistic Growth Model

In 1840, a Belgian Mathematician by name Verhulst, thought of population growth as not only depending on the population size, but also on how far this size is from an upper limit. He thus proposed a new model in the form:

$$\frac{dP}{dt} = rP(1-\frac{P}{K}) \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Where $r > 0$, is the population growth rate, and $K > 0$ is the carrying capacity or the maximum supportable population. This equation is also known as a *logistic difference equation*.

The curve resulting from the plot of the logistic equation resembles an S, and for that, it is sometimes called an S-shaped curve or a Sigmoid. In this model, the population initially experiences an exponential growth phase, but as the size approaches the carrying capacity, the growth slows down and then reaches a stable level. This gradual convergence to the carrying capacity is perhaps the result of some environmental conditions.

In Solving the Logistic difference equation, the constant solutions are $P = 0$ and $P = K$, while the non-constant solutions may be obtained by solving the appropriate difference equation under the initial conditions $P(0) = P_0$ (assuming that P_0 is not equal to both 0 or K), to get

$$P(t) = \frac{KP_0}{P_0 + (K - P_0)e^{-rt}} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

From this result, it is clearly seen that if $P(t)$ is the logistic formula and K is the carrying capacity, then $\lim_{t\to\infty} P(t) = K$.

This follows because

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{PoK}{Po+(K-Po)e^{-rt}} = \frac{PoK}{Po} = K \quad \ldots \quad \ldots \quad (7)$$

3.0 Projections of the Nigerian Population

In this section, we examine the official projections of the population of Nigeria from the year 1991 to 2050. The official projection was obtained from the published data of the Nigeria Population Commission, available at [4] and [5]. The Exponential and the Logistic growth models were then used to provide alternative projections of the population of Nigeria, with 1991 as the initial time period t_0, and an average value was computed. The population growth rate used in the official projection was adopted for the purpose of uniformity.

Applying the Exponential growth model, and using the information in Table 1 with $t = 0$ corresponding to the year 1991, we have $P_0 = 92.296$. We then solve for r, using the fact that $P = 140.432$ when $t = 15$, and by using the equation (2), we solve to obtain

$$r = 0.028 \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

This leads us to the general solution $P(t) = 92.296e^{0.028t}$ ... ... (9)

Substitute for values of t from 1 through 59 or as appropriate to obtain the exponential model projections of Table 1.

Similarly, applying the Logistic growth model, and using the information of Table 1, with $t = 0$ corresponding to the year 1991, we have $P_0 = 92.296$. We can solve for r, using the fact that

$P = 140.432$ when $t = 15$.

Now let us assume an economical, but arbitrary carrying capacity of 500million persons for the Nigerian State. By using (6), we have the general formula

$$P(t) = \frac{500(92.296)}{92.296 + (500 - 92.296)e^{-rt}} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

$$\text{i.e. } 140.432 = \frac{500(92.296)}{92.296 + (500 - 92.296)e^{-r15}}$$

$$e^{-15r} = \frac{33186.68813}{57254.68815} = 0.05796326764$$

$$\text{Hence, } r = 0.036 \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Substituting (11) in (10), we have the general solution for the population as:

$$P(t) = \frac{500(92.296)}{92.296 + (500 - 92.296)e^{-0.036t}} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

Evaluating $P(t)$, at time t = 0, 4, 9, 14, ... , 59, we obtain the logistic projection in Table 1.

Table 1: Official, Exponential, Logistic and Average projections for population of Nigeria between 1991 and 2050.

Time t	Date	Official Projection	Exponential Growth projection.	Logistic Growth projection.	Projection Average.
0	1991	92.296	92.296	92.296	92.926
4	1995	108.425	100.384	103.628	104.086
9	2000	122.877	115.469	119.194	119.180
14	2005	139.611	136.592	136.293	137.500
19	2010	159.425	157.119	154.848	157.131
24	2015	182.202	180.729	174.716	179.216
29	2020	206.830	207.888	195.681	203.466
34	2025	233.558	239.128	217.493	230.060
39	2030	262.599	275.063	239.813	259.158
44	2035	293.965	316.398	262.297	290.887
49	2040	327.406	363.944	284.584	325.311

54	2045	362.356	418.636	306.324	362.439
59	2050	398.508	481.546	327.203	402.419

4.0 Discussion

As at the year 1960, the population of Nigeria was put at 45.2 million by a census exercise which was more of a projection as in earlier pre-independence attempts at getting a reliable censusR1. This is because only the major cities and a few other accessible or known areas were captured in the enumeration exercise. Furthermore, in addition to cultural beliefs in some areas of the nation which, supposedly, was not in favour of counting of humans, the motive of the exercise was highly misunderstood and suspected by the general public to be solely for taxation purposes. So a lot of communities and individuals did much to evade the exercise or under - represent their populationR2. Moreover, as recent as 1985, more communities were still being discovered in Nigeria, whose existence were hitherto unknown to Government or her agencies. A celebrated case of this was the discovery of the Koma People on the mountains of the then Gongola State by an adventure of a Military Administrator in the middle 1980s. According to [6], the credibility question of the early population figures of Nigeria therefore, bothers on under – estimation. However, the general attitude to national census issues were soon to assume another dangerous and frightening dimension. With the discovery and exploitation of crude oil in commercial quantity in Nigeria, and the resultant sharing of centrally pooled revenue, and with the advent of representative democracy and the adoption of the policies of federal character and quota system, competitive manipulation of population figures between Regions, States, Local Government Areas and Communities became the order of the day. Inflation and bloating of figures from preferred areas by the coordinating agencies was always alluded to each time there was census in Nigeria. It is on record that till date there is no reliable and uncontentious census exercise successfully carried out in Nigeria.

By 2015, the population of Nigeria has risen to an estimate of about 182 million. Unarguably, Nigeria is actually the most populous country in Africa, and perhaps, the seventh most populous in the world. According to [7], Nigeria is considered a high fertility country, with obvious evidence that her large population has contributed to the inability of Government to meet the basic needs of her citizens. The population of the country was projected to grow at about 3% annually. According to [8], the global population growth rate is about 1.1%, which translates to roughly 75 million additions every year. This high population growth rate of Nigeria can be attributed to several factors. The most prominent of them are the huge investment in the health sector by various levels of Government and Non Governmental Agencies, which has resulted in safer delivery, reduced miscarriages, reduced infant mortality, improvement in life expectancy, and higher fertility rates. The global fight against killer diseases, and migration into Nigeria from neighbouring countries especially in the North also feature in the list of factors.

The Exponential Model of population growth appears to have found much credibility in the Nigerian context. Being a purely subsistent and agrarian economy, with high dependence on oil resources and massive importation of food, the challenges of a high population in a country like Nigeria are very daunting [9]. From the economic angle, there is prevalence of under-employment and unemployment, low per capital income, poor standard of living, excess pressure on and over subscription of the available basic amenities and huge external debt. On the social front there is food shortage, hunger and disease, internal and external migration, communal clashes and boundary disputes, poverty and quite dangerously, the provision of recruitment base for terror groups.

From experience, it is well accepted that Malthus's model applies fairly well over a short span of time. The predicted population sizes are quite close to those of the actual sizes. It starts to manifest serious differences thereafter. One of the weaknesses of this model is the assumption that the relative population growth rate is constant, and that population would grow without bound. But in reality, this cannot happen indefinitely..

In comparison, the prediction for the Logistic growth model appears closer to the official than the exponential. However, the relevance and utility of the logistic growth equation in real populations are limited because the dynamics of populations are complex and in practical context, it is difficult to come up with the real value for the carrying capacity in the study of human population. It is not a fixed number over time; it is always changing depending on many conditions, and it is often limited by the current level of technology, which is subject to change. More generally, species can sometimes alter and expand their niche. If the carrying capacity of a system happens to experience a change during a period of logistic growth, a second period of logistic growth with a different carrying capacity can superimpose on the first growth pulse.

The projection average of Table 1 refers to the average value of the official, exponential and logistic projections. This manipulated projection can be viewed as another estimate of the population at time t. It is a conservative, middle-of-the-road approach that has the capacity to moderate the extreme effects of the other projections. Both the explosive tendency of the exponential model and the inexplicably converging effects of the carrying capacity in the logistic model are cancelled out.

5.0 Conclusion

Models represent something that look real and relevant in our estimation, but they are just abstractions that must never be confused with the reality we are trying to mimic. Hence, if the outcome of prediction by our models does not reflect what we see or measure in the real world, it is the model that will need to be re-examined, and not the real world. It is based on this premise that the search for more consistent and credible means of estimating the population of Nigeria will continue until such a time when the nation will mature to the point of a dispassionate conduct of a truly reliable census.

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