Method Article

Measuring The Cost For Some Single Channel Waiting Line Models

ABSTRACT

Queuing models applications are centered on the question of finding the ideal level of services, waiting times and queue lengths. The aim of this study is to measure the cost for three models and compare the cost for the three single channel waiting line models instead of finding the ideal level of services, waiting times and queue lengths which calculated in many studies. Each model depends on two important parameters arrival rate (λ) and service rate (μ) which followed different distributions. The cost for the three single channel waiting line models is calculated when arrival rate (λ) is followed Poisson distribution and service rate (μ) is followed different distributions. The objective for the waiting line models is to minimize total expected costs by minimize the sum of service costs and waiting costs. Therefore, the study concerned with changing the distribution of the service rate (μ) and examining its impact on cost. This choice was made to emphasize the basic idea of the study (there is a relationship between the service rate (μ) distribution and the cost). The study results showed that there is a relationship between the service rate (μ) distribution and the cost.

Keywords: Exponential distribution; Gamma distribution; Poisson distribution; Single Channel models; Waiting line cost; Weibull distribution

1- Introduction

Queuing theory had its beginning in the research work of a Danish engineer named Anger Krarup Erlang In 1909; Erlang's experimented with fluctuating demand in telephone traffic. At the end of World War II, Erlang'searly work was extended to more general problems and to business applications of waiting lines[1].Queuing theory is basically a mathematical approach applied to the analysis of waiting lines. The queuing model is a very powerful tool for determining that how to manage a queuing system in the most effective manner[2].Queues or waiting lines are very common in everyday life whereby certain business situations require customers to wait in line for a service[3].

Uses models to represent the various types of queuing systems. Formula for each model indicates how the related queuing system should perform, under a variety of conditions. The queuing theory is also known as the random system theory, which studies the content of: the behavior problems, the optimization problem and the statistical inference of queuing system[4]. Queuing models applications are centered on the question of finding the ideal level of services, waiting times and queue lengths.

Applications of the queuing theory such as traffic flow (vehicles, aircraft, people, communications, transportation networks), scheduling (patients in hospitals, jobs on machines, programs on the computer), facility design (banks, post offices, supermarkets, manufacturing)[5].Most banks used queuing models. It is very useful to avoid standing in a queue for a long time to give tickets to all customers. Queuing is used to generate a sequence of customers' arrival time and to choose randomly between three different services: open an account, transaction, and balance, with a different period of time for each service. [6]

Mehri et al.[7] introduced the basic concepts of queuing models and showed how linear programming can be used to estimate the performance measures of a system. They studied Tunisian transport and found widespread use in the analysis of service facilities, production and many other situations where congestion or competition for scarce resources may occur.

Edith et al.[8] Regression analysis was employed to model the banks' queue system. They found that The Coefficient of determination, R^2value was close to unity for multiple linear regression and unity for non-linear regression. Also, the Degree of Correlation obtained was found to be 92%and 100% for the multiple linear regression and non-linear regression.

Dhar & Rahman [9]used queuing model to derive the arrival rate, service rate, utilization rate, waiting time in the queue and the average number of customers in the queue. Queuing can help bank ATM to increase its quality of service, by anticipating, if there are many customers in the queue. In ATM, bank customers arrive randomly and the service time.

Muruganantha and Usha [10] calculated average queue length, average number of customer in the system.Average customer waiting time and average number of customer time spent in the queue in kanyakumari district at various places are introduced.

Santhi and Saravanan [11] discussed several queuing model for cloud computing. These models are used to reduce waiting time of customer and increase the performance of the system. Furthermore, they presented a comparison of several queuing models results which are used for cloud computing environment.

The aim of this study is to measure the cost for three models and compare the cost for the three single channel waiting line models instead of finding the ideal level of services, waiting times and queue lengths which calculated in many studies.

The organization of the study is as follows: In Section 2 the study Identifying Models Using Kendall Notation. Section 3 described the Waiting Line costs. numerical study discusses in Section 4. Finally, discussion concluding remarks are provided in Section 5.

2-Kendall Notation:

This section identifying the three models which applied in this study using Kendall Notation.David. G. Kendall 1953 developed a notation that has been widely accepted for specifying the pattern of arrivals, the service time distribution, and the number of channels in a queuing model. There are three symbolsKendall notation as follows

Arrival distribution / Service time distribution/Number of service channels open.

The following letters are commonly used in Kendall notation:

G= general distribution with mean and variance known,

D= constant (deterministic) rate, and

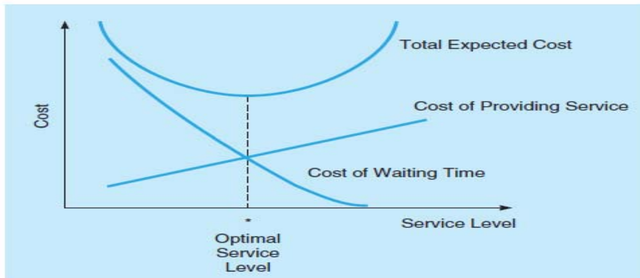
M= Poisson distribution for number of occurrences (or exponential times), [4].

Single-Channel$(M/M/1)$, Constant – Service Time Model$(M/D/1)$, General Service queuing model$(M/G/1)$.the previous models are the three models which will use in this study.

3- Cost for Waiting Line models

One of the goals of queuing analysis is finding the best level of service for an organization. Its objective is usually to find the medium between two extremes. On the other hand, a firm can retain a large staff and provide many service facilities. This can become expensive.The other extreme is to have the minimum possible number of checkout lines, such as gas pumps, or teller windows open. This keeps the service cost down but may result in customer dissatisfaction. As the average length of the queue increases and poor service results, customers and goodwill may be lost. Managers must deal with the trade-off betweenthe cost of providing good service and the cost of customer waiting time.One means of evaluating a service facility is thus to look at a total expected cost; this the sum of expected service costs plus expected to wait costs. As service improves in speed, however, the cost of time spent waiting in lines decreases. This waiting cost may reflect the lost productivity of workers while their tools or machines are awaiting repairs or may simply be an estimate of the costs of customers lost because of the poor service and long queues. The objective is to minimize total expected costs. by minimize the sum of service costs and waiting costs.[7].

Figure (3.1) queuing cost and service levels



Total expected service cost = (Number of channels)(Cost per channel) = $m\,C_s$(3.1)

Where

m = number of channels

C_s = service cost (labor cost) of each channel

The waiting cost when the waiting time cost is based on time in the system is

Total expected waiting cost = (Total time spent waiting by all arrivals) (Cost of waiting)

= (Number of arrivals) (Average wait per arrival) C_w

So,

Total expected waiting cost = $(\lambda W)C_w$ (3.2)

If the waiting time cost is based on time in the queue, this becomes

Total expected waiting cost = $(\lambda W_q)C_w$(3.3)

These costs are based on whatever time units (often hours) are used in determining λ. Adding

the total service cost to the total waiting cost, have the total cost of the queuing system.

When the waiting cost is based on the time in the system, this is

Total expected cost = Total expected service cost + Total expected waiting cost

Total expected cost = $m\,C_s$ + $\lambda W C_w$(3.4)

When the waiting cost is based on time in the queue, the total cost is

Total expected cost = $m\,C_s + \lambda W_q C_w$.(3.5)

4- **Numerical Study**

This section discusses the numerical studywhich used to evaluate the performance of the three waiting for lines models; Single-Channel, Constant – Service Time model$(M/D/1)$, and general - service queuing model$(M/G/1)$.Each model depends on two important parameters arrival rate (λ) and service rate (μ) which followed different distributions. The cost for the three single channel waiting line models is calculated when arrival rate (λ) is followed Poisson distribution and service rate (μ) is followed different distributions.

*The first model Single - Channel $(M/M/1)$ the arrival rate (λ) followed Poisson distribution and the service rate (μ) followed exponential distribution.

*The second model $(M/D/1)$ the arrival rate (λ) followed the Poisson distribution and the service rate (μ) followed exponential distribution and constant service rate model.

*The third model $(M/G/1)$ the arrival rate (λ) followed the Poisson distribution and the service rate (μ) followed exponential. The study evaluates the performance for the three waiting line single channel models when the cost for each model is calculated when three different distributions (exponential, Gamma and Weibull distributions) are used for service rate (μ). The study chosen three distributions related to exponential distribution or the three distributions are considered special cases of each other. The objective for the waiting line models are to minimize total expected costs by minimize the sum of service costs and waiting costs. Therefore, the study concerned with changing the distribution of the service rate (μ) and examining its impact on cost.

The numerical simulation study takes the following steps:

1- The study depends on the data which generated by Arinze *et al* [12] from NNPC mega petroleum station Owerri and NNPC mega petroleum station Enugu.

2-The study solved the three models for r =52 where r is the number or replication for each model with different values for the arrival rate (λ) and the service rate (μ). and showed the results in the following paragraph.

3- The first model Single - Channel $(M/M/1)$ applied when the arrival rate (λ) followed Poisson distribution and the service rate (μ) is followed an exponential distribution with different parameters.

4- The second model $(M/D/1)$ applied when the arrival rate (λ) is followed the Poisson distribution and when the service rate (μ) is followed two constant service rate model. This model applied first when (μ) service rate used as in Arinze *et al* [12]. Second, the model $(M/D/1)$ applied when (μ) was follow constant (deterministic) value. The study used other constant (deterministic) data to confirm that the distribution is related to cost.

5- The third model $(M/G/1)$ applied when the arrival rate (λ) is followed Poisson distribution and the service rate has followed any distribution. The study chosen three distributions related to exponential distribution or the three distributions are considered special cases of each other. This choice was made to emphasize the basic idea of study (there is a relationship between the service rate (μ) distribution and the cost)

6- The study generated N=350 is followed Gamma distribution and weibull distribution by Minitab program to choose number of replication r =52 which selected. The goodness of fit is used by easy fit program to be sure that the data which selected follow Gamma distribution and weibull distribution. The package program "QM for windows V5" is used to solve the three models under consideration.

7- To study the effect of distribution for each model the study suggests that the server cost = 4 and waiting cost = 2 as a constant for all cases.

Table (4.1) The three models with λ ~Poisson, μ~ exponential.

Day	λ	μ	M/M/1		M/D/1		M/G/1	
			Waiting cost	system cost	Waiting cost	system cost	Waiting cost	system cost
1	29	30	60.07	62.00	32.03	33.97	54.74	56.67
2	30	31	62.06	64.00	33.03	34.97	58.14	60.08
3	31	32	64.06	66.00	34.03	35.97	61.71	63.65
4	32	33	66.06	68.00	35.03	36.97	65.44	67.38
5	33	34	68.06	70.00	36.03	37.97	69.35	71.29
6	34	35	70.06	72.00	37.03	38.97	73.44	75.39
7	35	36	72.06	74.00	38.03	39.97	77.72	79.66
8	36	37	74.05	76.00	39.03	40.97	82.18	84.13
9	37	38	76.05	78.00	40.03	41.97	75.19	77.14
10	38	39	78.05	80.00	41.03	42.97	91.71	93.66
11	39	40	80.05	82.00	42.03	43.98	96.78	98.73
12	40	41	82.05	84.00	43.02	44.98	69.26	71.22
13	29	31	31.13	33.00	17.56	19.44	29.3	31.17
14	30	32	32.13	34.00	18.06	19.94	31.02	32.9
15	31	33	33.12	35.00	18.56	20.44	32.83	34.71
16	33	35	35.11	37.00	19.56	21.44	36.71	38.59
17	36	38	38.11	40.00	21.05	22.95	43.21	45.11
18	37	39	39.10	41.00	21.55	23.45	39.6	41.49
19	38	40	40.10	42.00	22.05	23.95	48.04	49.94
20	39	41	41.10	43.00	22.55	24.45	35.02	36.92
21	40	42	42.10	44.00	23.05	24.95	44.05	45.95
22	28	31	20.86	22.67	12.43	14.24	19.72	21.53

23	29	32	21.52	23.33	12.76	14.57	20.83	22.65
24	30	33	22.18	24.00	13.09	14.91	22	23.82
25	32	35	23.50	25.33	13.75	15.58	24.5	26.33
26	36	39	26.15	28.00	15.08	16.92	30.24	32.09
27	38	41	27.48	29.33	15.74	17.59	23.63	25.49
28	40	43	28.81	30.67	16.4	18.26	25.58	27.44
29	41	44	29.47	31.33	16.73	18.6	26.6	28.46
30	29	33	16.74	18.50	10.37	12.13	16.62	18.37
31	37	41	20.70	22.50	12.35	14.15	17.96	19.77
32	38	42	21.19	23.00	12.6	14.4	26.24	28.05
33	39	43	21.69	23.50	12.84	14.66	19.38	21.2
34	40	44	22.18	24.00	13.09	14.91	20.13	21.95
35	41	45	22.68	24.50	13.34	15.16	20.9	22.73
36	29	34	13.89	15.60	8.95	10.65	14.09	15.8
37	35	40	16.25	18.00	10.13	11.88	18.95	20.7
38	36	41	16.64	18.40	10.32	12.08	14.57	16.33
39	37	42	17.04	18.80	10.52	12.28	17.71	19.47
40	38	43	17.43	19.20	10.72	12.48	15.68	17.45
41	39	44	17.83	19.60	10.91	12.69	16.27	18.04
42	35	41	13.96	15.67	8.98	10.69	12.33	14.04
43	36	42	14.29	16.00	9.14	10.86	14.81	16.53
44	37	43	14.61	16.33	9.31	11.03	13.23	14.95
45	40	46	15.59	17.33	9.8	11.54	14.7	16.44
46	29	36	10.67	12.29	7.34	8.95	11.23	12.84
47	39	46	13.45	15.14	8.72	10.42	12.72	14.42
48	33	42	9.76	11.33	6.88	8.45	11.45	13.03
49	36	45	10.40	12.00	7.2	8.8	9.79	11.39
50	38	47	10.83	12.44	7.41	9.03	10.43	12.05
51	35	45	9.44	11.00	6.72	8.28	8.93	10.48
52	37	47	9.83	11.40	6.91	8.49	9.49	11.06

The table (4.1) showed the results for the three Single–Channel models $(M/M/1)$, $(M/D/1)$ and $(M/G/1)$when arrival rate (λ) is followed Poisson distribution and service rate (µ) is followed an exponential distribution. For the number of replication r =52 which are used with different values for, service rate (µ), arrival rate (λ). Cost for the system or the queue was decreased when different between arrival rate (λ) and service rate (µ) increased the reason for that refer to the utilization factor. The utilization factor or the probability that the service facility is used decrease when different between arrival rate (λ) and service rate (µ) increased. The cost which calculated from (M/D/1) model is less than the cost which calculated from the other two models when the same data are used. The study suggests that the server cost = 4 and waiting cost = 2 as a constant for all cases to study the behavior of each model.

Table (4.2) the (M/D/1) model with λ ~Poisson, µ ~ exponential and constant.

Day	M/D/1				M/D/1			
	λ	µ	Waiting cost	system cost	λ	µ	Waiting cost	system cost
1	29	30	32.03	33.97	29	30	32.03	33.97
2	30	31	33.03	34.97	30	31	33.03	34.97
3	31	32	34.03	35.97	31	32	34.03	35.97
4	32	33	35.03	36.97	33	34	36.03	37.97
5	33	34	36.03	37.97	35	36	38.03	39.97
6	34	35	37.03	38.97	39	40	42.03	43.98
7	35	36	38.03	39.97	44	45	47.02	48.98
8	36	37	39.03	40.97	28	30	17.07	18.93
9	37	38	40.03	41.97	29	31	17.56	19.44
10	38	39	41.03	42.97	30	32	18.06	19.94
11	39	40	42.03	43.98	32	34	19.06	20.94
12	40	41	43.02	44.98	34	36	20.06	21.94
13	29	31	17.56	19.44	38	40	22.05	23.95
14	30	32	18.06	19.94	43	45	24.54	26.46
15	31	33	18.56	20.44	28	31	12.43	14.24
16	33	35	19.56	21.44	29	32	12.76	14.57
17	36	38	21.05	22.95	31	34	13.42	15.25
18	37	39	21.55	23.45	33	36	14.08	15.92
19	38	40	22.05	23.95	37	40	15.41	17.26
20	39	41	22.55	24.45	42	45	17.07	18.93
21	40	42	23.05	24.95	28	32	10.13	11.88
22	28	31	12.43	14.24	30	34	10.62	12.38
23	29	32	12.76	14.57	32	36	11.11	12.89

24	30	33	13.09	14.91	36	40	12.1	13.9
25	32	35	13.75	15.58	41	45	13.34	15.16
26	36	39	15.08	16.92	29	34	8.95	10.65
27	38	41	15.74	17.59	31	36	9.34	11.06
28	40	43	16.4	18.26	35	40	10.13	11.88
29	41	44	16.73	18.6	40	45	11.11	12.89
30	29	33	10.37	12.13	28	34	7.84	9.49
31	37	41	12.35	14.15	30	36	8.17	9.83
32	38	42	12.6	14.4	34	40	8.82	10.52
33	39	43	12.84	14.66	39	45	9.63	11.37
34	40	44	13.09	14.91	29	36	7.34	8.95
35	41	45	13.34	15.16	33	40	7.89	9.54
36	29	34	8.95	10.65	38	45	8.58	10.27
37	35	40	10.13	11.88	28	36	6.72	8.28
38	36	41	10.32	12.08	32	40	7.2	8.8
39	37	42	10.52	12.28	37	45	7.8	9.45
40	38	43	10.72	12.48	31	40	6.67	8.22
41	39	44	10.91	12.69	36	45	7.2	8.8
42	35	41	8.98	10.69	30	40	6.25	7.75
43	36	42	9.14	10.86	35	45	6.72	8.28
44	37	43	9.31	11.03	29	40	5.91	7.36
45	40	46	9.8	11.54	34	45	6.34	7.85
46	29	36	7.34	8.95	28	40	5.63	7.03
47	39	46	8.72	10.42	33	45	6.02	7.48
48	33	42	6.88	8.45	32	45	5.75	7.17
49	36	45	7.2	8.8	31	45	5.53	6.9
50	38	47	7.41	9.03	30	45	5.33	6.67
51	35	45	6.72	8.28	29	45	5.17	6.46
52	37	47	6.91	8.49	28	45	5.02	6.27

The table (4.2) showed the results for the Constant – Service Time model(M/D/1) first when arrival rate (λ) is followed Poisson distribution and service rate (μ) is followed exponential distribution. For the number of replication r =52 which are used with different values for, service rate (μ), arrival rate (λ). Cost was decreased when the different between arrival rate (λ) and service rate (μ) increasedthe reason for that refer to the utilization factor. The utilization factor or the probability that the service facility is used decrease when different between arrival rate (λ) and service rate (μ) increased. Second, when Constant – Service Time model(M/D/1) applied with arrival rate (λ) is followed Poisson distribution and service rate (μ) is followed constant values chosen arbitrarily the cost for $(M/D/1)$ model is smallest when constant values were chosen arbitrarily. The reason for that refer to the distribution for the data is differ. The study made goodness of fit test by the easy fit program for the data. The study suggests that the server cost = 4 and waiting cost = 2 as a constant for all cases to study the behavior for each model.

Table (4.3) the M/G/1 model with λ ~Poisson, μ~ exponential, Gamma and Weibull.

Day	M/G/1				M/G/1				M/G/1			
	λ	μ	Waiting cost	system cost	λ	μ	Waiting cost	system cost	λ	μ	Waiting cost	system cost
1	29	30	54.74	56.67	31	32	61.71	63.65	40	41	69.26	71.22
2	30	31	58.14	60.08	39	40	96.78	98.73	41	42	72.26	74.22
3	31	32	61.71	63.65	40	41	69.26	71.22	36	37	82.18	84.13
4	32	33	65.44	67.38	41	42	72.26	74.22	39	40	96.78	98.73
5	33	34	69.35	71.29	42	43	75.36	77.32	39	41	35.02	36.92
6	34	35	73.44	75.39	43	44	78.57	80.52	36	38	43.21	45.11
7	35	36	77.72	79.66	29	31	29.3	31.17	37	39	45.58	47.47
8	36	37	82.18	84.13	33	35	36.71	38.59	37	39	45.58	47.47
9	37	38	75.19	77.14	34	36	38.78	40.67	35	37	40.95	42.84
10	38	39	91.71	93.66	35	37	40.95	42.84	40	42	36.49	38.39
11	39	40	96.78	98.73	37	39	45.58	47.47	33	36	25.84	27.68
12	40	41	69.26	71.22	38	40	48.04	49.94	31	34	23.22	25.05
13	29	31	29.3	31.17	39	41	35.02	36.92	41	44	26.6	28.46
14	30	32	31.02	32.9	40	42	36.49	38.39	29	32	20.83	22.65
15	31	33	32.83	34.71	42	44	39.57	41.48	35	38	28.71	30.55
16	33	35	36.71	38.59	43	45	41.19	43.1	38	41	23.63	25.49
17	36	38	43.21	45.11	30	33	22	23.82	39	42	24.59	26.45
18	37	39	39.6	41.49	35	38	28.71	30.55	38	42	18.66	20.47
19	38	40	48.04	49.94	36	39	30.24	32.09	40	44	20.13	21.95
20	39	41	35.02	36.92	42	45	27.65	29.52	37	41	17.96	19.77
21	40	42	44.05	45.95	31	35	18.43	20.2	33	37	20.42	22.21
22	28	31	19.72	21.53	39	43	19.38	21.2	29	33	16.62	18.37
23	29	32	20.83	22.65	40	44	20.13	21.95	40	45	16.87	18.65
24	30	33	22	23.82	43	47	22.53	24.36	37	42	15.12	16.88
25	32	35	24.5	26.33	44	48	23.38	25.21	38	43	15.68	17.45

26	36	39	30.24	32.09	45	49	24.25	26.09	36	41	14.57	16.33
27	38	41	23.63	25.49	29	34	14.09	15.8	28	33	13.41	15.11
28	40	43	25.58	27.44	30	35	14.81	16.53	39	45	14.2	15.93
29	41	44	26.6	28.46	37	42	15.12	16.88	35	41	12.33	14.04
30	29	33	16.62	18.37	40	45	16.87	18.65	37	43	13.23	14.95
31	37	41	17.96	19.77	35	41	12.33	14.04	30	36	13.03	14.69
32	38	42	26.24	28.05	36	42	12.77	14.49	38	44	13.71	15.43
33	39	43	19.38	21.2	37	43	13.23	14.95	36	42	12.77	14.49
34	40	44	20.13	21.95	37	43	13.23	14.95	29	35	12.42	14.08
35	41	45	20.9	22.73	40	46	14.7	16.44	38	45	12.3	13.99
36	29	34	14.09	15.8	29	36	11.23	12.84	33	40	13.49	15.14
37	35	40	18.95	20.7	29	36	11.23	12.84	36	43	11.49	13.16
38	36	41	14.57	16.33	35	42	11.11	12.77	40	47	13.16	14.86
39	37	42	17.71	19.47	36	43	11.49	13.16	29	36	11.23	12.84
40	38	43	15.68	17.45	40	47	13.16	14.86	35	42	11.11	12.77
41	39	44	16.27	18.04	29	37	10.34	11.91	39	46	12.72	14.42
42	35	41	12.33	14.04	38	46	11.25	12.9	34	41	10.74	12.39
43	36	42	14.81	16.53	40	48	12.01	13.67	37	45	10.88	12.53
44	37	43	13.23	14.95	36	45	9.79	11.39	32	40	11.81	13.41
45	40	46	14.7	16.44	38	47	10.43	12.05	29	37	10.34	11.91
46	29	36	11.23	12.84	40	49	11.11	12.75	38	46	11.25	12.9
47	39	46	12.72	14.42	41	50	11.47	13.11	30	39	10.07	11.61
48	33	42	11.45	13.03	32	42	8.16	9.68	31	40	10.51	12.06
49	36	45	9.79	11.39	36	46	9.2	10.77	29	38	9.65	11.18
50	38	47	10.43	12.05	37	47	9.49	11.06	36	46	9.2	10.77
51	35	45	8.93	10.48	38	48	9.78	11.36	30	40	9.49	10.99
52	37	47	9.49	11.06	44	54	11.77	13.4	32	42	8.16	9.68
average			**33.77**	**35.59**			**26.78**	**28.55**			**23.07**	**24.81**

The table (4.3) showedthe results for general - service queuing model $(M/G/1)$when first arrival rate (λ) is followed Poisson distribution and service rate (μ) is followed exponential distribution, Second witharrival rate (λ) is followed Poisson distribution and service rate (μ) is followed Gamma distribution, third witharrival rate (λ) is followed Poisson distribution and service rate (μ) is followed Weibull distribution. For the number of replicationr =52 which are used with different values for, service rate (μ), arrival rate (λ). Cost was decreased when different between arrival rate (λ) and service rate (μ) increased the reason for that refer to the utilization factor. The utilization factor or the probability that the service facility is used decrease when different between arrival rate (λ) and service rate (μ) increased. The averagecost is decreased when service rate (μ) is followed Weibull distribution than the same model with different distributions(exponential and Gamma)which used. the results showed that the cost which calculated for the (M/G/1) model when the service rate (μ) is followed weibull distribution is less than the same model when the service rate (μ) is followed exponential and gamma distributions.Although the study chose three distributions related with exponential distribution. However, the difference in the distribution used for the same method led to a difference in the cost values resulting and emphasized the objective of the study is that the distribution of data in the waiting line models will affect the cost

5- Conclusion

This section concerned with the results related with numerical study for the three single channel waiting lines models; Single - Channel (M/M/1) , Constant – Service Time model (M/D/1) and general - service queuing model (M/G/1) when different values for arrival rate (λ) and service rate (μ) are used.

The study comparison between three single channel waiting line models. First when Arinze *et al* [12] data are used for the three models.

The cost which calculated for(M/D/1) model is less than the cost for the other two models when the same data are used.The study suggests that the server cost = 4 and waiting cost = 2 as a constant for all cases to study the behavior of each model.

Second when (M/D/1) used Arinze *et al* [12]data and when data are chosen arbitrarily. the results showed that the cost which calculated for (M/D/1) modelis less than the cost for the same model when the data are chosen arbitrarily. The study made goodness of fit testby easy fit program for the data. The distribution for the Arinze *et al* [12] data and the distribution for the data which chosen arbitrarily are differ. The reason for that refer to the distribution for the data is differ. The study suggests that the server cost = 4 and waiting cost = 2 as a constant for all cases to study the behavior for each model.

Third when (M/G/1) used Arinze *et al* [12]data and generate two distributions used as service rate (μ) the results showed that the cost which calculated from (M/G/1) model when the service rate (μ) is followed weibull distribution is less than the cost which calculated from exponential and gamma distributions which used.Although the study chose three distributions related to exponential distribution. However, the difference in the distribution used for the same method led to a difference in the cost values resulting and emphasized the objective of the study is that the distribution of data in the waiting line models will affect the costThe results emphasize the basic idea of study (there is a relationship between the service rate (μ) distribution and the cost).

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