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ANOTHER GENERALIZED TRANSMUTED INVERSE EXPONENTIAL DISTRIBUTION: PROPERTIES AND APPLICATION

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ABSTRACT

In this study, we proposed another generalized transmuted inverse exponential distribution with three parameters and have transmuted inverse exponential and inverse exponential distributions as sub models. The hazard function of the distribution is nonmonotonic, unimodal and inverted bathtub shaped making it suitable for modelling lifetime data. We derived the moment, moment generating function, quantile function, maximum likelihood estimates of the parameters, Renyi entropy and order statistics of the distribution. A real life data set is used to illustrate the usefulness of the proposed model.

Keywords: Another generalized transmuted-G; Inverse exponential distribution; Entropy; Maximum likelihood

1.0 INTRODUCTION

Probability distributions have so many applications in the description of real world phenomenon. Classical distributions have been developed over decades to model data in engineering, actuarial, environmental and medical sciences, finance, demography and economics. Several probability distributions have been proposed for modeling of lifetime data in the literature, one of such distributions is inverse exponential distribution. The inverse exponential distribution was first introduced by [1], for modeling lifetime data. The cumulative density function (cdf) of inverse exponential distribution is

$$G(x) = \exp(-\frac{\beta}{x}) \qquad x > 0$$

(1)

Where $\beta > 0$ is the scale parameter. The corresponding density function (pdf) to (1) is

$$g(x) = \frac{\beta}{x^2}\exp(-\frac{\beta}{x}) \qquad x > 0$$

(2)

Many univariate distributions have been extended for accuracy and flexibility purposes in modelling. Based on this understanding, a number of authors have proposed and generalized the inverse exponential distribution. The kumaraswamy-Inverse Exponential distribution [2], Generalized inverse exponential distribution [3], Exponentiated generalized inverse exponential distribution [4], transmuted inverse exponential distribution [5], Transmuted generalized inverted exponential distribution [6], Beta inverted exponential distribution [7] and Beta generalized inverted exponential distribution [8].

In this paper, we introduced Another Generalized Transmuted nverse-Exponential distribution (AGT-IE). This distribution can be used to model data with inverted bathtub failure rate. It provides greater flexibility then generalized inverse exponential and inverse exponential distribution.

The rest of this paper is organised as: section 2 states the distribution function, density function; hazard function of Another Generalized Transmuted Inverse Exponential (AGT-IE) distribution. We derived its mathematical properties such as the moments and quantile function in section 3, while section 4 deals with maximum likelihood estimates of the parameters of the distribution. The expressions for entropy and order statistics of Another Generalized Transmuted Inverse Exponential

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(AGT-IE) were obtained in section 5 and 6 respectively. Lastly, in section 7 we provided the real life application of the proposed distribution.

2.0 ANOTHER GENERALIZED TRANSMUTED INVERSE EXPONENTIAL (AGT-IE) DISTRIBUTION

The cumulative distribution function (cdf) of Another Generalalized Transmuted class (AGT-G) of distributions according to [9] is

$$F(x)=(1+\lambda)\left[1-\left(\bar{G}(x)\right)^{\alpha}\right]-\lambda\left[1-\left(G(x)\right)^{\alpha}\right]^{2}$$

(3)

With the corresponding PDF

$$f(x)=\alpha g(x)\left(\bar{G}(x)\right)^{\alpha-1}\left\{1+\lambda-2\lambda\left[1-\left(\bar{G}(x)\right)^{\alpha}\right]\right\}$$

(4)

Where $\alpha>0,\ |\lambda|\le 1$ and $G(x)$ is the CDF of the baseline PDF. $\bar{G}(x)$ is the survival function of the baseline PDF and its is given by

$$\bar{G}(x)=1-G(x)$$

(5)

Substituting (1), (2) and (5) into (3) and (4) we obtained the cumulative density function CDF

$$F(x)=(1+\lambda)\left[1-\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\alpha}\right]-\lambda\left[1-\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\alpha}\right]^{2}$$

(6)

and probability density function PDF

$$f(x)=\frac{\alpha\beta}{x^{2}}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left\{1+\lambda-2\lambda\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]\right\}$$

(7)

of Another Generalized Transmuted Inverse Exponential $AGT-IE(x,\alpha,\beta,\lambda)$ distribution.

Where $\alpha>0$, $\beta>0$, $|\lambda|\le 1$

2.1 Sub-Models of AGT-IE Distribution

The sub-models that can be derived from AGT-IE distribution are as follows

I. If $\alpha=1$, equation (7) reduces to transmuted inverse exponential (T-IE) distribution with parameters β and λ [5]

II. If $\lambda=0$ and $\alpha=1$, (6) reduces to one parameter inverse exponential distribution with parameter β [1]

2.2 Hazard Function

The hazard function $h(x)$ otherwise known as hazard rate of Another Generalized Transmuted Inverse Exponential $AGT-IE(x,\alpha,\beta,\lambda)$ is given by

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$$h(x)=\frac{f(x)}{1-F(x)}$$

$$h(x)=\frac{\frac{\alpha\beta}{x^2}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left\{1+\lambda-2\lambda\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]\right\}}{1-(1+\lambda)\left[1-\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\alpha}\right]+\lambda\left[1-\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\alpha}\right]^2}$$

(8)

2.3 Shapes of Density and Hazard Functions

The possible shapes of the density and hazard functions of Another Generalized Transmuted Inverse Exponential ($AGT-IE$) distribution for selected values of α,β and λ are illustrated in figures below. The hazard function in figure 2 shows that $AGT-IE$ distribution allows for increasing, unimodal decreasing failure rate.

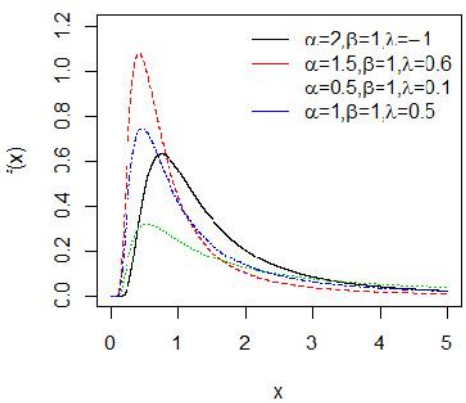

Figure.1 Plots of density function of $AGT-IE$

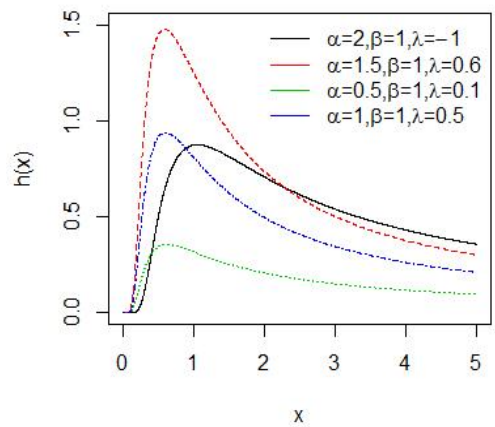

Figure.2 The plots of the hazard rate function of $AGT-IE$

3.0 MOMENTS AND QUANTILES

In this section, we present the rth moment, moment generating function (mgf) and the quantile function of the $AGT-IE(x,\alpha,\beta,\lambda)$ distribution.

3.1 Moments

Theorem 1: If X has the $AGT-IE(x,\alpha,\beta,\lambda)$ with $|\lambda|\le 1$, then the rth non central moment of X is

$$E(X^r)=\alpha\beta^r\Gamma(1-r)\left\{(1-\lambda)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}(j+1)^{r-1}+2\lambda\sum_{j=0}^{\infty}(-1)^j\binom{2\alpha-1}{j}(j+1)^{r-1}\right\}$$

(9)

Proof:

By definition

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$$E\left(X^r\right)=\int_{-\infty}^{\infty} x^r f(x)dx$$

$$E\left(X^r\right)=\int_{-\infty}^{\infty} x^r \frac{\alpha\beta}{x^2}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left\{(1+\lambda)-2\lambda\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]\right\}dx$$

$$E(X^r)=\alpha\beta(1+\lambda)\int_0^{\infty} x^{r-2}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}dx-2\alpha\beta\lambda\int_0^{\infty} x^{r-2}\exp\left(-\frac{\beta}{x}\right)$$

$$\times\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]dx$$

Considering the power series expansion

$$(1-z)^b=\sum_{j=0}^{\infty}(-1)^j\binom{b}{j}z^j \quad \text{where} \quad |z|\le 1$$

$$E(X^r)=\alpha\beta(1-\lambda)\beta^{r-1}\Gamma(1-r)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}(j+1)^{r-1}+2\alpha\beta\lambda\beta^{r-1}\Gamma(1-r)\sum_{j=0}^{\infty}(-1)^j\binom{2\alpha-1}{j}(j+1)^{r-1}$$

$$E(X^r)=\alpha\beta^r\Gamma(1-r)\left\{(1-\lambda)\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}(j+1)^{r-1}+2\lambda\sum_{j=0}^{\infty}(-1)^j\binom{2\alpha-1}{j}(j+1)^{r-1}\right\}$$

3.2 Moment Generating Function (MGF)

Theorem 2: If X has the $AGT-IE(x,\alpha,\beta,\lambda)$ with $|\lambda|\le 1$, then the Moment generating function (MGF) of x is

$$M_x(t)=\alpha\beta Z_{jk}\left[(1-\lambda)\binom{\alpha-1}{j}+2\lambda\binom{2\alpha-1}{j}\right]$$

(10)

Where $Z_{jk}=\sum_{j=0}^{\infty}\sum_{j=0}^{\infty}(-1)^j\frac{t^k}{k!}\left[\beta(j+1)\right]^{k-1}\Gamma(1-k)$

Proof:
By definition

$$M_x(t)=\int_0^{\infty} e^{tx} f(x)dx$$

$$M_x(t)=\int_0^{\infty} e^{tx}\left\{\frac{\alpha\beta}{x^2}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left\{(1+\lambda)-2\lambda\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]\right\}\right\}dx$$

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$$M_x(t)=\alpha\beta\left\{\begin{array}{l}(1+\lambda)\int_0^\infty \frac{e^{tx}}{x^2}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}-2\lambda\int_0^\infty \frac{e^{tx}}{x^2}\exp\left(-\frac{\beta}{x}\right)\\ \times\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]\end{array}\right\}dx$$

Using binomial expansion

$$\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}=\sum_{j=0}^{\infty}(-1)^j\binom{\alpha-1}{j}\exp\left(-\frac{\beta j}{x}\right)$$

And the expansion of $\exp(tx)$

$$\exp(tx)=1+tx+\frac{(tx)^2}{2!}+\frac{(tx)^3}{3!}+\cdots=\sum_{k=0}^{\infty}\frac{(tx)^k}{k!}$$

We now have that

$$M_x(t)=\alpha\beta(1-\lambda)\sum_{j=0}^{\infty}\sum_{j=0}^{\infty}\left\{(-1)^j\binom{\alpha-1}{j}\frac{t^k}{k!}\beta(j+1)^{k-1}\Gamma(1-k)\right\}$$

$$+2\alpha\beta\lambda\sum_{j=0}^{\infty}\sum_{j=0}^{\infty}\left\{(-1)^j\binom{2\alpha-1}{j}\frac{t^k}{k!}\beta(j+1)^{k-1}\Gamma(1-k)\right\}$$

Therefore

$$M_x(t)=\alpha\beta Z_{jk}\left[(1-\lambda)\binom{\alpha-1}{j}+2\lambda\binom{2\alpha-1}{j}\right]$$

Where $Z_{jk}=\sum_{j=0}^{\infty}\sum_{j=0}^{\infty}(-1)^j\frac{t^k}{k!}[\beta(j+1)]^{k-1}\Gamma(1-k)$

3.3 Quantile Function

The quantile function of corresponding to Another Generalized Transmuted Inverse Exponential $AGT-IE(x,\alpha,\beta,\lambda)$ distribution is the real solution of

$$x=Q(q)=\frac{\beta}{\ln\left\{1-\left[\frac{(\lambda-1)+\sqrt{(1-\lambda)^2+4\lambda(1-q)}}{2\lambda}\right]^{\frac{1}{\alpha}}\right\}^{-1}},\qquad 0<q<1$$

(11)

Where q is uniformly distributed within the interval $U(0,1)$ by letting $q=\frac{1}{2}$ in (11) we obtain the median of $AGT-IE(x,\alpha,\beta,\lambda)$

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$$x_{0.5} = \frac{\beta}{\ln\left\{1-\left[\frac{(\lambda-1)+\sqrt{\lambda^2+1}}{2\lambda}\right]^{\frac{1}{\alpha}}\right\}^{-1}}, \lambda \neq 0$$

(12)

4.0 ESITIMATION

The method of maximum likelihood is used for the estimation of the parameters of the $AGT-IE(x,\alpha,\beta,\lambda)$ distribution. The likelihood function is given by

$$L(\alpha,\beta,\lambda/x) = \prod_{i=1}^{n}\left\{\frac{\alpha\beta}{x^2}\exp\left(-\frac{\beta}{x}\right)\left[1-\exp\left(-\frac{\beta}{x}\right)\right]^{\alpha-1}\left\{1+\lambda-2\lambda\left[1-\left(1-\exp(-\frac{\beta}{x})\right)^{\alpha}\right]\right\}\right\}$$

(13)

Taking the natural log of both sides we and Let $\ln L(\alpha,\beta,\lambda/x) = l$ then

$$l = n\ln\alpha + n\ln\beta - 2\sum_{i=1}^{n}\ln x_i - \beta\sum_{i=1}^{n}\frac{1}{x_i} + (\alpha-1)\sum_{i=1}^{n}\ln\left[1-\exp\left(-\frac{\beta}{x_i}\right)\right]$$
$$+\sum_{i=1}^{n}\ln\left\{1+\lambda-2\lambda\left[1-\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha}\right]\right\}$$

(14)

The maximum likelihood estimates $\hat{\omega} = \left(\hat{\alpha},\hat{\beta},\hat{\lambda}\right)^T$ of $AGT-IE(x,\alpha,\beta,\lambda)$ is obtained by differentiating (15) partially with respect to α,β,λ and equating to zero.

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\ln\left[1-\exp\left(-\frac{\beta}{x_i}\right)\right] + \sum_{i=1}^{n}\left\{\frac{2\lambda\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha}\ln\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)}{1+\lambda-2\lambda\left[1-\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha}\right]}\right\}$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} - \sum_{i=1}^{n}\frac{1}{x_i} + (\alpha-1)\sum_{i=1}^{n}\left[\frac{\exp\left(-\frac{\beta}{x_i}\right)}{x_i\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)}\right] + \sum_{i=1}^{n}\left[\frac{2\lambda\alpha\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha-1}\exp\left(-\frac{\beta}{x_i}\right)}{x_i\left[1+\lambda-2\lambda\left[1-\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha}\right]\right]}\right]$$

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$$\frac{\partial l}{\partial \lambda}=\sum_{i=1}^{n}\left\{\frac{1-2\left[1-\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha}\right]}{1+\lambda-2\lambda\left[1-\left(1-\exp\left(-\frac{\beta}{x_i}\right)\right)^{\alpha}\right]}\right\}$$

The solutions to the nonlinear system of equations above will yield the ML estimators $\hat{\alpha},\hat{\beta}$ and $\hat{\lambda}$. Using large sample property of MLE estimators, estimators of ω can be treated as being approximately trivariate normal. The asymptotic distribution of the elements of the 3×3 information matrix for the $AGT-IE$ distribution is $\sqrt{n\left(\hat{\omega}-\omega\right)} \sqcup N_3\left(0,V^{-1}\right)$.where V is the expected information matrix. The expected information matrix is

$$V^{-1}=-E\begin{bmatrix}\frac{\partial^2 l}{\partial\alpha^2} & \frac{\partial^2 l}{\partial\alpha\partial\beta} & \frac{\partial^2 l}{\partial\alpha\partial\lambda}\\ \frac{\partial^2 l}{\partial\beta\partial\alpha} & \frac{\partial^2 l}{\partial\beta^2} & \frac{\partial^2 l}{\partial\beta\partial\lambda}\\ \frac{\partial^2 l}{\partial\lambda\partial\alpha} & \frac{\partial^2 l}{\partial\lambda\partial\beta} & \frac{\partial^2 l}{\partial\lambda^2}\end{bmatrix}$$

(15)

Solving the inverse matrix of the observed information matrix (15) gives the asymptotic variance and co-variance of the MLE $\hat{\alpha},\hat{\beta}$ and $\hat{\lambda}$. Hence approximate $100\left(1-\alpha\right)\%$, asymptotic confidence intervals for α,β and λ are

$$\hat{\alpha}\pm Z_{\frac{\alpha}{2}}\sqrt{\hat{V}_{11}}, \qquad \hat{\beta}\pm Z_{\frac{\alpha}{2}}\sqrt{\hat{V}_{22}}, \qquad \hat{\lambda}\pm Z_{\frac{\alpha}{2}}\sqrt{\hat{V}_{33}}$$

where $Z_{\frac{\alpha}{2}}$ is the upper αth percentile of the standard normal distribution.

5.0 RENYI ENTROPY

The entropy of a random variable X with density function $f(x)$ is a measure of variation of uncertainty. For any real parameter $\gamma>0$ and $\gamma\neq 1$. The Renyi entropy is given by

$$I_R(\gamma)=\frac{1}{1-\gamma}\log\left\{\int_{-\infty}^{\infty} f^{\gamma}(x)dx\right\}$$

(16)

The Renyi entropy of $AGT-IE\left(x,\alpha,\beta,\lambda\right)$ is by substituting (7) in (16)

$$f^{\gamma}(x)=\left[\frac{\alpha\beta}{x^2}\exp\left(-\frac{\gamma\beta}{x}\right)\left[1-\exp\left(-\frac{\gamma\beta}{x}\right)\right]^{\gamma(\alpha-1)}\left\{1+\lambda-2\lambda\left[1-\left(1-\exp\left(-\frac{\gamma\beta}{x}\right)\right)^{\alpha}\right]\right\}\right]^{\gamma}$$

$$f^{\gamma}\left(x\right)=\left(\frac{\alpha\beta}{x^2}\right)^{\gamma}\exp\left(-\frac{\gamma\beta}{x}\right)\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\gamma(\alpha-1)}\left(1+\lambda\right)^{\gamma}\left\{1-\frac{2\lambda}{(1+\lambda)}\left[1-\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\alpha}\right]\right\}^{\gamma}$$

(17)

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Applying the general binomial expansion on $\left\{1-\frac{2\lambda}{(1+\lambda)}\left[1-\left(1-\exp\left(-\frac{\beta}{x}\right)\right)^{\alpha}\right]\right\}^{\gamma}$, equation (17) becomes

$$I_R(\gamma)=\frac{1}{1-\gamma}\log\left\{(\alpha\beta)^{\gamma}(1+\lambda)^{\gamma}\sum_{i,j,k=0}^{\infty}(-1)^{i+j+k}\binom{\gamma}{i}\binom{i}{j}\binom{\gamma\alpha+j\alpha-\gamma}{k}\left(\frac{2\lambda}{1+\lambda}\right)\int_0^{\infty}x^{-2\gamma}\exp\left(-\frac{\beta(k+\gamma)}{x}\right)dx\right\}$$

$$I_R(\gamma)=\frac{\gamma}{1-\gamma}\left[\log(\alpha\theta)+\log(1+\lambda)\right]+\frac{1}{1+\lambda}\log\left[\sum_{k=0}^{\infty}w_k\Gamma(2\gamma-1)\right]$$

(18)

where $w_k=w_k(\alpha,\lambda,\gamma)=(-1)^k\sum_{i,j=0}^{\infty}(-1)^{i+j}\binom{\gamma}{i}\binom{i}{j}\binom{\gamma\alpha+j\alpha-\gamma}{k}\left(\frac{2\lambda}{1+\lambda}\right)$

6.0 ORDER STATISTICS

Consider a random sample $x_1,x_2,x_3,...,x_n$ drawn from $AGT-IE(x,\alpha,\beta,\lambda)$ distribution with probability density function and cumulative distribution function as defined in (7) and (6) respectively. The ith order statistics denoted by $x_{i:n}$ is given by

$$f(x_{i:n})=\frac{n!}{(i-1)!(n-i)!}f(x)F(x)^{i-1}\left(1-F(x)\right)^{n-i}$$

$f(x_{i,n})=Kf(x)F(x)^{i+j-1}$ where $K=\frac{n!}{(i-1)!(n-i)!}$, $F(x)$ and $f(x)$ are as defined in equations (6) and (7) respectively.

Applying the general binomial expansion, we have

$$f(x_{i:n})=K\left[\sum_{p=0}^{\infty}w_{k,m}x^{-2}\exp\left(-\frac{\beta(p+1)}{x}\right)-\sum_{q=0}^{\infty}h_{k,n}\exp\left(-\frac{\beta(p+1)}{x}\right)\right]$$

(19)

where

$$w_{k,m}=w_{k,m}(\alpha,\beta,\lambda)=(1+\lambda)\alpha\beta\sum_{k,m}^{\infty}(-1)^{k+m+p}\lambda^k(1+\lambda)^{i+j-k-1}\binom{i+j-1}{k}\binom{i+j+k-1}{m}\binom{\alpha(m+1)-1}{p}$$

and

$$h_{k,n}=h_{k,n}(\alpha,\beta,\lambda)=2\lambda\alpha\beta\sum_{k,n}^{\infty}(-1)^{k+n+q}\lambda^k(1+\lambda)^{i+j-k-1}\binom{i+j-1}{k}\binom{i+j+k}{n}\binom{\alpha(n+1)-1}{q}$$

7.0 APPLICATION

In this section application of AGT-IE to real life data set is provided. The data set is from [10] with respect to breaking strength of carbon fibres. The maximum likelihood method of estimation of parameters is used for estimation. The fit is compared with other models based on the maximised log-likelihood, Komogorov-Smirnov test (K-S) and Akaike Information Criterion (AIC). The histogram of the data set, pdf and cdf of the fitted models are also presented for graphical illustration of the goodness of fit.

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The estimate of the parameters of AGT-IE for the data set is as shown in the table 1. From table 1 the AGT-IE provides a better fit compared Inverse exponential (IE) distribution, Inverse Rayleigh (IR) distribution and generalized inverse exponential (GIE) distribution. The AGT-IE distribution has the smallest K-S and AIC statistics and the largest log-likelihood value. This proves that the three parameter AGT-IE seems to be a better model than other models considered in the table.

Table 1 Estimated parameters of the AGT-IE, IE, IR and GIE distribution for the data set

Models	**Parameter estimates (Standard errors)**			**Statistics**		
	α	β	λ	**K-S Statistics**	**Log L**	**AIC**
$AGT-IE$	8.175 (1.930)	5.215 (0.750)	-0.575	0.117	-149.12	304.23
IE	$-$	2.139 (0.214)	$-$	0.345	-199.39	400.79
IR	$-$	3.276 (0.329)	$-$	0.182	-175.24	352.48
GIE	9.059 (2.088)	6.197 (0.604)	$-$	0.130	-151.24	306.48

8.0 CONCLUSION

In this paper we proposed a new generalization of the in inverse exponential distribution, called the Another generalized transmuted inverse exponential distribution (AGT-IE). We derived the explicit expression moments, moment generating functions, quantile function, maximum likelihood estimate of the parameters, Renyi entropy and order statistics. An application to show that the proposed model provides a better fit than other competing models is done. We hope that this distribution will attract more applications in statistics.

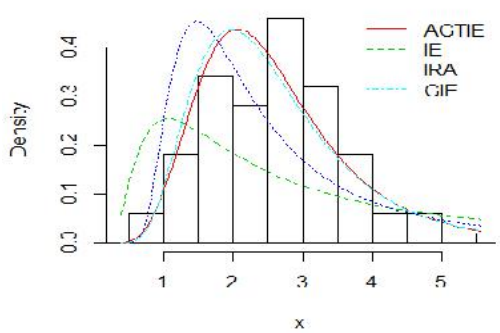

Figure 3: The fitted pdfs for the data set

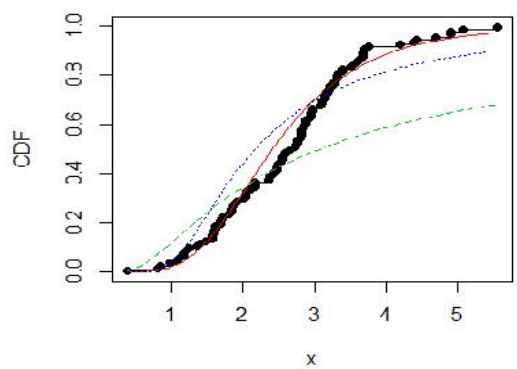

Figure 4: The fitted cdfs for the data set

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