

On the Properties and Applications of Transmuted Odd Generalized Exponential-Exponential Distribution

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ABSTRACT

This article proposed a new distribution referred to as the transmuted odd generalized exponential-exponential distribution (*TOGEED*) as an extension of the popular odd generalized exponential-exponential distribution by using the Quadratic rank transmutation map (*QRTM*) proposed and studied by [28]. Using the transmutation map, we defined the probability density function (*pdf*) and cumulative distribution function (*cdf*) of the transmuted odd generalized Exponential- Exponential distribution. Some properties of the new distribution were extensively studied after derivation. The estimation of the distribution's parameters was also done using the method of maximum likelihood estimation. The performance of the proposed probability distribution was checked in comparison with some other generalizations of Exponential distribution using a real life dataset.

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1. INTRODUCTION

An Exponential distribution which can be used in Poisson processes gives a description of the time between events. The distribution has been applied widely in life testing experiments. The distribution exhibits memoryless property with a constant failure rate which makes the distribution unsuitable for real life problems and hence creating a vital problem in statistical modeling and applications.

There are several ways of adding one or more parameters to a distribution function which makes the resulting distribution richer and more flexible for modeling data. A recap of some of these ways or methods include the Beta generalized family (Beta-G) by [13], the Kumaraswamy-G family by [9], Transmuted family of distributions by [28], Gamma-G (type 1) by [34], McDonald-G by [1], Gamma-G (type 2) by [26], Gamma-G (type 3) by [31], Log-gamma-G by [6], Exponentiated T-X by [5], Exponentiated-G (EG) by [10], Logistic-G by [32], Gamma-X by [4], Logistic-X by [29], Weibull-X by [3], Weibull-G by [7], a new Weibull-G family by [30], a Lomax-G family by [11], a new generalized Weibull family by [12] and Beta Marshall-Olkin family of distributions by [2] and several other studies. As a result of the above mentioned methods, some studies have been done to extend the exponential distribution and some of the recent studies on the generalization of exponential distribution are; a quasi exponential distribution by [27], the Gompertz inverse exponential distribution by [23], the Marshall-olkin Logistic-exponential distribution by [18], the transmuted exponential distribution by [24], transmuted inverse exponential distribution by [21], the odd generalized exponential-exponential distribution by [17] and the Weibull-Exponential distribution by [22]. Of interest to us in this article is the odd generalised exponential-exponential distribution (*OGEED*) which can be use in various fields to model variables whose chances of success decreases with time whereas those of failure increases as time increases. It was also discovered that the *OGEED* is positively skewed and performed better than some existing distributions like Gamma, exponentiated exponential, Weibull and Pareto distributions.

According to [17], a random variable X is said to follow a odd generalized exponential-exponential

distribution with parameters α and θ if its probability density function (*pdf*) is given as

$$f(x) = \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} \quad (1.1)$$

and the corresponding cumulative distribution function (*cdf*) is given as

$$F(x) = 1 - e^{-\alpha(e^{\theta x} - 1)} \quad (1.2)$$

The *pdf* and *cdf* of the transmuted *OGEED* are obtained using the steps proposed by [28]. According to [28], a random variable X is said to have a transmuted distribution function if its *pdf* and *cdf* are respectively given by;

$$f(x) = g(x)[1 + \lambda - 2\lambda G(x)] \quad (1.3)$$

and

$$F(x) = (1 + \lambda)G(x) - \lambda[G(x)]^2 \quad (1.4)$$

respectively.

where; $x > 0$, and $-1 \leq \lambda \leq 1$ is the transmuted parameter, $G(x)$ is the *cdf* of any continuous distribution while $f(x)$ and $g(x)$ are the associated *pdf* of $F(x)$ and $G(x)$, respectively.

The aim of this paper is to introduce a new continuous distribution called the Transmuted Odd Generalized Exponential-Exponential distribution (*TOGEED*) with a decreasing failure rate useful for analysing real life data using the proposed quadratic rank transmutation map by [28]. The remaining parts of this paper are presented in sections as follows: We defined the new distribution and give its plots in section 2. Section 3 derived some properties of the new distribution. The estimation of parameters using maximum likelihood estimation (MLE) is provided in section 4. An application of the new model with other two existing one to a real life data is done in section 5 and some useful conclusions are made in section 5.

2. THE TRANSMUTED ODD GENERALIZED EXPONENTIAL-EXPONENTIAL DISTRIBUTION (*TOGEED*)

Substituting equation (1.1) and (1.2) in (1.3) and (1.4) and simplifying, we obtain the *cdf* and *pdf* of the *TOGEED* as follows:

$$F(x) = 1 - e^{-\alpha(e^{\theta x} - 1)} \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta x} - 1)} \right] \quad (2.1)$$

and

$$f(x) = \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x} - 1)} \right] \quad (2.2)$$

respectively. Where, $x > 0, \alpha > 0, \theta > 0, -1 \leq \lambda \leq 1$, α is the shape parameters respectively and θ is the exponential parameter while λ is called the transmuted parameter.

The *pdf* and *cdf* of the *TOGEED* using some parameter values are displayed in Figs 2.1 and 2.2 as follows.

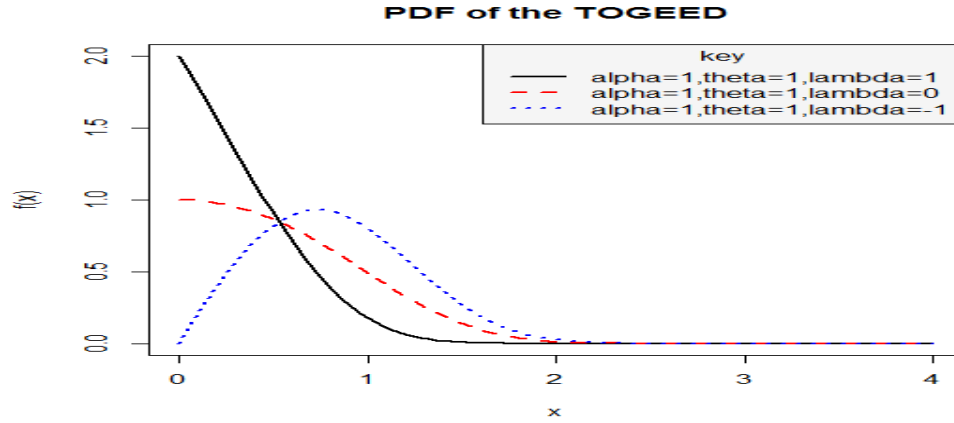


Fig. 2.1. PDF of the *TOGEED* for different values of the parameters α, θ – and $-\lambda$ as displayed in the key on the graph

Fig. 2.1 indicates that the *TOGEED* distribution could be positively skewed and otherwise depending on the parameter values. This means that distribution can be very useful for datasets with right-tailed shapes.

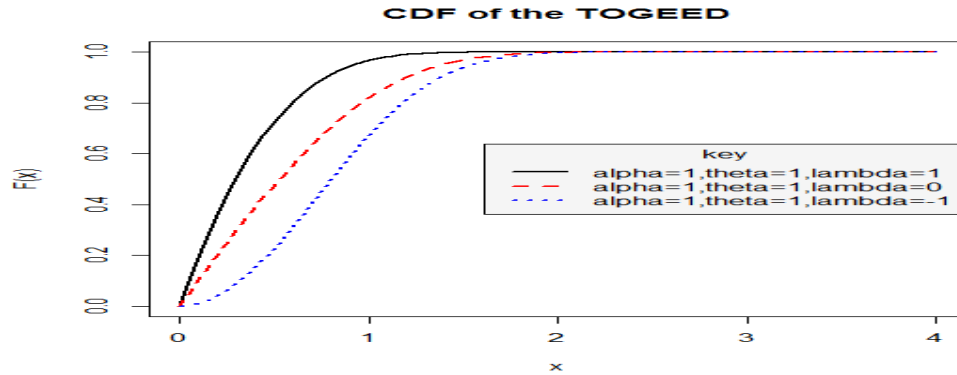


Fig. 2.2. CDF of the *TOGEED* for different values of the parameters α, θ – and $-\lambda$ as displayed in the key on the graph

From the above *cdf* plot, the *cdf* increases when X increases, and approaches 1 when X becomes large, as expected.

3. PROPERTIES

In this section, we defined and discuss some properties of the *TOGEED* distribution.

3.1 Moments

Let X denote a continuous random variable, the n^{th} moment of X is given by;

$$\mu'_n = E(X^n) = \int_0^{\infty} x^n f(x) dx \quad (3.1.1)$$

where $f(x)$ the *pdf* of the *TOGEED* is as given in equation (3.1.1).

$$f(x) = \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x} - 1)} \right]$$

Expansion and simplification of the *pdf* gives:

$$f(x) = (1 - \lambda) \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} + 2\lambda \alpha \theta e^{\theta x} e^{-2\alpha(e^{\theta x} - 1)} \quad (3.1.2)$$

Hence,

$$\begin{aligned} \mu'_n &= E(X^n) = \int_0^{\infty} x^n f(x) dx = \int_0^{\infty} x^n \left\{ (1 - \lambda) \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} + 2\lambda \alpha \theta e^{\theta x} e^{-2\alpha(e^{\theta x} - 1)} \right\} dx \\ \mu'_n &= \int_0^{\infty} x^n (1 - \lambda) \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} dx + \int_0^{\infty} x^n 2\lambda \alpha \theta e^{\theta x} e^{-2\alpha(e^{\theta x} - 1)} dx \\ \mu'_n &= (1 - \lambda) \alpha \theta \int_0^{\infty} x^n e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} dx + 2\lambda \alpha \theta \int_0^{\infty} x^n e^{\theta x} e^{-2\alpha(e^{\theta x} - 1)} dx \end{aligned} \quad (3.1.3)$$

Using integration by substitution in (3.1.3) above we have,

Let

$$\begin{aligned} w &= e^{\theta x} - 1 \Rightarrow x = \frac{\ln(w)}{\theta} \\ \frac{dw}{dx} &= \theta e^{\theta x} \Rightarrow dx = \frac{dw}{\theta e^{\theta x}} \end{aligned}$$

Therefore, substituting for w and dx in equation (3.1.3) above and simplifying, we have:

$$\mu_n' = \frac{(1-\lambda)\alpha}{\theta^n} \int_0^\infty (\ln(w))^n e^{-\alpha w} dw + \frac{2\lambda\alpha}{\theta^n} \int_0^\infty (\ln(w))^n e^{-2\alpha w} dw \quad (3.1.4)$$

Using a result by [20], [8] and [33],

$$\int_0^\infty (\ln(w))^n e^{-\alpha w} dw = I(n, \alpha)$$

Reduces to

$$I(n, \alpha) = \left(\frac{\partial}{\partial a} \right)^n \left[(\alpha)^{-a} \Gamma(a) \right] \big|_{a=1}$$

Hence, substituting and simplifying in (3.1.4), we have:

$$\mu_n' = \frac{(1-\lambda)\alpha}{\theta^n} \left(\frac{\partial}{\partial a} \right)^n \left[(\alpha)^{-a} \Gamma(a) \right] \big|_{a=1} + \frac{2\lambda\alpha}{\theta^n} \left(\frac{\partial}{\partial a} \right)^n \left[(2\alpha)^{-a} \Gamma(a) \right] \big|_{a=1} \quad (3.1.5)$$

The Mean

The mean of the *TOGEED* can be obtained from the n^{th} moment of the distribution when $n=1$ as follows:

$$\mu_1' = \frac{(1-\lambda)\alpha}{\theta} \left(\frac{\partial}{\partial a} \right) \left[(\alpha)^{-a} \Gamma(a) \right] \big|_{a=1} + \frac{2\lambda\alpha}{\theta} \left(\frac{\partial}{\partial a} \right) \left[(2\alpha)^{-a} \Gamma(a) \right] \big|_{a=1} \quad (3.1.6)$$

Also the second moment of the *TOGEED* is obtained from the n^{th} moment of the distribution when $n=2$ as

$$\mu_2' = \frac{(1-\lambda)\alpha}{\theta^2} \left(\frac{\partial}{\partial a} \right)^2 \left[(\alpha)^{-a} \Gamma(a) \right] \big|_{a=1} + \frac{2\lambda\alpha}{\theta^2} \left(\frac{\partial}{\partial a} \right)^2 \left[(2\alpha)^{-a} \Gamma(a) \right] \big|_{a=1} \quad (3.1.7)$$

The Variance

The n^{th} central moment or moment about the mean of X , say μ_n , can be obtained as

$$\mu_n = E(X - \mu_1')^n = \sum_{i=0}^n (-1)^i \binom{n}{i} \mu_1'^i \mu_{n-i}' \quad (3.1.8)$$

By obtaining the central moment when $n=2$, the variance of X for *TOGEED* is obtained as follows:

$$Var(X) = E(X^2) - \{E(X)\}^2 \quad (3.1.9)$$

$$Var(X) = \mu_2' - \{\mu_1'\}^2 \quad (3.1.10)$$

Where μ_1' and μ_2' are the mean and second moment of the *TOGEED* all obtainable from equation (3.1.5).

3.2 Moment Generating Function

The *mgf* of a random variable X can be defined and obtained as given below:

$$M_x(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} f(x) dx \quad (3.2.1)$$

$$M_x(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \mu_n' = \sum_{n=0}^{\infty} \frac{t^n}{n!} \left\{ \frac{(1-\lambda)\alpha}{\theta^n} \left(\frac{\partial}{\partial a} \right)^n \left[(\alpha)^{-a} \Gamma(a) \right] \Big|_{a=1} + \frac{2\lambda\alpha}{\theta^n} \left(\frac{\partial}{\partial a} \right)^n \left[(2\alpha)^{-a} \Gamma(a) \right] \Big|_{a=1} \right\} \quad (3.2.2)$$

3.3 Characteristics Function

The characteristics function of a random variable X is defined and given as follows;

$$\phi_x(t) = E(e^{itx}) = E[\cos(tx) + i \sin(tx)] = E[\cos(tx)] + E[i \sin(tx)] \quad (3.3.1)$$

Simple algebra and power series expansion proves that

$$\phi_x(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} \mu_{2n}' + i \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} \mu_{2n+1}' \quad (3.3.2)$$

Where μ_{2n}' and μ_{2n+1}' are the moments of X for $n=2n$ and $n=2n+1$ respectively and can be obtained from μ_n' in equation (3.1.5).

3.4 Reliability Analysis of the *TOGEED*.

The reliability function in engineering is the characteristic of a variable that maps a set of events, usually associated with mortality or failure of some system onto time. It is used to study the efficiency or life span of products or machines during the production process in engineering. The Survival function describes the likelihood that a system or an individual or component will not fail after a given time. Mathematically, the survival function is given by:

$$S(t) = 1 - F(t) \quad (3.4.1)$$

Applying the *cdf* of the *TOGEED* in (2.1), the survival function for the *TOGEED* is obtained as:

$$S(t) = e^{-\alpha(e^{\theta t}-1)} \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta t}-1)} \right] \quad (3.4.2)$$

The following is a plot for the survival function of the *TOGEED* using different parameter values as shown in Fig.3.4.1 below;

The graph in Fig. **3.4.1** shows that the probability of the survival equals one (1) at initial time or early age and it decreases as t increases and equals zero (0) as t becomes larger.

Hazard function is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1 - F(t)} \quad (3.4.3)$$

Meanwhile, the expression for the hazard rate of the *TOGEED* is given by

$$h(t) = \frac{\alpha \theta e^{\theta t} \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta t}-1)} \right]}{\left[1 - \lambda + \lambda e^{-\alpha(e^{\theta t}-1)} \right]} \quad (3.4.4)$$

where $\alpha, \theta > 0$ and $-1 \leq \lambda \leq 1$.

The following is a plot of the hazard function at chosen parameter values in Fig. 3.4.2

Interpretation: the figure above revealed that the *TOGEED* has a decreasing failure rate which implies that the probability of failure for any random variable following a *TOGEED* decreases as time increases, that is, as time goes on, the probability of death decreases

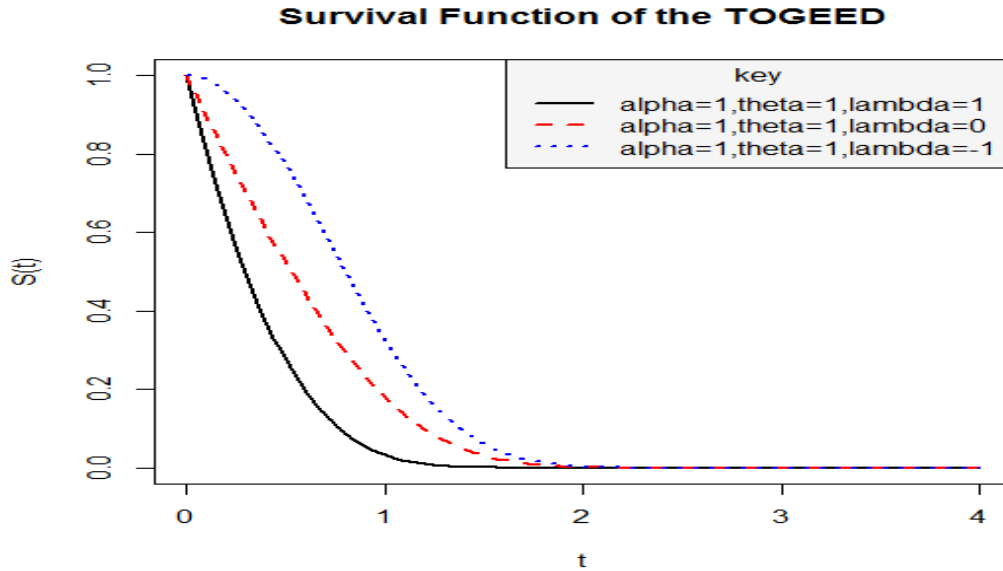


Fig. 3.4.1. Survival function of the *TOGEED* at different parameter values

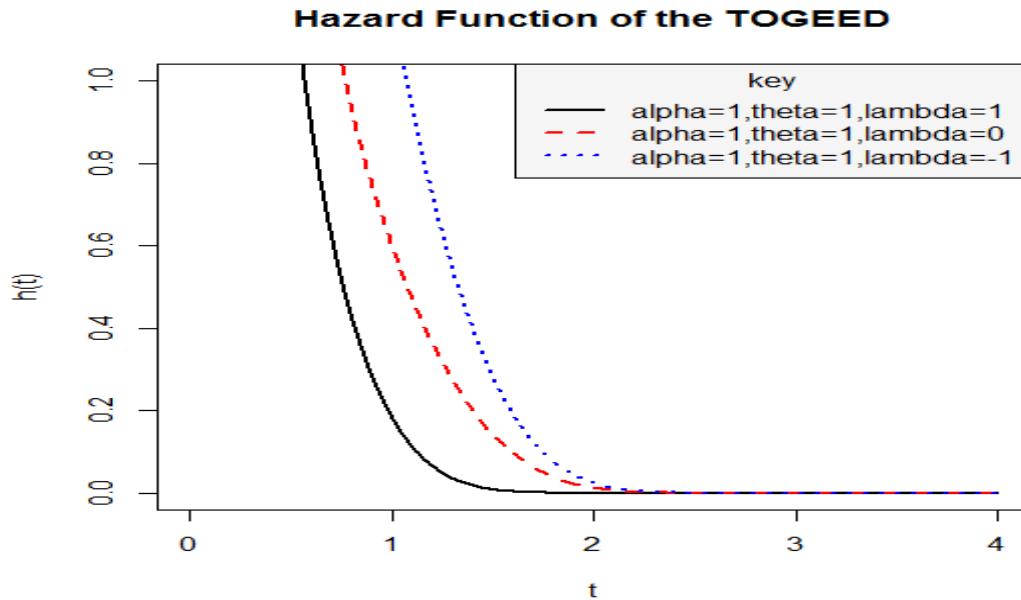


Fig. 3.4.2. The hazard function of the *TOGEED* for different values of the parameters as displayed in the key on the graph

3.5 Quantile Function

[14] defined the quantile function for any distribution in the form

$Q(u) = F^{-1}(u)$ where $Q(u)$ is the quantile function of $F(x)$ for $0 < u < 1$

Taking $F(x)$ to be the *cdf* of the *TOGEED* and inverting it as above will give us the quantile function as follows:

$$F(x) = 1 - e^{-\alpha(e^{\theta x} - 1)} \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta x} - 1)} \right] = u \quad (3.5.1)$$

Simplifying equation (3.5.1) above, we obtain:

$$Q(u) = X_q = \left\{ \left(-\frac{\ln(1-u)}{2\theta} \right)^{\frac{1}{\alpha}} - 1 \right\} \quad (3.5.2)$$

This function as derived above is used for obtaining some moments like skewness and kurtosis as well as the median and for generation of random variables from the distribution in question.

3.6 Skewness and Kurtosis

This paper presents the quantile based measures of skewness and kurtosis due to non-

$$f_{in}(x) = \frac{n!}{(i-1)!(n-i)!} f(x) F(x)^{i-1} [1 - F(x)]^{n-i} \quad (3.7.1)$$

Where $f(x)$ and $F(x)$ are the *pdf* and *cdf* of the *TOGEED* respectively.

Using (2.1) and (2.2), the *pdf* of the i^{th} order statistics X_{in} , can be expressed from (3.7.1) as;

$$f_{in}(x) = \frac{n!}{(i-1)!(n-i)!} \sum_{k=0}^{n-i} (-1)^k \binom{n-i}{k} \left[\alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x} - 1)} \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta x} - 1)} \right] \right] \left[1 - e^{-\alpha(e^{\theta x} - 1)} \right] \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta x} - 1)} \right]^{i+k-1} \quad (3.7.2)$$

existence of the classical measures in some cases.

The Bowley's measure of skewness by [15] based on quartiles is given by;

$$SK = \frac{Q\left(\frac{3}{4}\right) - 2Q\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)} \quad (3.6.1)$$

And the [19] kurtosis is on octiles and is given by;

$$KT = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + Q\left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{1}{8}\right)} \quad (3.6.2)$$

Where $Q(.)$ is obtainable with the help of equation (3.5.2).

3.7 Order Statistics

Let $X_{(1)}$ denote the smallest of $X_1, X_2, \dots, X_n$,

$X_{(2)}$ denote the second smallest of

$X_1, X_2, \dots, X_n$, and similarly $X_{(i)}$ denote the i^{th}

smallest of $X_1, X_2, \dots, X_n$. Then the random

variables $X_{(1)}, X_{(2)}, \dots, X_{(n)}$, called the order

statistics of the sample $X_1, X_2, \dots, X_n$, has

probability density function of the i^{th} order

statistic, $X_{(i)}$, as:

Hence, the *pdf* of the minimum order statistic $X_{(1)}$ and maximum order statistic $X_{(n)}$ of the *TOGEED* are given by;

$$f_{1:n}(x) = n \sum_{k=0}^{n-1} (-1)^k \binom{n-1}{k} \left[\alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x}-1)} \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x}-1)} \right] \right] \left[1 - e^{-\alpha(e^{\theta x}-1)} \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta x}-1)} \right] \right]^k \quad (3.7.3)$$

And

$$f_{n:n}(x) = n \left[\alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x}-1)} \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x}-1)} \right] \right] \left[1 - e^{-\alpha(e^{\theta x}-1)} \left[1 - \lambda + \lambda e^{-\alpha(e^{\theta x}-1)} \right] \right]^{n-1} \quad (3.7.4)$$

respectively.

4. ESTIMATION OF PARAMETERS

Let $X_1, \dots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the *TOGEED* with unknown parameters α , θ , and λ defined previously. The *pdf* of the *TOGEED* is given as

$$f(x) = \alpha \theta e^{\theta x} e^{-\alpha(e^{\theta x}-1)} \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x}-1)} \right]$$

The likelihood function is given by;

$$L(X_1, X_2, \dots, X_n / \alpha, \theta, \lambda) = (\alpha \theta)^n e^{\theta \sum_{i=1}^n x_i} e^{-\alpha \sum_{i=1}^n (e^{\theta x_i} - 1)} \sum_{i=1}^n \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x_i} - 1)} \right] \quad (4.1)$$

Let the log-likelihood function, $l = \log L(X_1, X_2, \dots, X_n / \alpha, \theta, \lambda)$, therefore

$$l = n \log \alpha + n \log \theta + \theta \sum_{i=1}^n x_i - \alpha \sum_{i=1}^n (e^{\theta x_i} - 1) + \sum_{i=1}^n \log \left[1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x_i} - 1)} \right] \quad (4.2)$$

Differentiating l partially with respect to α , θ and λ respectively gives;

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^n (e^{\theta x_i} - 1) - 2\lambda \sum_{i=1}^n \left\{ \frac{e^{-\alpha(e^{\theta x_i} - 1)} (e^{\theta x_i} - 1)}{1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x_i} - 1)}} \right\} \quad (4.3)$$

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^n x_i - \alpha \sum_{i=1}^n \left\{ \frac{x_i e^{-\alpha(e^{\theta x_i} - 1)} e^{-\theta x_i}}{1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x_i} - 1)}} \right\} \quad (4.4)$$

$$\frac{\partial l}{\partial \lambda} = \sum_{i=1}^n \left\{ \frac{2e^{-\alpha(e^{\theta x_i} - 1)} - 1}{1 - \lambda + 2\lambda e^{-\alpha(e^{\theta x_i} - 1)}} \right\} \quad (4.5)$$

Equating equations (4.3), (4.4) and (4.5) to zero and solving for the solution of the non-linear system of equations will give us the maximum likelihood estimates of parameters α, θ , and λ respectively. However, the solution cannot be obtained analytically except numerically with the aid of suitable statistical software like Python, R, SAS, e.t.c when data sets are given.

5. APPLICATIONS

In this section, we have applied and compared the performance of the Transmuted odd generalised exponential exponential distribution (*TOGEED*) to that of odd generalised exponential exponential distribution (*OGEED*) and Weibull-

exponential distribution (*WED*) using the following dataset.

Data set: This data represents the remission times (in months) of a random sample of 128 bladder cancer patients. It has previously been used by [16] and [25]. It's summarised as follows.

From the descriptive statistics in table 5.1, we observed that the data set is positively skewed with a very high coefficient of kurtosis and therefore suitable for flexible and skewed distributions.

To compare the distributions listed above, we have used several measures of model fit such as *AIC* (Akaike Information Criterion), Cram'er-Von Mises (W^*), Anderson Darling (A^*) Kolmogorov smirnov ($K-S$) statistics.

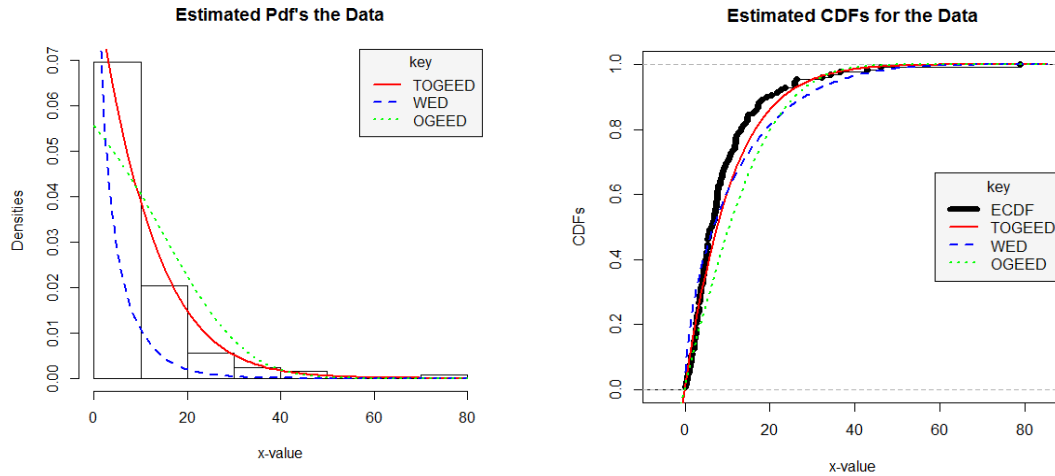


Fig. 5.1. Histogram and estimated densities and empirical and estimated CDFs of the *TOGEED*, *OGEED* and *WED* for the bladder cancer dataset

Table 5.1. Summary Statistics for the dataset

parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	128	0.0800	3.348	6.395	11.840	9.366	79.05	110.425	3.3257	19.1537

Table 5.2. The statistics ll , AIC, A^* , W^* and $K-S$ for the fitted models to the dataset

Distributions	Parameter estimates	ll =(log-likelihood value)	AIC	A^*	W^*	K-S	P-Value (K-S)	Ranks
TOGEED	$\hat{\theta}=0.0182$ $\hat{\alpha}=2.7822$ $\hat{\lambda}=0.7591$	416.5186	839.0372	1.0381	0.1747	0.1079	0.1014	1
WED	$\hat{\theta}=0.0293$ $\hat{\alpha}=1.9653$ $\hat{\beta}=0.6759$	546.5931	1099.186	1.4123	0.2357	0.1476	0.0076	2
OGEED	$\hat{\theta}=0.0346$ $\hat{\alpha}=1.6066$	439.5273	883.0546	3.2153	0.5463	0.2341	1.6e-06	3

Note that the model with the lowest values of these statistics shall be chosen as the best model to fit the data.

It is shown from Table 5.2 above that the Transmuted odd generalized exponential distribution (*TOGEED*) corresponds to the lower values of II , AIC , A , W and $K-S$ compared to those of the odd generalized exponential-exponential distribution (*OGEED*) and the Weibull-exponential distribution (*WED*) and therefore we chose the *TOGEED* as the best model the fits the data.

The following figure displayed the histogram and estimated densities as well as empirical cumulative distribution function (ECDF) and estimated CDFs of the fitted models for the bladder cancer patients' data set under study.

From the estimated density plots in figures 5.1 it is very clear that the performance of the *TOGEED* is better than the other two Models.

5. CONCLUSION

In this article, we proposed a new distribution, derived and study some of its properties with graphical analysis and discussion on its usefulness and applications. Hence, having demonstrated earlier in the previous section, we have a conclusion based on our applications of the model to a real life data that the new distribution (*TOGEED*) has a better fit compared to the other two already existing models based on the data set considered in this study.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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