MODULATED FEEDBACK AND COUPLING TIME DELAYS, AND ALL-TO-ALL CHAOS SYNCHRONIZATION IN A NETWORK OF NETWORKS:ONE OF THE SIMPLEST CASES

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ABSTRACT

We report on all- to- all chaos synchronization in a network of networks based on the Ikeda model. We consider one of the simplest cases. We find the existence and stability conditions for such a synchronization regime. Numerical simulations validate the analytical findings. The results can be of certain importance in achieving high level output for the coupled systems and information processing.

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Key words: Network of networks; Ikeda model; time-delay systems; modulated feedback and coupling delay times; all-to-all chaos synchronization; existence and stability conditions.

I.INTRODUCTION

Networks or a network of networks is a widespread concept in a world-wide-web, population dynamics, neuroscience, power grids, communication, social and computer systems, etc. Research of such interacting systems is a very hot topic in nonlinear dynamics, see e.g. [1-6] and references there-in.

Chaos synchronization [1] as a control method is of fundamental importance in a variety of complex physical, chemical and biological systems [7]. Synchronization of chaos refers to a process wherein two (or many) chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behavior due to a coupling or to a forcing (periodical or noisy)[7]. In the context of coupled chaotic elements, many different synchronization states have been studied, namely complete or identical synchronization, phase synchronization, lag synchronization, generalized synchronization, anticipating synchronization, etc.[7-9]. Complete synchronization [10] was the first to be discovered and is the simplest form of synchronization in chaotic systems. It consists in a perfect hooking of the chaotic trajectories of two systems which is achieved by means of a coupling signal, in such a way that they remain in step with each other in the course of the time. Generalized synchronization [11] goes further in using completely different systems and associating the output of one system to a given function of the output of the other system. Coupled non-identical oscillatory or rotatory systems can reach an intermediate regime of phase

synchronization [12-14], wherein a locking of phases occurs, while correlation in the amplitudes remain weak. Lag synchronization [15] is a step between phase synchronization and complete synchronization . It implies the asymptotic boundedness of the difference between the output of one system at time t and the output of the other shifted in time (lag time). This implies that the two outputs lock their phases and amplitudes, but with the presence of a time lag. In anticipating synchronization [16-17] the driven system state is synchronized to the future state of the driver system. For some other types of synchronization see also [18-21] and references there-in. Synchronization in complex systems is of certain importance in governing and performance improving point of view, e.g. enhancing emission power from such systems [7]. Additionally, from the fundamental point of view synchronization of coupled (chaotic) systems eliminates some degrees of freedom of the coupled system and so produces a significant reduction of complexity, thus allowing for significant simplification of computational and theoretical analysis of the system.

As synchronization in a wider sense is associated with communication, a study of existence and stability conditions for synchronization is of paramount importance in networks. Synchronization is important in chaos based communication system to decode the transmitted message [7,17]: At the transmitter part of the communication system a message is masked with chaos, then chaos masked message is transmitted to the receiver system. At the receiver part of the communication system due to the chaos synchronization between the transmitter and the receiver systems chaos is regenerated. Finally, deducting the receiver input and the receiver output one can decode the transmitted message, Figure 1.

In this paper we study chaos synchronization in one of the simplest cases of the network of networks based on the Ikeda system-paradigmatic model of chaotic dynamics in time delay systems [22]. In case of constant time delays we derive analytically the existence and sufficient stability conditions for complete synchronization between all the constituents of the network. We support our analytical findings with the numerical simulations. We also present examples of chaos synchronization between the constituent Ikeda models in case of variable time delay systems.

The organization of the rest of this paper is as follows. In Sec. II we introduce our model. In Sec. III we present the results of analytical study. Section IV is dedicated to the numerical simulations of all-to-all chaos synchronization between the Ikeda models, including the case of modulated time delays. We summarize our results in Sec. IV.

II. SYSTEM MODEL

Consider all-to-all synchronization between the chaotic Ikeda systems with the following coupling topology (see, Figure 2):x-Ikeda system governs both networks $((y, z$ and $u, w))$ which consists of only two unidirectionally coupled Ikeda systems. For simplicity consider the case when all the

Ikeda systems are identical and time delays in the network is constant.

$$\frac{dx}{dt} = -\alpha x + m_1 \sin x_\tau \tag{1}$$

$$\frac{dy}{dt} = -\alpha y + m_2 \sin y_\tau + m_6 \sin x_{\tau_1} \tag{2}$$

$$\frac{dz}{dt} = -\alpha z + m_3 \sin z_\tau + m_8 \sin y_{\tau_1} \tag{3}$$

$$\frac{du}{dt} = -\alpha u + m_4 \sin u_\tau + m_7 \sin x_{\tau_1} \tag{4}$$

$$\frac{dw}{dt} = -\alpha w + m_5 \sin w_\tau + m_9 \sin u_{\tau_1} \tag{5}$$

Here $x_\tau \equiv x(t-\tau)$. The same is valid for the other dynamical variables y, z, u, w. Initally the Ikeda model was introduced to describe the dynamics of an optical bistable resonator, playing an important role in electronics and physiological studies and is well-known for delay-induced chaotic behavior, see e.g.[22] and references there-in. Later it was established that the Ikeda model or its modifications can be used to describe the dynamics of an opto-electronical, an acousto-optical systems and even the dynamics of the wavelength of the Distributed Bragg Reflector (DBR) Laser [22]. Furthermore, this investigation is of considerable practical importance, as the equations of the class B lasers with feedback (typical representatives of class B are solid-state, semiconductor, and low pressure CO_2 lasers [23]) can be reduced to an equation of the Ikeda type [24].
Physically x is the phase lag of the electric field across the resonator (it should be noted that in the opto-electronical and acousto-optical systems x is proportinal to the voltage fed to a modulator [12]); α is the relaxation coefficient for the driving x and driven y, z, u, w dynamical variables; τ is the feedback loop time delay; τ_1 is the coupling time delay between x and y,y and z, x and u,u and w; Below we will consider the case $\tau = \tau_1$;m_1, m_2, m_3, m_4, m_5 are the feedback strengths for the Ikeda systems x, y, z, u, w respectively; m_6, m_8, m_7, m_9 are the coupling strengths between the systems x and y,y and z, x and u, u and w, respectively. It is noted that system x is directly connected to system y and connection to system z occurs via system y. Analogously, system x is directly connected to system u and connection to system w occurs via system u. It should also be emphasized that there is no direct connection between the networks (y, z) and (u, w).

As mentioned above we will consider the all-to-all synchronization for the coupling topology presented in Fig.2. First we consider the complete synchronization case between the variables x and y. It is straightforward to establish that the synchronization error $\Delta_{x,y} = x - y$ under the condition

$$m_2 = m_1 - m_6 \tag{6}$$

obeys the dynamics

$$\frac{d\Delta_{x,y}}{dt} = -\alpha\Delta_{x,y} + m_2\Delta_{x,y}\cos x_\tau \tag{7}$$

Obviously $\Delta_{x,y} = 0$ is a solution of system (7).
The sufficient stability condition of the synchronization regime

$$x = y \tag{8}$$

can be found by applying the Lyapunov-Krasovskii functional approach [25-26]:

$$\alpha > |m_2| \tag{9}$$

By applying this procedure to synchronization between the dynamical variables y and z, x and z,x and u, u and w, x and w,y and u, z and u,y and w, z and w we establish that for the configuration in **Fig.2** all-to-all complete synchronization

$$x = y = z = u = w \tag{10}$$

occurs under the following conditions:

$$m_1 = 2m_2, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 \tag{11}$$

We note that formula (11) is the existence condition and formula (9) is the stability (9) condition for all-to-all complete synchronization (10). In the next Section we present the results of the numerical simulations of this synchronization regime.

III. NUMERICAL SIMULATIONS AND DISCUSSION

In this Section we numerically demonstrate how the analytical findings of the previous Section are validated. Synchronization quality is characterized by the cross-correlation coefficient C [27] between the dynamical variables say x and y:

$$C(\Delta t) = \frac{< (x(t)- < x >)(y(t+\Delta t)- < y >) >}{\sqrt{< (x(t)- < x >)^2 >< (y(t+\Delta t)- < y >)^2 >}}, \tag{12}$$

where the brackets $< . >$ represent the time average; Δt is a time shift between the dynamical variables.In our case $\Delta t = 0$. This coefficient indicates the quality of synchronization: $C = 1$ means perfect complete synchronization.

Figure 3 portrays time series of the system z for parameter values $\alpha = 8.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$. Figure 4 presents synchronization error dynamics $\Delta_{z,w} = z - w$ versus time for parameters as in figure 3. $C_{z,w} = 0.99$ is the cross-correlation coefficient between the systems z and w.For parameter values as in for Figure 3 the other cross-correlation coefficients are $C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = 0.99$. The value of the cross-correlation coefficients testify to the high quality chaos synchronization, which is vital for information processing in chaos-based communication systems and other possible applications. In numerical simulations we mainly presented the synchronization case between the most outer Ikeda models- z and w.

It should be noted that the approach based on the Lyapunov-Krasovskii method gives a sufficient stability condition for synchronization, but does not forbid synchronization [25] when the condition (9) is not met. In Figures 5 and 6 we present the case of chaos synchronization when the stability condition for all-to-all synchronization (9) is violated. Figure 5 shows the dynamics of the system z for parameter values $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau_5$. Error $\Delta_{z,w} = z - w$ dynamics is presented in Figure 6.It is seen that despite the fact that condition (10) is violated, there is a high degree of synchronization. $C_{z,w} = 1$ is the cross-correlation coefficient between the systems x and z. For this case the other cross-correlation coefficients are $C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = 1$.

We notice that larger values of the relaxation coefficient α decrease the amplitude of the chaotic vibrations.Comparing the dynamics of the variable z (Figures 3 and 5) and the error $z-w$ dynamics (Figures 4 and 6) one should pay attention to the scale on the ordinate axis.

Next we consider the case of variable time delays in the constituent Ikeda models, e.g. both the feedback and coupling time delays are variable. The role of modulated feedback and coupling time delays in controlling chaos in some laser systems was studied in [28].

We will consider three cases of time delay modulations: a) sinusoidal modulation of time delays; b)chaotic modulation of time delays; c)combined sinusoidal and chaotic modulation of time delays. For sinusoidal modulations we take

$$\tau(t) = \tau + \tau_a \sin(\omega_m t), \tag{13}$$

where τ is the zero-frequency component (constant time delay),τ_a is the amplitude,ω_m is the frequency of the modulation. For this case we use the following set of the new parameters: $\tau = 5$, $\tau_a = 1$,$\omega_m = 0.1$. Figure 7 shows dynamics of the Ikeda model x.Numerical simulations show that for this case the correlation coefficients between the junctions are:$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = 1$. Figure 8 demonstrate highest quality synchronization between Ikeda models z and w:Correlation coefficient $C_{z,w} = 1$. **Figure 8(a) pictures the dynamics of variables x (solid line) and y (dotted line) in one plot. It is clear that after some transient processes the dynamics of both variables coinside with each other.Correlation coefficient $C_{x,y} = 1$.**

For the case of chaotic modulations of the coupling time delays we choose the following form:

$$\tau(t) = 5 + 0.8x_1(t), \tag{14}$$

where $x_1(t)$ is the chaotic solution of the Ikeda model:

$$\frac{dx_1}{dt} = -2x_1(t) + 10\sin x_1(t-5) \tag{15}$$

Chaotic dynamics of x for parameters as in Eq.(15) and $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau_5$ is shown in Figure 9.
According to the numerical simulations, for the case of chaotically modulated feedback and coupling time delays the correlation coefficients between the Ikeda models are:$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = 1 = C_{z,w} = 1$.

Finally, we consider the case of the combined sinusoidal and chaotic modulations of the coupling time delays:

$$\tau(t) = 5 + 0.5x_1(t)\sin(0.1 * t). \tag{16}$$

The results of the numerical modeling for this case are:$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = C_{z,w} = 1$.
In support of high quality synchronization between the driven Ikeda models, in Fig.10 dependence of z on w is demonstrated.
We have also numerically experimented with different amplitudes and frequencies of the modulation and obtained that the synchronization quality is quite robust to such modulations. As shown by the numerical simulations the effect of dithering coupling and feedback time delays on the synchronization quality between the Ikeda models is not pronounced. In other words, the studied configuration of Ikeda models is quite robust to the modulation of the coupling and feedback delays. Thus, these results testify that driven Ikeda models, although are not coupled directly between themselves, can be synchronized quite robustly by a single driver model even under the conditions of the dithered feedback and coupling time delays.

We also note that complete synchronization between two Ikeda models was investigated in previous work [29] where the authors considered the case of sinusoidal modulation of the feedback time delays. In this paper we considered complete synchronization under the modulation of both feedback and coupling time delays (including the case of chaotic modulation) in a network (however simple) Ikeda systems.

It should also mentioned that chaos synchronization is not the only phenomenon observed in an ensemble of chaotic systems. Another very interesting phenomenon is the realization of chimera states in chaotic systems. In [30] the authors have studied dynamical properties of one-dimensional ensembles of identical chaotic oscillators with non-local coupling. The authors have established that such systems can demonstrate the transition from complete chaotic synchronization to spatiotemporal chaos when the coupling coefficient decreases. This transition is called the coherence incoherence transition and, for certain networks, is accompanied by the appearance of chimera states.

Apart from this, breathers and traveling waves can also be observed in some networks [31]. The study of these very interesting phenomena is beyond the scope of this research.

IV. CONCLUSIONS

To summarize, we have reported on all-to-all complete chaos synchronization in unidirectionally nonlinearly coupled Ikeda systems. We have considered both constant time delays (feedback and coupling times) and variable time delays cases. In case of constant time delays we have derived analytically the existence and stability conditions for complete chaos synchronization. Numerical simulations fully support the analytical findings.As synchronization is vital in communication systems, these results are of certain importance for information processing purposes. Additionally the results are useful for obtaining high emission power from such networks. Besides these results testify that driven Ikeda models, although are not coupled directly between themselves, can be synchronized quite robustly by a single driver model even under the conditions of the dithered feedback and coupling time delays. This studied configuration can serve as a *motif*(building block) for much more complex networks.

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Figure captions

FIG.1. Schematic view of chaos based communication system. For details, see,text.

FIG.2. Schematic view of the system under consideration, see text for details.

FIG.3. Numerical simulation of all-to-all synchronization between Ikeda systems with the coupling scheme described in Fig.2, Eqs.(1-5) for $\alpha = 8.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$.Dynamics of the system z is shown. Dimensionless units.

FIG.4. Error dynamics $\Delta_{z,w} = z - w$ versus time t for parameters as in FIG.3. $C_{z,w}$ is the cross-correlation coefficient between the systems z and w. Dimensionless units.

FIG.5. Numerical simulation of all-to-all synchronization between Ikeda systems with the coupling scheme described in Fig.2, Eqs.(1-5) for $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$. Note that stability condition (4) is not fulfilled.Time series of the system z is shown. Dimensionless units.

FIG.6. Error dynamics $\Delta_{z,w} = z - w$ versus time t for parameters as in FIG.5. $C_{z,w}$ is the cross-correlation coefficient between the systems z and w. Dimensionless units.

FIG.7. Chaotic dynamics of Ikeda model x for sinusoidal modulations of the feedback and coupling time delays.Dimensionless units.

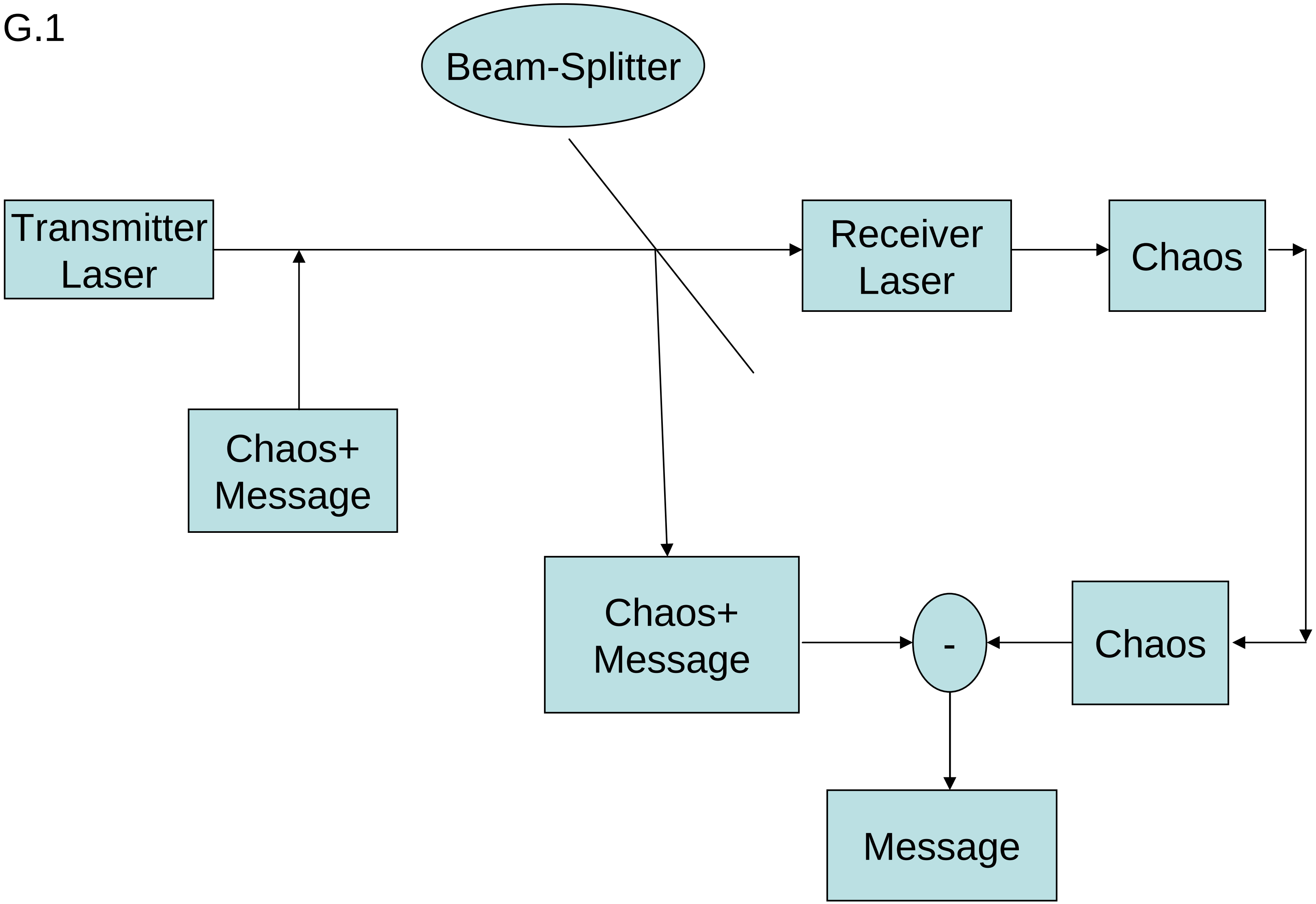
FIG.8.Synchronization between Ikeda models z and w in case of sinusoidal modulations of the feedback and coupling time delays:z versus w for parameters $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5, \tau_a = 1$,$\omega_m = 0.1$. Correlation coefficient $C_{z,w} = 1$.Dimensionless units.

FIG.8(a).Dynamics of Ikeda models x (solid line) and y(dotted line) for the parameter values as in Figure (8). Correlation coefficient $C_{x,y} = 1$.

FIG.9.Chaotic dynamics of Ikeda model x for chaotic modulations $\tau(t) = 5 + 0.8x_1(t)$ of the feedback and coupling time delays. Dimensionless units.

FIG.10. Synchronization between Ikeda models z and w in case of combined chaotic and sinusoidal modulations of the feedback and coupling time delays: z versus w for parameters $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5, \tau(t) = 5 + 0.5x_1(t)\sin(0.1 * t)$.Correlation coefficient $C_{z,w} = 1$. Dimensionless units.

FIG.1



Beam-Splitter
Transmitter Laser
Receiver Laser
Chaos
Chaos+ Message
Chaos+ Message
-
Chaos
Message

FIG.2

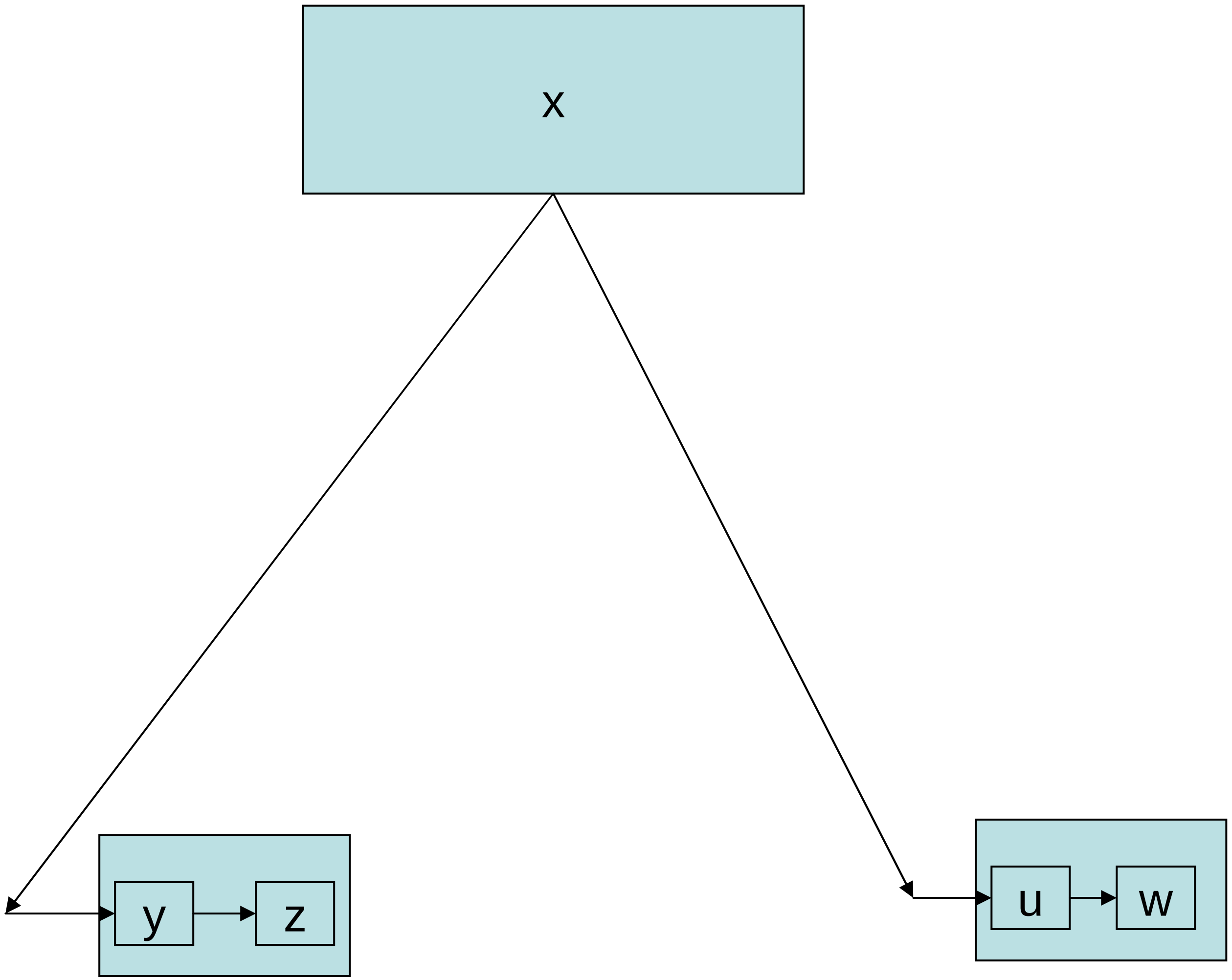
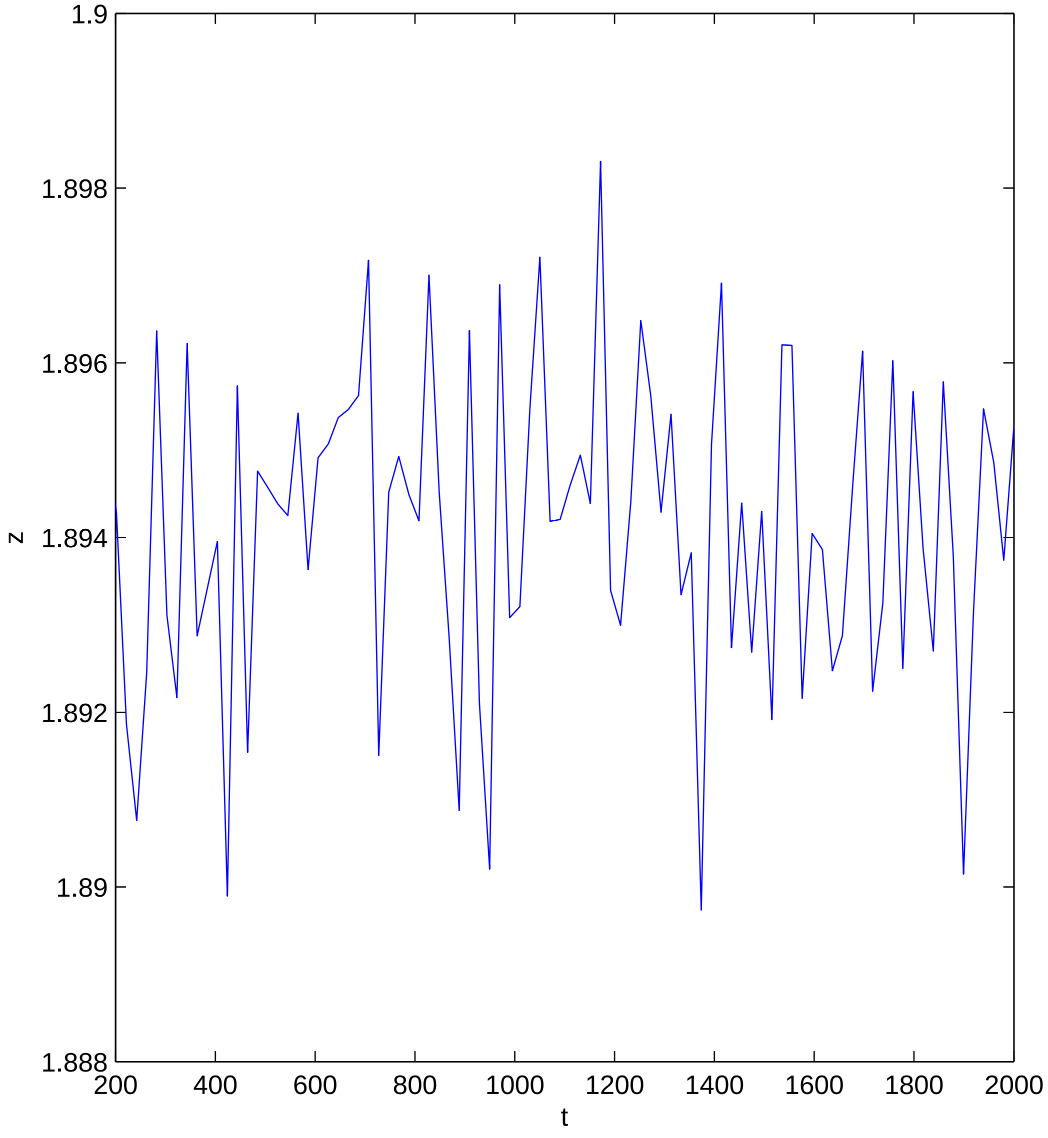


FIG.3



1.9
1.898
1.896
1.894
1.892
1.89
1.888
z
200
400
600
800
1000
1200
1400
1600
1800
2000
t

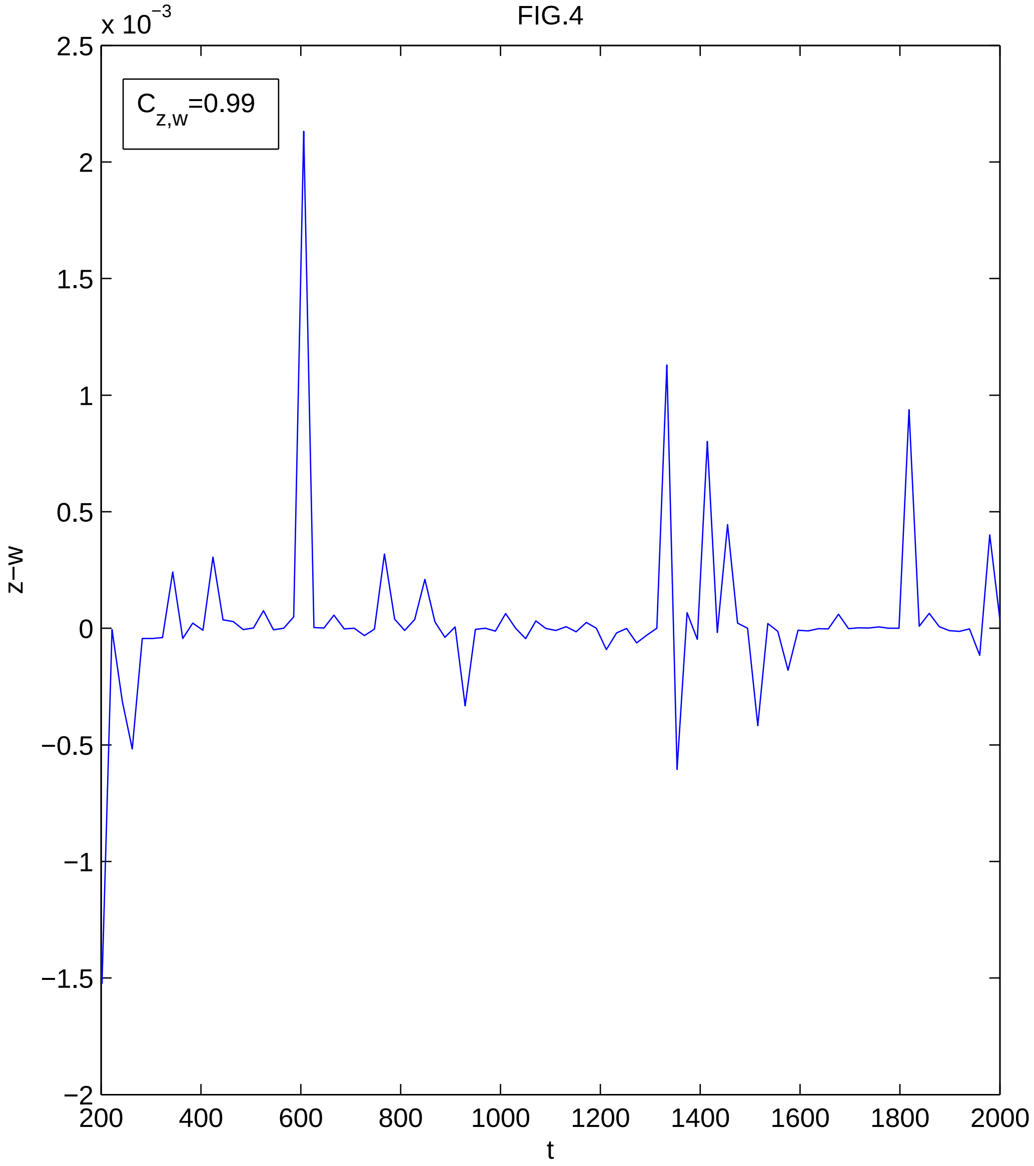


FIG.4
x 10^−3
C_z,w=0.99
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
z−w
200
400
600
800
1000
1200
1400
1600
1800
2000
t

FIG.5

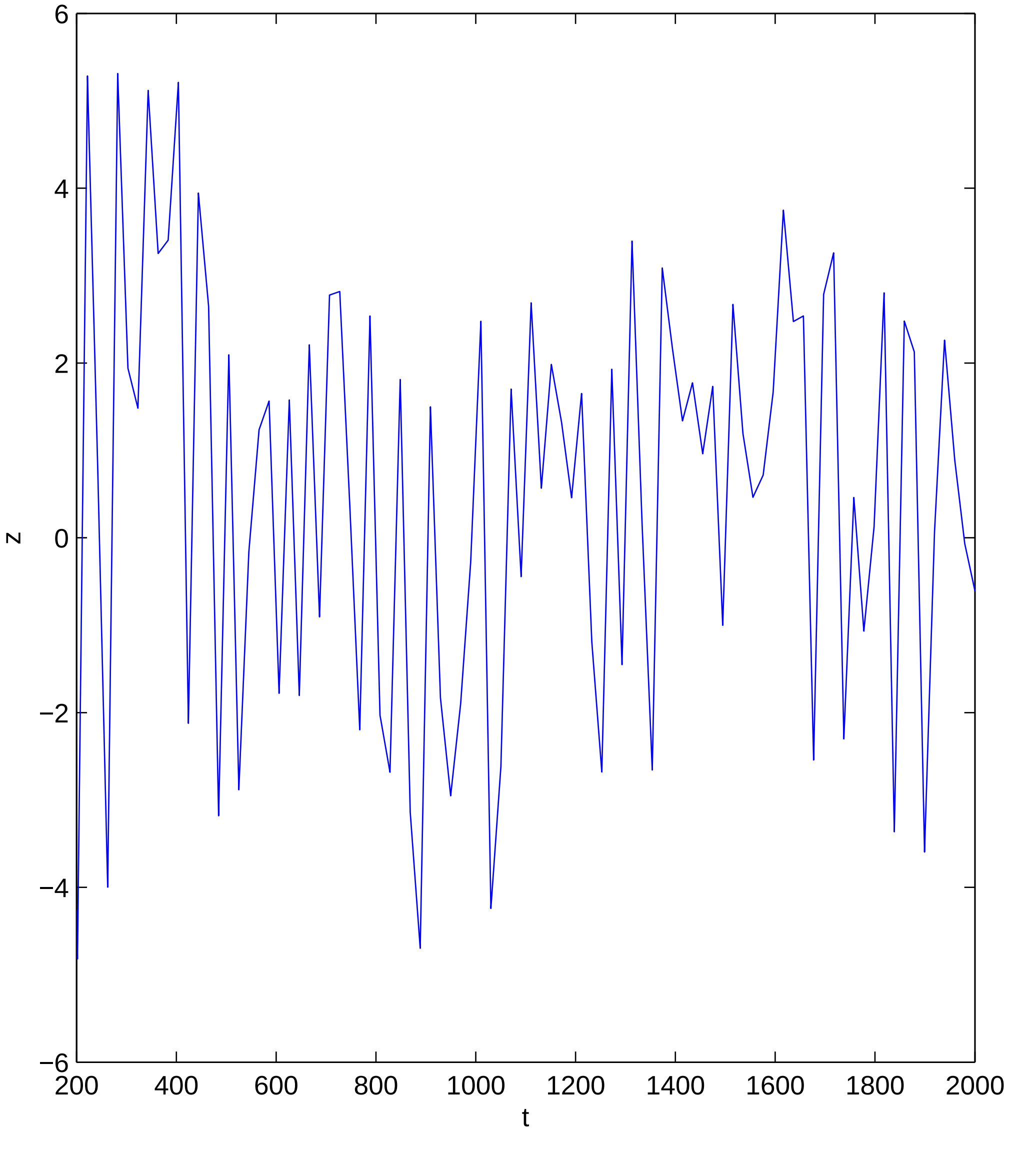
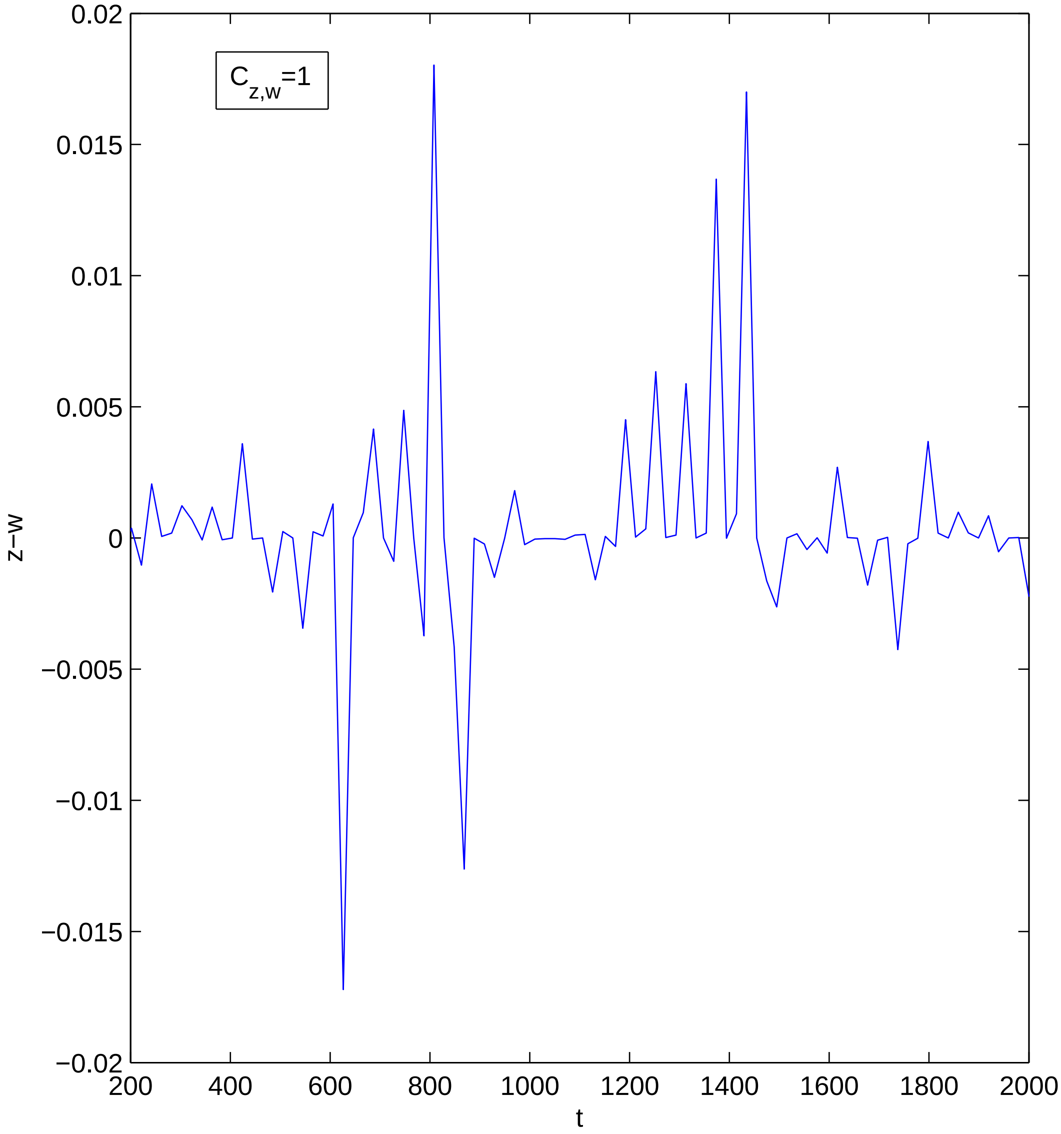


FIG.6



$C_{z,w}$=1
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
z−w
200
400
600
800
1000
1200
1400
1600
1800
2000
t

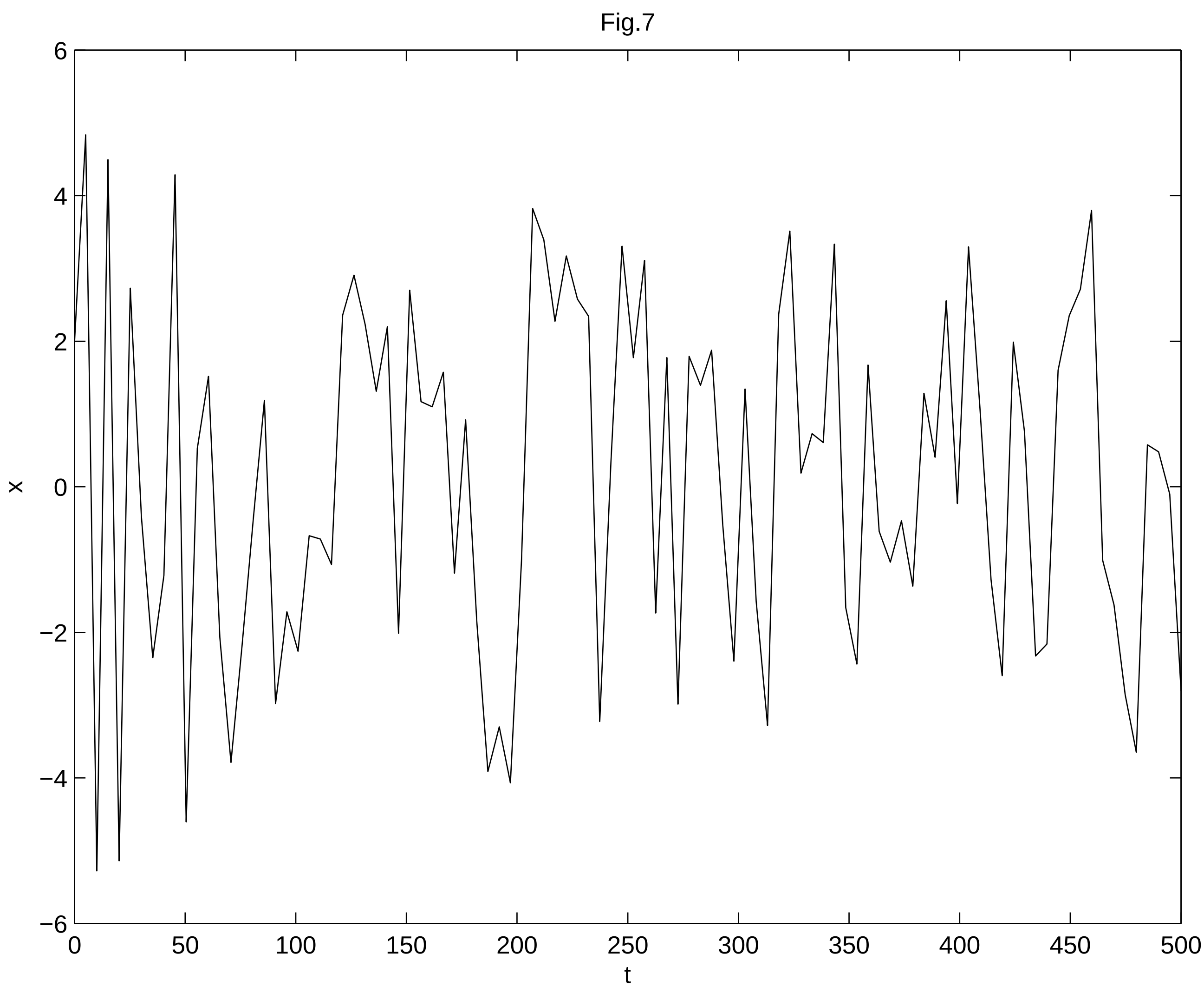


Fig.7
6
4
2
0
−2
−4
−6
x
0
50
100
150
200
250
300
350
400
450
500
t

Fig.8

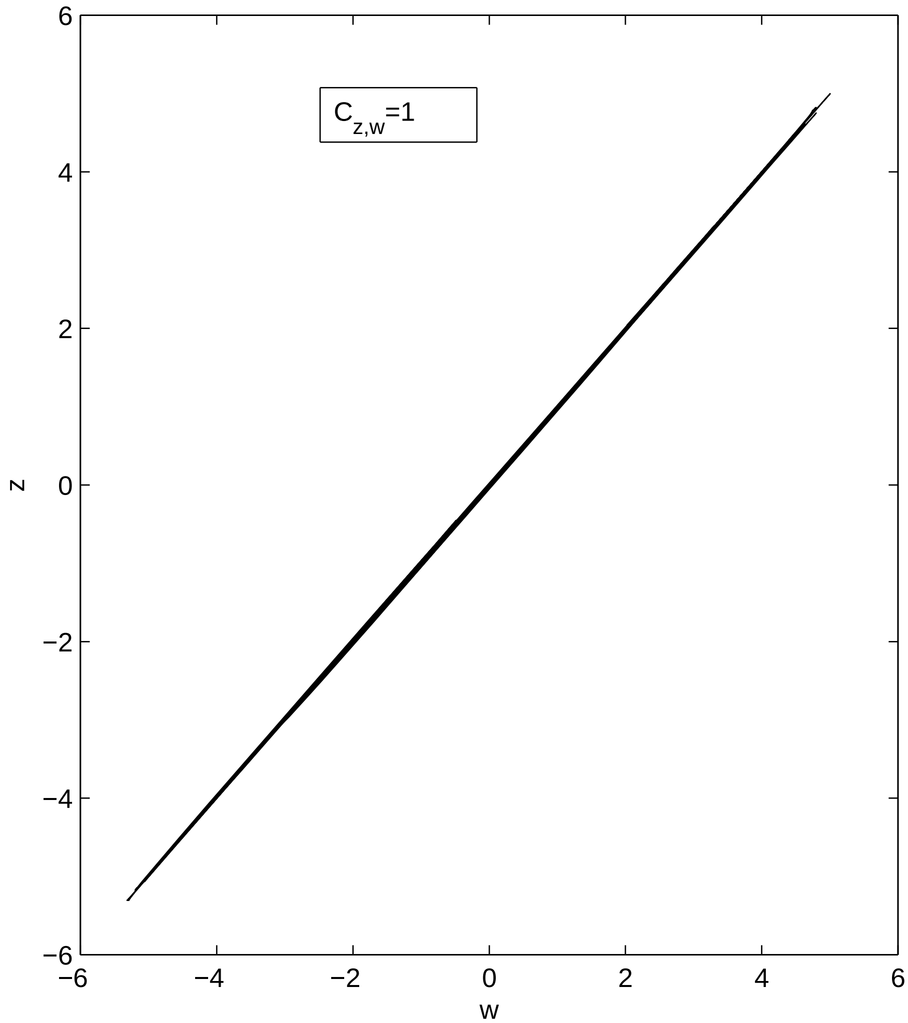


Fig.8(a)

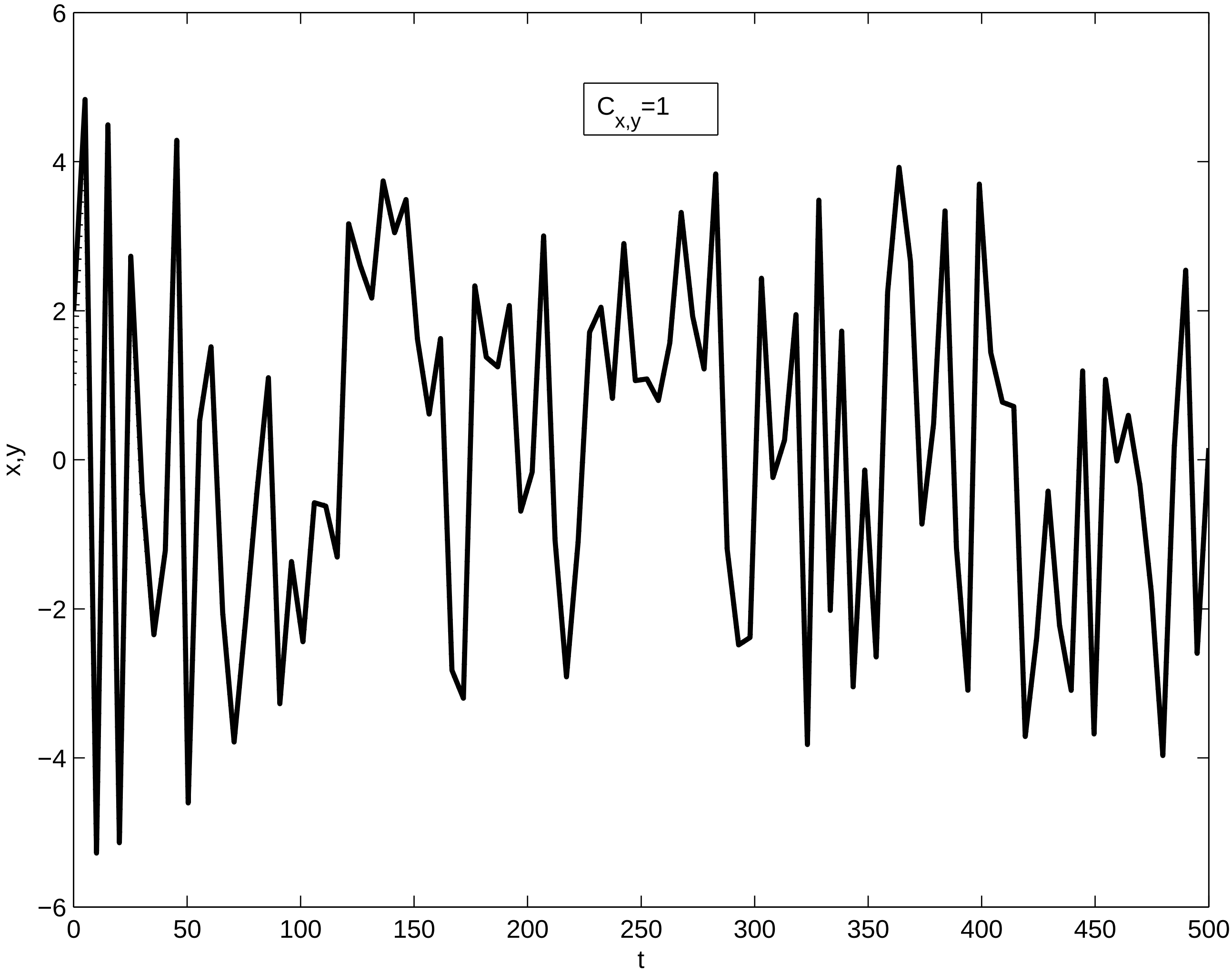


Fig.9

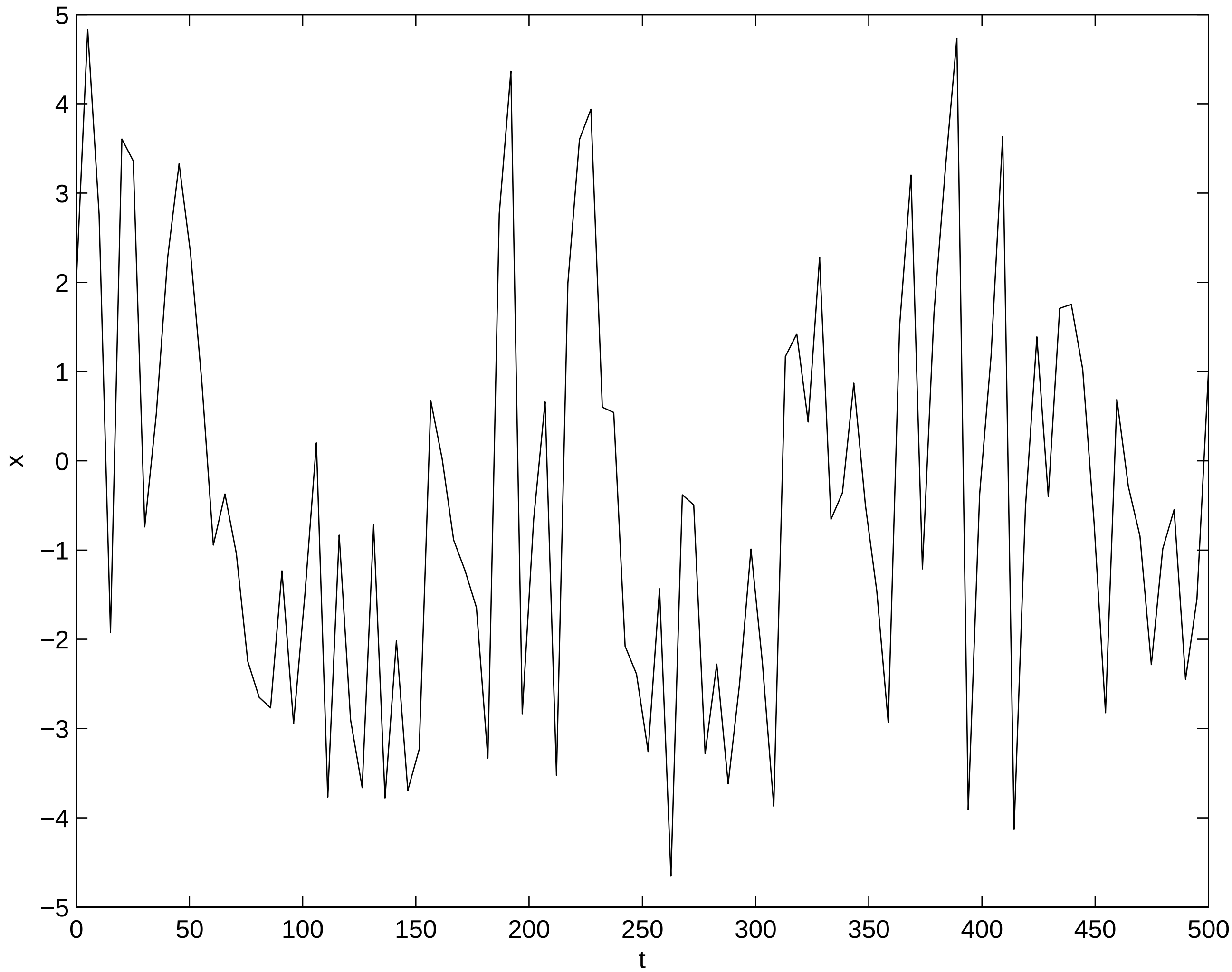


Fig.10