

Original Research Article

ON MAKING AN INFORMED CHOICE BETWEEN TWO LOMAX-BASED CONTINUOUS
PROBABILITY DISTRIBUTIONS USING LIFETIME DATA

Abstract

Probability distributions and their generalizations have contributed greatly in modeling and analysis of random variables. However, due to the increased introduction of new distributions there has been a major problem with choosing and applying the right distribution for a given set of data. Most times, it is discovered that, the data set in question fits two or more probability distributions and hence one has to be chosen among the others. The Lomax-Weibull and Lomax-Log-Logistic distributions introduced in an earlier study using a Lomax-based generator were found to be positively skewed and may be victims of this situation especially when modeling positively skewed datasets. In this article, we apply the two distributions to some selected datasets to compare their performance and provide useful insight on how to select the most fit among them when dealing with a real life situation. We used the value of the log-likelihood function, AIC, CAIC, BIC, HQIC, Cram'er-Von Mises (W^*) and Anderson Darling (A^*) statistics as performance evaluation tools for selecting between the two distributions.

Keywords: Lomax-Weibull distribution, Lomax-Log-Logistic distribution, Lomax-based generator, Performance Evaluation.

1. Introduction

Previously, researchers have developed compound probability distributions which have been shown and taken to have performed much better than the well known standard probability distributions. Some and most of these studies are meant and do purposefully introduce a higher level of skewness and flexibility in the existing probability distributions by generalizing on a well-known distribution considering some facts and assumptions. Further work on some of these studies stated above have led to the production of some compound probability distributions such as skew normal distribution by [3], the generalized Weibull distribution by [20], the exponentiated Weibull distribution by [21], the beta-Weibull distribution by [8], the kumaraswamy normal distribution by [5], the Lomax-Frechet distribution by [13] and the Lomax-Gumbel distribution by [12] e.t.c all of which have been proven to be better than their parent or baseline counterparts.

Making a clear choice between two related probability distribution functions is very vital and has been done by some researchers under the following topics and considerations; “a test of discriminating between models” by [1], “discrimination between the Log-normal and Weibull distribution” by [7], “a method for discriminating between models with discussion” by [2],

“discrimination between the Log-normal and Gamma distribution” by [15], “On Modeling of Lifetimes Data Using Exponential and Lindley Distributions” by [24], “A Study of Probability Models in Monitoring Environmental Pollution in Nigeria” by [23] as well as “discriminating between the Weibull-normal and the generalized Weibull-normal distributions” by [14].

Cordeiro et al [6] proposed a new family of distributions called the Lomax generator with two extra positive parameters for extending continuous distributions and in their study, some special models such as the Lomax-normal, Lomax–Weibull, Lomax-log-logistic and Lomax–Pareto distributions were derived and discussed appropriately. Some mathematical properties of the new generator including ordinary and incomplete moments, quantile function, moment generating function, mean and median deviations, distribution of the order statistics and some entropy measures were presented. They discussed the estimation of the model parameters using the method of maximum likelihood and also proposed a minification process based on the marginal Lomax-exponential distribution. They also defined a log-Lomax–Weibull regression model for censored data. The point of interest and attraction for the authors here is to compare the performance of the Lomax-Weibull distribution to that of the Lomax-Log-logistic distribution because of the following reasons: (i) both distributions have the same shape pattern and are skewed to the right according to the graphical representation of the two distributions by [6]; (ii) applications to three real life datasets show that the Lomax-Weibull distribution has a better fit compare to beta-Weibull, Kummaraswamy-Weibull, Lomax-exponential, beta-pareto, Weibull and Burr distributions among others however, its performance has not been compared to that of the Lomax-Log-logistic distribution which seems to be related to the Lomax-Weibull distribution by graphical observations. Therefore the aim of this paper is to compare the fitness of the Lomax-Weibull distribution to that of the Lomax-Log-logistic distribution defined by [6] using some statistical measures and seven real life datasets.

The rest of this article is presented thus: in Section 2 we state the definition of Lomax distribution, Lomax-G family, Lomax-Weibull and Lomax-Log-logistic distributions as well as some statistics and goodness-of-fit measures. In section 3, we present some datasets, their summary and analysis and discussions. Finally, we offer some concluding remarks in section 4.

2 Materials and Methods

2.1 The Lomax Distribution and Lomax-G family of distributions.

The Lomax distribution was developed to tackle modeling of business failure data by [19]. The Lomax distribution is useful for a wide range application such as income and wealth inequality, size of towns, actuarial studies, medical and biological sciences, engineering, lifetime and reliability modeling.

A random variable X is said to follow a Lomax distribution with parameter α and β if its probability density function (*pdf*) is given by

$$f(x) = \frac{\alpha}{\beta} \left[1 + \left(\frac{x}{\beta} \right) \right]^{-(\alpha+1)} \quad (2.1)$$

and the corresponding cumulative distribution function (*cdf*) is given as

$$F(x) = 1 - \left[1 + \left(\frac{x}{\beta} \right) \right]^{-\alpha} \quad (2.2)$$

where $x > 0, \alpha > 0, \beta > 0$ where α and β are the shape and scale parameters respectively.

According to [6], the *cdf* and *pdf* of the Lomax-G family distributions (based on a Lomax generator) are respectively given by:

$$F(x) = 1 - \left\{ \frac{\beta}{\beta - \log[1 - G(x)]} \right\}^{\alpha} \quad (2.3)$$

and

$$f(x) = \alpha \beta^{\alpha} \frac{g(x)}{[1 - G(x)] \{\beta - \log[1 - G(x)]\}^{\alpha+1}}, \quad (2.4)$$

where $g(x)$ and $G(x)$ are the *pdf* and *cdf* of any continuous distribution to be extended, while $\alpha > 0$ and $\beta > 0$ are the additional new parameters responsible for the scale and shape of the distribution respectively.

2.2 The Lomax-Weibull Distribution (*LWD*)

The *cdf* and *pdf* of a random variable X taking a Weibull distribution with scale parameter $a > 0$ and shape parameter $b > 0$ are respectively given by (2.5) and (2.6):

$$G(x) = 1 - e^{-ax^b} \quad (2.5)$$

$$g(x) = abx^{b-1} e^{-ax^b} \quad (2.6)$$

where $x > 0, a > 0, b > 0$ where a and b are the scale and shape parameters respectively.

By substituting equations (2.5) and (2.6) into (2.3) and (2.4) and simplifying, we obtain the *cdf* and *pdf* of the Lomax-Weibull distribution respectively as:

$$F(x) = 1 - \frac{\beta^{\alpha}}{\left[\beta - \log \left[1 - \left(1 - e^{-ax^b} \right) \right] \right]^{\alpha}} \quad (2.7)$$

$$f(x) = \frac{\alpha \beta^\alpha a b x^{b-1}}{\left(\beta - \log \left[1 - \left(1 - e^{-a x^b} \right) \right] \right)^{\alpha+1}} \quad (2.8)$$

The following is a plot the *pdf* of the *LWD* at different parameter values.

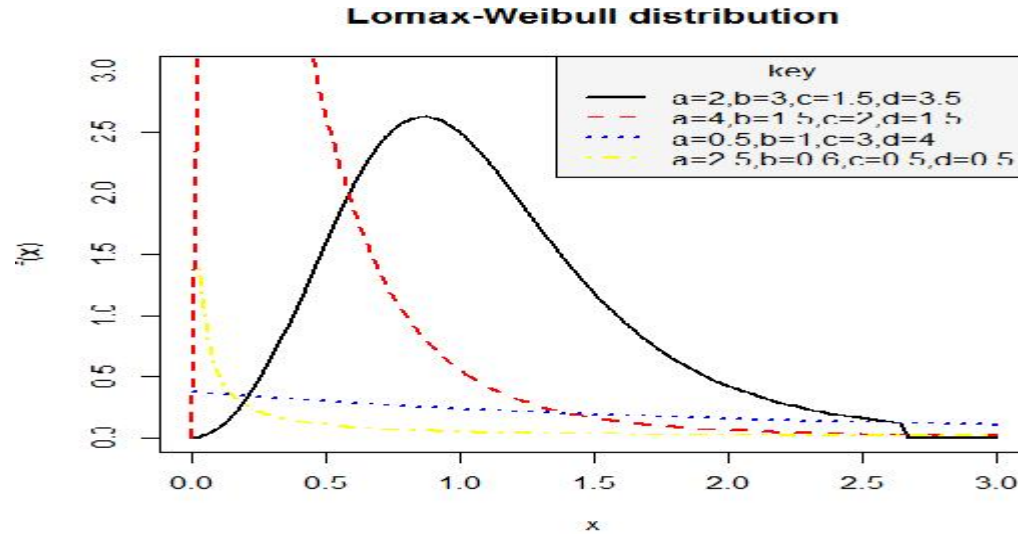


Figure 2.1: The graph of *pdf* of the *LWD* at different parameter values where $c = \alpha$ and $d = \beta$.

Considering the plot above, we can rightly say that the *LWD* is skewed to the right with a very high degree of peakedness and could also exhibit other shapes depending on the parameter values which are useful for modeling real life data sets.

2.3 The Lomax-Log-Logistic Distribution (LLD)

The *cdf* and *pdf* of the Log-logistic distribution are respectively given by:

$$G(x) = 1 - \left[1 + \left(\frac{x}{a} \right)^b \right]^{-1} \quad (2.9)$$

and

$$g(x) = \frac{b}{a^b} x^{b-1} \left[1 + \left(\frac{x}{a} \right)^b \right]^{-2} \quad (2.10)$$

For $x > 0$, where $a > 0$ and $b > 0$ are the scale and shape parameters respectively.

By substituting equations (2.9) and (2.10) into (2.3) and (2.4) and simplifying, we obtain the *cdf* and *pdf* of the Lomax-Log-logistic distribution as follows:

$$F(x) = 1 - \frac{\beta^\alpha}{\left[\beta + \log \left[1 + \left(\frac{x}{a} \right)^b \right] \right]^\alpha} \quad (2.11)$$

$$f(x) = \frac{\alpha \beta^\alpha b}{a^b} x^{b-1} \frac{\left[1 + \left(\frac{x}{a} \right)^b \right]^{-1}}{\left(\beta + \log \left[1 + \left(\frac{x}{a} \right)^b \right] \right)^{\alpha+1}} \quad (2.12)$$

Below is a graph of the *pdf* of the *LLD* for some selected values of the model parameters.

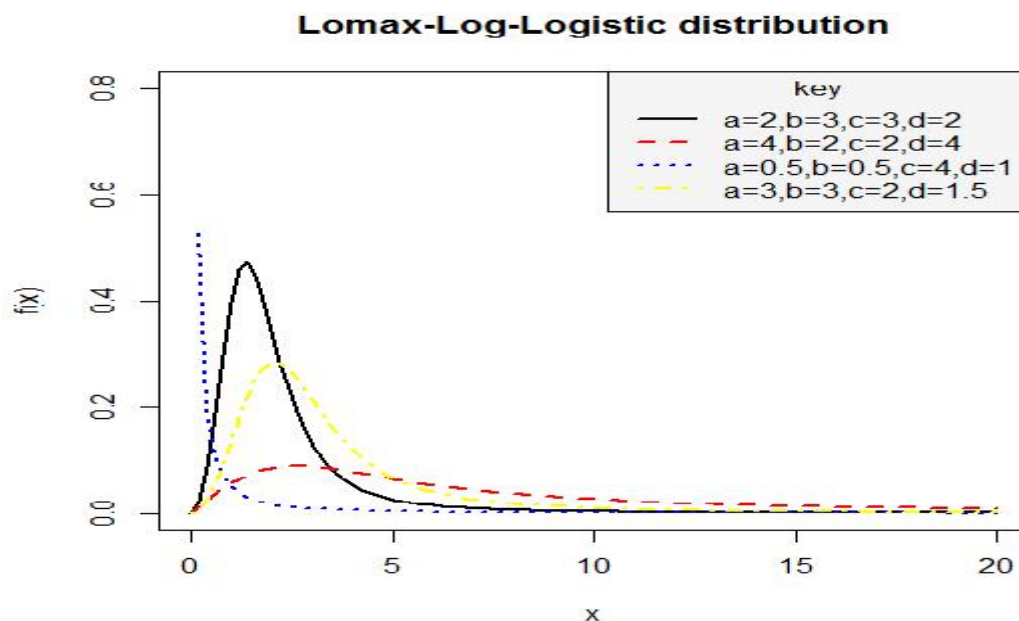


Figure 2.2: The graph of *pdf* of the *LLD* at different parameter values where $c = \alpha$ and $d = \beta$.

The plot for the *pdf* shows that the *LLD* is positively skewed and can take various shapes good for modeling lifetime datasets.

2.4 Goodness-of-Fit Test

To compare these two distributions, we have considered some criteria: the value of the log-likelihood function evaluated at the *MLEs* (*ll*), *AIC* (Akaike Information Criterion), *CAIC* (Consistent Akaike Information Criterion), *BIC* (Bayesian Information Criterion), and *HQIC* (Hannan Quin Information Criterion). These statistics are given as:

$$AIC = -2ll + 2k$$

128 $BIC = -2ll + k \log(n),$

129 $CAIC = -2ll + \frac{2kn}{(n-k-1)}$ and

130 $HQIC = -2ll + 2k \log[\log(n)]$

131 Where ll denotes the log-likelihood function evaluated at the $MLEs$, k is the number of model
132 parameters and n is the sample size.

133 We also used goodness-of-fit tests in order to know which distribution fits the data better, we
134 apply the Cram'er-Von Mises (W^*), and Anderson Darling (A^*) test statistics. Additional
135 information about these statistics can be obtained from [4]. These statistics can be computed as:

136 $W^* = w^2 \left(1 + 0.5/n\right)$ and

137 $A^* = A^2 \left(1 + 0.75/n + 2.25/n^2\right)$

138 Where

139 $w^2 = \sum_{i=1}^n \left\{ \frac{u_i - (2_i - 1)}{(2n)} \right\}^2 + \frac{1}{(12n)},$

140 $A^2 = -n - \frac{1}{n} \sum_{i=1}^n \left\{ (2_i - 1) \log(u_i) + (2n + 1 - 2_i) \log(1 - u_i) \right\},$

141 $V_i = F(x_i, \hat{\theta})$ is the known *cdf* with $\hat{\theta}$ (a k -dimensional parameter vector), $y_i = \Phi^{-1}(V_i)$ is the
142 standard quantile function, $u_i = \Phi\left\{\frac{y_i - \bar{y}}{s_y}\right\}$, $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ and $s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2$.

143 Note: In decision making process, the model with the lowest values for these statistics would be
144 chosen as the best fit model.

145

146 **3 Results and Discussions**

147 **3.1 Analysis of Data**

148 In this section, seven different datasets were used to fit both the *LWD* and Lomax-Log-Logistic
149 distribution by applying the formulas of the test statistics in section 4 in order to discriminating
150 between the two mentioned distributions. The available data sets and their respective summary
151 statistics are provided in as follows;

Dataset I: This dataset represents the remission times (in months) of a random sample of 128 bladder cancer patients. It has previously been used by [18]. It is summarized as follows:

Table 3.1: Summary statistics for dataset I

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537

Dataset II: This dataset is the strength data of glass of the aircraft window reported by [9].

Table 3.2: Summary statistics for dataset II

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	31	18.83	25.51	29.90	35.83	30.81	45.38	52.61	0.43	2.38

Dataset III: This dataset represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by [10] for fitting the Lindley distribution.

Table 3.3: Summary statistics for dataset III

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	100	0.80	4.675	8.10	13.020	9.877	38.500	52.3741	1.4953	5.7345

Dataset IV: This dataset represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by [11] and has been used by [25].

Table 3.4: Summary statistics for dataset IV

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	20	1.10	1.475	1.70	2.05	1.90	4.10	0.4958	1.8625	7.1854

Dataset V: This data represent the survival times in weeks for male rats from [16].

Table 3.5: Summary statistics for dataset V

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	20	40.00	86.75	119.00	140.80	113.45	165.00	1280.892	-0.3552	2.2120

Dataset VI: The dataset is from [17]. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests. Its summary is given as follows:

173 **Table 3.6: Summary statistics for dataset VI**

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	23	17.88	47.20	67.80	95.88	72.23	173.40	1404.78	1.0089	3.9288

174

175 **Dataset VII:** This dataset represents 66 observations of the breaking stress of carbon fibres of
 176 50mm length (in GPa) given by [22]. The descriptive statistics for this data are as follows:

177 **Table 3.7: Descriptive statistics for dataset VII**

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	66	0.390	2.178	2.835	3.278	2.760	4.900	0.795	-0.1285	3.2230

178 From the summary statistics of the seven data sets, we found that data sets I, II, III, IV and VI are
 179 positively skewed, while V is approximately normal. Also, data sets I, III and IV have higher
 180 kurtosis while others have moderate level of peakness.

181 **Table 3.8:** Performance of the distributions using their *AIC*, *CAIC*, *BIC* and *HQIC* values of the
 182 models' *MLEs* based on datasets **I-VII**.

Datasets	Models	Log-likelihood value	Parameter Estimates	Statistics	Model Ranks
Dataset I	LWD	420.7675	$\hat{\alpha}=0.3728$ $\hat{b}=0.8728$ $\hat{\alpha}=4.4735$ $\hat{\beta}=6.5702$ $\hat{\alpha}=7.9706$	$AIC=849.5355$ $CAIC=849.8607$ $BIC=860.9437$ $HQIC=854.1707$	2
	LLD	411.4727	$\hat{\alpha}=7.9706$ $\hat{b}=1.6719$ $\hat{\alpha}=8.1752$ $\hat{\beta}=5.4754$ $\hat{\alpha}=0.0717$	$AIC=830.9454$ $CAIC=831.2707$ $BIC=842.3536$ $HQIC=835.5806$	1
	LWD	146.435	$\hat{\alpha}=0.0917$ $\hat{b}=0.7887$ $\hat{\alpha}=7.1732$ $\hat{\beta}=5.3711$ $\hat{\alpha}=0.5706$	$AIC=300.8701$ $CAIC=302.4085$ $BIC=306.606$ $HQIC=302.7398$	1
	LLD	148.548	$\hat{\alpha}=9.5706$ $\hat{b}=3.3745$ $\hat{\alpha}=2.2712$ $\hat{\beta}=6.2711$ $\hat{\alpha}=0.5759$	$AIC=305.096$ $CAIC=306.6345$ $BIC=310.832$ $HQIC=306.9658$	2
Dataset II	LWD	342.2547	$\hat{\alpha}=0.5759$ $\hat{b}=0.7410$ $\hat{\alpha}=3.4755$ $\hat{\beta}=8.6739$ $\hat{\alpha}=0.5794$	$AIC=692.5095$ $CAIC=692.9305$ $BIC=702.9302$ $HQIC=696.7269$	2
	LLD	319.8772	$\hat{\alpha}=9.5894$ $\hat{b}=2.2864$ $\hat{\alpha}=7.5868$ $\hat{\beta}=4.8884$ $\hat{\alpha}=4.0761$	$AIC=647.7543$ $CAIC=648.1754$ $BIC=658.175$ $HQIC=651.9718$	1
	LWD	10.3037	$\hat{\alpha}=4.0761$ $\hat{b}=1.5707$ $\hat{\alpha}=0.9788$ $\hat{\beta}=2.9716$ $\hat{\alpha}=1.6779$	$AIC=28.6075$ $CAIC=31.2741$ $BIC=32.5904$ $HQIC=29.3849$	1
	LLD	15.7405	$\hat{\alpha}=1.6779$ $\hat{b}=1.6779$ $\hat{\alpha}=1.6779$ $\hat{\beta}=1.6779$ $\hat{\alpha}=1.6779$	$AIC=39.4809$	2

Dataset V	LWD	132.1458	$\hat{\alpha}=7.78^{19}$	CAIC=42.1476	1
			$\hat{\alpha}=5.26^{19}$	BIC=43.4639	
			$\hat{\beta}=7.08^{92}$	HQIC=40.2585	
	LLD	138.5343	$\hat{\alpha}=0.45^{03}$	AIC=272.2916	2
			$\hat{\alpha}=0.45^{03}$	CAIC=274.9582	
			$\hat{\beta}=0.82^{11}$	BIC=276.2745	
Dataset VI	LWD	128.6364	$\hat{\alpha}=1.21^{17}$	CAIC=273.0691	1
			$\hat{\beta}=6.52^{86}$	BIC=285.0687	
			$\hat{\alpha}=7.32^{57}$	HQIC=287.7354	
	LLD	138.7535	$\hat{\alpha}=7.32^{57}$	AIC=289.0516	2
			$\hat{\beta}=1.99^{88}$	BIC=289.0516	
			$\hat{\alpha}=1.59^{15}$	HQIC=285.8462	
Dataset VII	LWD	83.5572	$\hat{\beta}=9.07^{32}$	AIC=265.2728	1
			$\hat{\alpha}=0.21^{09}$	CAIC=267.495	
			$\hat{\alpha}=0.21^{09}$	BIC=269.8148	
	LLD	86.7655	$\hat{\beta}=4.29^{29}$	HQIC=266.4151	2
			$\hat{\alpha}=8.83^{91}$	AIC=285.507	
			$\hat{\alpha}=8.83^{91}$	CAIC=287.7292	
Dataset VIII	LWD	83.5572	$\hat{\beta}=2.64^{49}$	BIC=290.0489	1
			$\hat{\beta}=9.42^{49}$	HQIC=286.6492	
			$\hat{\alpha}=2.99^{33}$	AIC=175.1145	
	LLD	86.7655	$\hat{\beta}=1.54^{20}$	CAIC=175.7702	2
			$\hat{\alpha}=1.32^{82}$	BIC=183.8731	
			$\hat{\beta}=7.22^{01}$	HQIC=178.5754	
Dataset IX	LWD	83.5572	$\hat{\alpha}=6.94^{16}$	AIC=181.531	1
			$\hat{\alpha}=6.94^{16}$	CAIC=182.1867	
			$\hat{\beta}=3.52^{29}$	BIC=190.2896	
	LLD	86.7655	$\hat{\alpha}=9.21^{94}$	HQIC=184.9917	2
			$\hat{\beta}=0.47^{01}$		

Table 3.8 shows parameter MLEs to each one of the two fitted distributions for the seven data sets (Datasets I-VII), the table also shows the corresponding values of ll , AIC , BIC , $CAIC$ and $HQIC$ for each model. The values in Table 3.8 show that the LWD performs better for five datasets while the LLD performs better for just two datasets. We also notice that the five datasets for which the LWD performs better than LLD are those with low degree of kurtosis and the two datasets for which the LLD performs better are the ones with higher degree of kurtosis. Hence, we can say at this point that the LWD should be used for modeling positively skewed datasets most especially those with moderate or low kurtosis while the LLD should be applied when the datasets are skewed to the right with higher degree of peakedness.

Table 3.9: Performance of the distributions using the W^* and A^* values of the models based on dataset I, III, V and VI.

Datasets	Models	W^*	A^*	Model Ranks
Dataset I	LWD	0.0312	0.2084	1
	LLD	0.0382	0.2856	2
Dataset III	LWD	0.0212	0.1663	1
	LLD	0.0536	0.3742	2
Dataset V	LWD	0.0872	0.5949	1
	LLD	0.1136	0.7662	2
Dataset VI	LWD	0.0302	0.1867	1

VI	LLD	0.0521	0.3937	2
----	-----	--------	--------	---

194

195 Table 3.9 displays the values of goodness-of-fit statistics W^* and A^* for the two distributions
 196 under four selected datasets (I, III, V and VI). The results from table 3.9 confirm that irrespective
 197 of the coefficient of kurtosis, the LWD performs better than the LLD . Based on the values of
 198 these statistics in table 3.9, we can confidently say that the LWD is better than the LLD and hence
 199 should be used for analyzing positively skewed datasets. Hence, the statement above is in line
 200 with [6] who also said that the LWD is better than the Beta-Weibull, Kummaraswamy-Weibull,
 201 Weibull and the Burr distributions.

202 4 Conclusion

203 In this article, we discriminated between two Lomax-based continuous probability distributions
 204 called the LWD and LLD . We considered seven real life data sets of different status and used the
 205 value of the log-likelihood function, AIC , $CAIC$, BIC , $HQIC$, Cram'er-Von Mises(W^*) and
 206 Anderson Darling (A^*) statistics as test tools for selecting between these two distributions. Our
 207 analysis and results proved that the LWD has better performance compared to the LLD
 208 irrespective of the level of skewness and kurtosis.

209

210 References

- 211 [1] Atkinson A. A test of discriminating between models, *Biometrika*, 1969; 56: 337-341.
- 212 [2] Atkinson A. A method for discriminating between models with Discussion, *J. Royal Stat.*
 213 *Society*, Ser. B, 1970; 32: 323-353.
- 214 [3] Azzalini A. A class of distributions Which Includes the Normal Ones, *Scandinavian J. of*
 215 *Stat.*, 1985; 31: 171-178.
- 216 [4] Chen G, Balakrishnan N. A general purpose approximate goodness-of-fit test. *J. Quality*
 217 *Techn.*, 1995; 27,154-161.
- 218 [5] Cordeiro GM, De Castro M. A new family of generalized distributions, *J. of Stat. Comp. and*
 219 *Sim.*, 2009; 1-17.
- 220 [6] Cordeiro GM, Ortega EMM, Popovic BV, Pescim RR. The Lomax generator of
 221 distributions: Properties, minification process and regression model. *Appl. Math. Comp.*,
 222 2014; 247:465-486
- 223 [7] Dumonceaux R, Antle CE. Discriminating between the Log-Normal and Weibull
 224 Distribution, *Techn.*, 1973; 15: 923-926.
- 225 [8] Famoye F, Lee C, Olumolade O. The Beta-Weibull Distribution. *J. of Stat. Th. and Appl.*,
 226 2005; 4(2): 121-136.

- 227 [9] Fuller EJ, Frieman S, Quinn J, Quinn G, Carter W. Fracture mechanics approach to the
228 design of glass aircraft windows: A case study. *SPIE Proc.*, 1994; 2286: 419-430.
- 229 [10] Ghitany M, Al-Mutairi D, Balakrishnan N, Al-Enezi I. Power Lindley distribution and
230 associated inference. *Comp. Stat. Data Analy*, 2013; 64: 20-33.
- 231 [11] Gross AJ, Clark VA. Survival distributions reliability applications in the Biometrical
232 Sciences. John Wiley, New York, USA, 1975.
- 233 [12] Gupta J, Garg M, Gupta M. The Lomax-Gumbel Distribution. *Palestine J. of Math.*, 2016;
234 5(1): 35–42.
- 235 [13] Gupta V, Bhatt M, Gupta J. The Lomax-Frechet distribution. *J. of Rajasthan Acad. of Phy.*
236 *Sci.*, 2015; 14(1): 25-43
- 237 [14] Ieren TG, Yahaya A. Discrimination between the Weibull-Normal and the Generalized
238 Weibull-Normal Distributions, *Abacus: The J. of Math. Ass. of Nigeria*, 2017; 44(1): 270-
239 274.
240
- 241 [15] Kundu D, Manglick A. Discriminating between the Log-Normal and Gamma Distributions,
242 *Noval Res. Log.*, 2004; 51:893-905.
- 243 [16] Lawless JF. Statistical Models and Methods for Lifetime Data, 2nd Ed., Wiley, New Jersey,
244 2003.
- 245 [17] Lawless JF. Statistical models and methods for lifetime data. John Wiley and Sons, New
246 York, USA, 1982.
- 247 [18] Lee ET, Wang JW. Statistical Methods for Survival Data Analysis. 3rd Ed., John Wiley and
248 Sons, New York, ISBN: 9780471458555, 2003; Pages: 534.
- 249 [19] Lomax KS. Business failures: Another example of the analysis of failure data, *J. American*
250 *Stat. Ass.*, 1954; 49: 847–852.
- 251 [20] Mudholkar GS, Kollia GD. Generalized Weibull Family: a Structural Analysis. *Comm.*
252 *Stat.: Th. and Meth.*, 1994; 23(4): 1149-1171.
- 253 [21] Mudholkar GS, Srivastava DK, Freimaer M. The Exponentiated Weibull Family: an
254 analysis of the Bus-Motor-Failure Data. *Technometrics.*, 1995; 37(4): 436-445.
- 255 [22] Nichols MD, Padgett WJ. A bootstrap control chart for Weibull percentiles. *Quality Rel.*
256 *Eng. Int.*, 2006; 22: 141-151.

- 257 [23] Oguntunde PE, Odetunmibi OA, Adejumo AO. A Study of Probability Models in
 258 Monitoring Environmental Pollution in Nigeria. *Journal of Probability and Statistics*,
 259 2014; Article ID: 864965, 6 Pages
- 260 [24] Shanker R, Fesshaye H, Selvaraj S. On Modeling of Lifetimes Data Using Exponential and
 261 Lindley Distributions. *Biometrics & Biostatistics International Journal*, 2015; 2(5):
 262 00042.
- 263 [25] Shanker R, Fesshaye H, Sharma S. On two-Parameter Lindley distribution and its
 264 applications to model lifetime data. *Biom. Biostat. Int. J.*, 2016; 3(1): 00056. DOI:
 265 10.15406/bbij.2016.03.00056