

## **Spectrum of sex ratios in Denmark**

**Abstract.** The sex ratio at birth is usually defined as the number of males per 100 females (SR) or, as in this study, the proportion of males among all births ( $P_M$ ). Among newborns, there is typically a slight excess of boys. Consequently, the SR is greater than 100, mainly around 106, and the chance of males is around 0.515. Attempts have been made to identify factors influencing the level of the  $P_M$ . It has been stated that where prenatal losses are low, as in Western countries, the SRs are usually around 105 to 106, but in areas where the frequencies of prenatal losses are relatively high the SRs are around 102. Later interest has focused on temporal, regional and seasonal fluctuations in the SR. In general, factors that affect the SR within families remain poorly understood. Attempts to identify such factors in national birth registers have been unsuccessful. Recently, SR studies have mainly concentrated on the identification of general but occasional factors. In this study, we identify any effects of issues associated with births such as maternal age and type of delivery (live- and stillborn, singletons and multiples). There is no significant difference between live- and stillborn and no significant maternal age effect was identified. The SR is higher among singletons than among multiples, but there is no significant difference between SR among twins and triplets. Among singletons the temporal differences are non-significant, but for twins and triplets, we obtained significant temporal differences,

**Key words:** Maximum likelihood; confidence intervals; temporal trends; maternal age; multiple maternities

### **Introduction**

The sex ratio at birth is usually defined as the number of males per 100 females (SR), or the ratio of males to females, or, as in this study, the proportion of males among all births ( $P_M$ ). Among newborns, there is typically a slight excess of boys. Consequently, the SR is greater than 100, mainly around 106 and the  $P_M$  around 0.515.

In the 17<sup>th</sup> century, lively discussion ensued on the causes of this excess. One argument was that this balance between male and female births is not the result of chance but of Divine Providence. Opponents argued that even if the chance of a male,  $P_M$ , differs from 0.5

sufficiently many repeated trials will result in a relative frequency very close to  $P_M$ . A description of the history of SR studies is given elsewhere [1] [2]. Today, the study of secondary sex ratio is based on a more stable foundation of statistical theory. This enables scientists *i.a.* to identify influential factors. The interest has mainly focused on temporal, regional and seasonal fluctuations in the SR. Temporal fluctuations have often been connected to war and peace. No family parameters can explain the time trends. In general, factors that affect the SR within families remain poorly understood. Attempts to identify such factors in national birth registers have failed [3]. Recently, SR studies have concentrated on the identification of general but occasional factors. Grech tried to identify such factors in a series of papers e.g. [4]. In this paper, we study the sex ratio using the variable  $P_M$ . The data for this study were collected over periods too short to allow the effect of temporal factors to be determined.

## Methods and Materials

**Maximum likelihood estimation.** If the theoretical proportion of males is  $p_0$ , then the observed relative frequency of males  $p$  is a maximum likelihood (ML) estimator of  $p_0$ , being unbiased, consistent, efficient and asymptotically normal with  $E(p) = p_0$  and  $\text{Var}(p) = p_0(1 - p_0)/N$ . This finding speaks in favour of the use of  $P_M$  as a sex ratio measure. According to the ML theory, the sex ratio  $\text{SR} = p/(1 - p)$  is a ML estimator of the transformed parameter  $\text{SR}_0 = p_0/(1 - p_0)$ , but SR is not unbiased. When  $N \rightarrow \infty$ , then  $p \rightarrow p_0$  and  $\text{SR} \rightarrow \text{SR}_0$ , and the estimate SR is consistent, biased, but asymptotically unbiased, and normally distributed [5].

**Standard deviations and confidence intervals.** Visaria in [6] pointed out that random errors influence the variation in the SR. Therefore, he presented a numerical table of how the confidence intervals (CIs) of the SR depend on the observed SR and the number of births. He gave no formula for the intervals, but stated that “the standard error of an observed sex ratio can be estimated as the standard error of the proportion  $p$  of male births among the total”. Fellman and Eriksson [5] interpreted Visaria’s statement such that he constructed CIs for  $p$ , that is

$$\left( p - k\sqrt{\frac{p(1-p)}{N}}, p + k\sqrt{\frac{p(1-p)}{N}} \right), \quad (1)$$

62 where  $k$  corresponds to the confidence level. After that, Visaria defined the CI for the SR,

63  $(SR_L, SR_U)$ , so that  $SR_L = p_L/(1 - p_L)$  and  $SR_U = p_U/(1 - p_U)$ , where

$$64 \quad p_L = p - k\sqrt{\frac{p(1-p)}{N}} \text{ and } p_U = p + k\sqrt{\frac{p(1-p)}{N}}. \quad (2)$$

65 Visaria's attempt is based on the fact that SR is a monotonously increasing function of  $p$ .

66 The length of Visaria's CI is

$$67 \quad CI_1 = \frac{p_U}{1 - p_U} - \frac{p_L}{1 - p_L} = \frac{2h}{(1-p)^2 - h^2} = \frac{2k\sqrt{\frac{p(1-p)}{N}}}{(1-p)^2 - k^2 \frac{p(1-p)}{N}} = \frac{2k\sqrt{\frac{p}{(1-p)^3 N}}}{1 - k^2 \frac{p}{(1-p)N}}. \quad (3)$$

68 Fellman and Eriksson [5] gave an alternative confidence interval for SR. According to the  
69 ML theory, the variance of SR is

$$70 \quad \text{Var}(\text{SR}) = \left( \frac{df(p)}{dx} \right)^2 \text{Var}(p), \quad (4)$$

71 where  $f(p) = \text{SR} = p/(1 - p)$ . Hence,

$$72 \quad \text{Var}(\text{SR}) = \left( \frac{d\left(\frac{p}{1-p}\right)}{dp} \right)^2 \text{Var}(p) = \left( \frac{1}{(1-p)^2} \right)^2 \frac{p(1-p)}{N} = \frac{p}{(1-p)^3 N}. \quad (5)$$

73 From (5), it follows that the standard deviation (SD) of SR is  $SD(\text{SR}) = \frac{1}{(1-p)} \sqrt{\frac{p}{(1-p)N}}$

74 and the CI is

$$75 \quad \left( \text{SR} - \frac{k}{(1-p)} \sqrt{\frac{p}{(1-p)N}}, \quad \text{SR} + \frac{k}{(1-p)} \sqrt{\frac{p}{(1-p)N}} \right). \quad (6)$$

76 Obviously, the centre of the CI is SR. The length of the CI is

$$77 \quad CI_2 = \frac{2k}{(1-p)} \sqrt{\frac{p}{(1-p)N}} \quad (7)$$

78 In addition to the result that the centre of the Visaria CI converges towards SR, Fellman and  
 79 Eriksson [5] obtained

$$80 \quad \frac{CI_2}{CI_1} = 1 - k^2 \frac{p}{(1-p)N} \leq 1. \quad (8)$$

81 Hence,  $CI_2 \leq CI_1$ , but the ratio  $CI_2/CI_1 \rightarrow 1$  when  $N \rightarrow \infty$ . Thus, the CIs are asymptotically  
 82 identical, and although the observed SRs are biased, both are applicable for large  $N$ .

83 **Statistical tests.** Fellman and Eriksson [5] presented also a new  $\chi^2$  test. The simultaneous  
 84 test of several SRs results in  $\chi^2$  test. Let the null hypothesis be that the sets have a common  
 85  $SR_0$ . According to (5), the variance  $\text{Var}(SR_t) = p_0 / [(1-p_0)^3 N_t]$ , where  $p_0 = SR_0 / (1+SR_0)$   
 86 and  $N_t$  is the sample size for  $t=1, \dots, T$  and  $T \geq 2$ . The variable  
 87  $(SR - SR_0) / \sqrt{p_0 / [(1-p_0)^3 N_t]}$  is a standardized variable, and consequently, asymptotically  
 88 as  $N(0, 1)$ . Hence,

$$89 \quad \chi^2 = \frac{(1-p_0)^3}{p_0} \sum_{t=1}^T N_t (SR_t - SR_0)^2, \quad (9)$$

90 is  $\chi^2$  asymptotically distributed with  $T$  degrees of freedom. If  $p_0$  is unknown, we introduce  
 91 the weighted mean  $\overline{SR} = \sum_{t=1}^T (N_t SR_t) / N_t$ , which under the null hypothesis is the most  
 92 efficient estimate of  $SR_0$ , and estimate  $p_0 = \overline{SR} / (1+\overline{SR})$ . Consequently, we have estimated  
 93 one parameter, and

$$94 \quad \chi^2 = \frac{1}{\overline{p}_0} (1 - \overline{p}_0) \sum_{t=1}^T N_t (SR_t - \overline{SR})^2 \quad (10)$$

95 is  $\chi^2$  distributed with  $T-1$  degrees of freedom.

96 Krackow et al. [7] presented an analogous  $\chi^2$  test based on the proportion of males. Their  
 97 method follows the same ideas as the Fellman and Eriksson test based on the SR.  
 98 Although we observed above that sufficient statistical methods exist for the analyses of the  
 99 SR, we note that the proportion of males births among the total ( $P_M$ ) is a more convenient

alternative for the statistical study of the proportions of males and females. We therefore consider this variable here.

**Material.** This study is based on data sets obtained from the official Danish StatBank produced by Statistics Denmark. Some of these data were submitted for this study by Caroline Østerholm-Jørgensen of Statistics Denmark. The goal of our investigations is to examine the effects of different possible factors. Consequently, we have to base our studies on different Danish data sets. Denmark is a small country and in order to obtain sufficiently large data sets, especially data concerning multiple maternities, the data must be collected over whole periods. Accordingly, one must investigate whether there are temporal effects on the SRs.

## Results

**Live- and stillborn.** In Fig. 1, we compare the temporal trends of  $P_M$  for live- and stillborn children. When we test temporal effects of  $P_M$  on live births, we obtain  $\chi^2 = 5.16$  with 6 DF, and the temporal effect is non-significant ( $P > 0.05$ ). For the stillborn, the corresponding test results are  $\chi^2 = 8.38$  with 6 DF and  $P > 0.05$ . When we compare the total data sets for live-born and stillborn children we observe that the difference between the total means is  $z = 0.00433$  with the  $z = 0.00433$  and  $t = 0.581$ , and consequently, the difference is non-significant ( $P > 0.05$ ). For the total data set,  $P_M = 0.51305$  with  $SD = 0.000445$ .

## Figure 1

**Maternal age.** We studied the effect of maternal age based on Danish data for the period 1973-2016. The results are presented in Figure 2. No significant maternal age effect was identified ( $\chi^2 = 3.299$  with 6 DF and  $P > 0.05$ ). The total mean is  $P_M = 0.5133$  with  $SD = 0.000302$ . Significant temporal differences were not obtained. In Fig. 2 is included a 95% confidence band. Note how the size of the data set influences the band.

## Figure 2

**Singletons and multiples.** Based on Danish data for the period 1997–2016, we can compare the temporal trends of  $P_M$  among singletons and multiples. For singletons, we obtain  $\chi^2 = 3.437$  with 6 DF, and the temporal differences are non-significant ( $P > 0.05$ ). For multiple maternities,  $\chi^2 = 9.969$  with 6 DF and  $P > 0.05$ . When we compare singletons and

129 multiples for the total data sets, we observe that the mean difference is 0.0055 with  
130 SD = 0.00158 and  $t = 3.49$ , and consequently, the difference is significant ( $P < 0.001$ ), and  
131 the  $P_M$  is higher among singletons than among multiples (see Fig. 3).

### 132 Figure 3

133 **Twins and triplets.** Based on Danish data for the period 1911 – 2016, we can compare the  
134 temporal trends of  $P_M$  among twins and triplets (see Fig. 4). For twins, we obtain  $\chi^2 = 43.71$   
135 with 20 DF, and the temporal differences are significant ( $P < 0.01$ ). For triplets, the test  
136 result is  $\chi^2 = 56.134$  with 20 DF and  $P < 0.001$ . When we compare twins and triplets for the  
137 total data sets, we observe that the mean difference is 0.0007057 with SD = 0.007923 and the  
138 difference is non-significant. Hence, there is no significant difference between  $P_M$  among  
139 twins and triplets. For the total set of data,  $P_M = 0.509568$  with SD = 0.001066.  
140 Furthermore, we observe that there is no significant correlation between time and  $P_M$  for  
141 either twins ( $r = -0.0410$ ) or triplets ( $r = 0.1168$ ).

### 142 Figure 4

### 143 Discussion

144 In the 17<sup>th</sup> century, lively discussion ensued on the causes of the excess of boys over girls.  
145 One argument was that this balance between male and female births is not the result of  
146 chance but of Divine Providence. Opponents argued that even if  $P_M$  differs from 0.5,  
147 sufficiently many repeated trials will result in a relative frequency very close to  $P_M$ . Berg [8]  
148 presented a detailed analysis of the SR in Sweden. Although the study was published as late  
149 as 1871, he connected the SR to the blessing in the Holy Bible that said, “Be fruitful and  
150 multiply, and fill the earth.”

151 Attempts have later been made to identify factors influencing the level of the SR. Hawley [9]  
152 stated that where prenatal losses are low, as in Western countries, the SRs are usually around  
153 105 to 106, but in areas where the frequencies of prenatal losses are relatively high the SRs  
154 are around 102. Visaria [6] stated that racial differences exist in the SR. Furthermore, he  
155 commented that random errors influence the variation in the SR. Therefore, he presented a  
156 numerical table of how the confidence intervals (CIs) of the SR depend on the observed SR  
157 and the number of births. He gave no formula for the intervals, but stated that “the standard  
158 error of an observed sex ratio can be estimated as the standard error of the proportion  $p$  of

male births among the total”. As Visaria [6] pointed out, the CIs are crucial when differences in the SRs must be interpreted. Fellman and Eriksson [5] presented Visaria’s and their CIs with respect to the sample size  $N$  given on a logarithmic scale. They noted that for small data sets the CIs are broad, and consequently, it is difficult to identify statistically significant differences.

Statistical analyses have shown that comparisons between SRs demand large data sets because random fluctuations in moderate data are marked. Consequently, reliable results presuppose national birth data. The interest has focused on temporal, spatial and seasonal fluctuations in the SR. Temporal fluctuations have often been connected to periods of war and peace. Temporal trends in SR were obtained in the Nordic countries, including Denmark, and linear trends for increasing SRs up to 1950 have been noted [10], [11].

Fellman and Eriksson [12] analysed the effect of total fertility rate (TFR) and crude birth rate (CBR) on the SR. They analysed historical demographic data and the regional variations between counties in Sweden, and they built spatial models and as regressors they used geographical coordinates for the provincial capitals of the counties. The SR among the live born in 1749–1869 and the twinning rate (TWR) in 1751–1860 showed slight spatial variations. The influence of CBR and TFR on the SR and TWR was examined, and statistically significant effects were found.

Recently, SR studies have mainly concentrated on the identification of general but occasional factors. Grech has tried to identify such factors in a series of papers; see, for instance, Grech and Borg [4].

## **Conclusion**

The variation in the SR cannot be finally solved. Attempts to identify influential factors in national birth registers have been unsuccessful. Fellman [13] noted a slight increasing trend in SR in Sweden in 1751-1870. This is probably associated with the country’s increasing standard of living. A minor trough was observed between 1800 and 1860. This result combined with other evidence points to a mild subsistence crisis in western Sweden during this period. In general, factors that affect the SR within families remain poorly understood. Attempts to identify such factors in national birth registers have been unsuccessful.

## **References**

[1] Fellman, J. Glimpses at the History of Sex Ratio Studies. *Sci J of Public Health* 5a (2015a) 291-302.

- [2] Fellman, J. Some aspects of sex ratio studies, *Acta Paediatrica* 105 (2016) 1371.
- [3] Fellman, J. Sex ratio at birth. In *International Encyclopedia of Statistical Science*, (Ed. Miodrag Lovric) Part 19 (2011) 1314-1316
- [4] Grech V. and Borg T. Seasonality of sex ratio at births in Iceland and effects of the 2010 Eyjafjallajökull volcanic eruption. *Acta Paediatrica* 105 (2016) 369–70.
- [5] Fellman, J. & Eriksson, A. W. Secondary sex ratio in multiple births. *Twin Res. and Hum. Genetics*, 13(1) (2010) 101-108.
- [6] Visaria P. M. Sex ratio at birth in territories with a relatively complete registration. *Eugenics Quarterly* 14 (1967) 132-142.
- [7] Krackow, S., Meelis, E. & Hardy, I. C. W. Analysis of sex ratio variances and sequences of sex allocation. In I. C. W. Hardy (Ed.). *Sex Ratios Concepts and Research Methods* Cambridge, UK: Cambridge University Press 2002 pp. 112–131.
- [8] Berg, F. T. Proportionen mellan könen bland de födde och inom den stående befolkningen med hänsyn till Sverige och dess provinciella olikheter (Sex ratio at birth and in the population with respect to Sweden and its regions). *Kungl Svenska Vetenskapsakademiens Handlingar*, 10(6) (1871) 1–40.
- [9] Hawley, A. H. Population composition. In: *The Study of Population: An Inventory and Appraisal*. Ed. P. M. Hauser and O. Dudley Duncan. Chicago, University of Chicago Press, 1959 pp. 361-382.
- [10] Fellman, J. & Eriksson, A. W. Temporal trends in the secondary sex ratio in Nordic countries. *Biodemography and Soc. Biology* 57:2 (2011) 143-154.
- [11] Fellman, J. Temporal and regional trends in the secondary sex ratio: the Nordic experience. *Early Hum. Development*, 91 (2015c) 801–806.
- [12] Fellman, J. & Eriksson, A. W. Effect of fertility on secondary sex ratio and twinning rate in Sweden, 1749-1870. *Twin Res. and Hum. Genetics*, 18 (2014) 92-99. doi:10.1017/thg.2014.71.
- [13] Fellman J. Regional variation in the secondary sex ratio in Sweden, 1749-1869. *Br. J. of Appl. Sci. & Technology* 9(6) (2015b) 566-575.

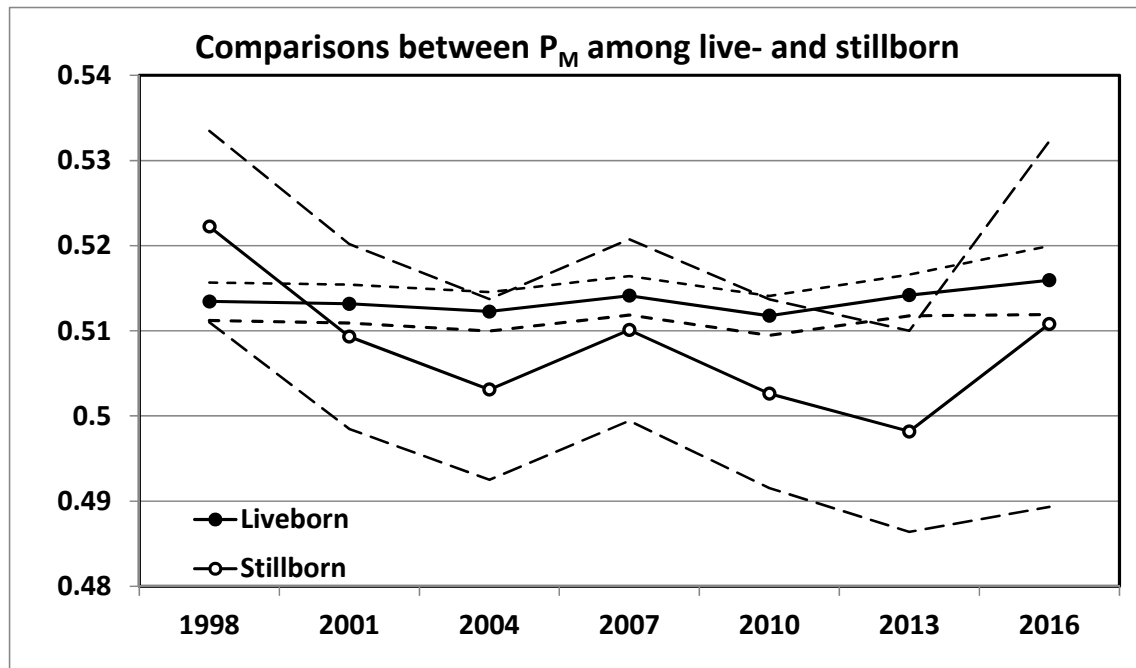


**Figure 1.** Temporal trends for proportions of males,  $P_M$ , among live- and stillborn children. The figure includes 95% confidence bands. The difference between live- and stillborn is non-significant. The study is based on Danish birth data for the period 1997-2016.

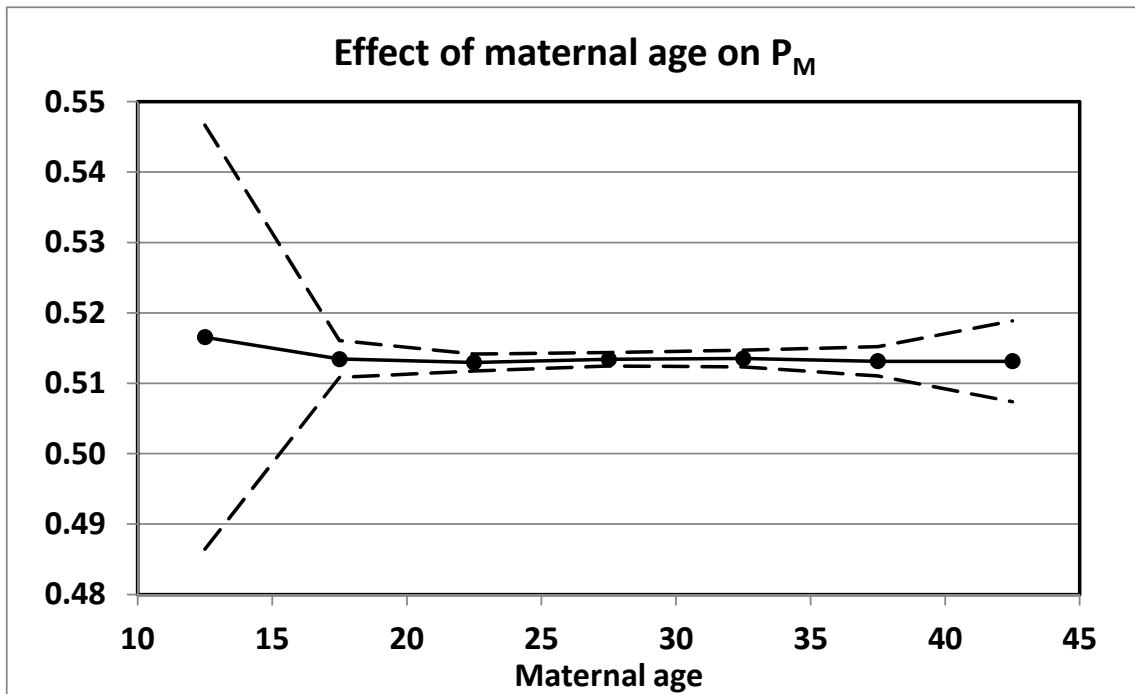


**Figure 2.** Effect of maternal age on proportion of males  $P_M$ . The figure includes a 95% confidence band. The analysis is based on Danish data for the period 1973-2016. The figure includes a 95% confidence band.

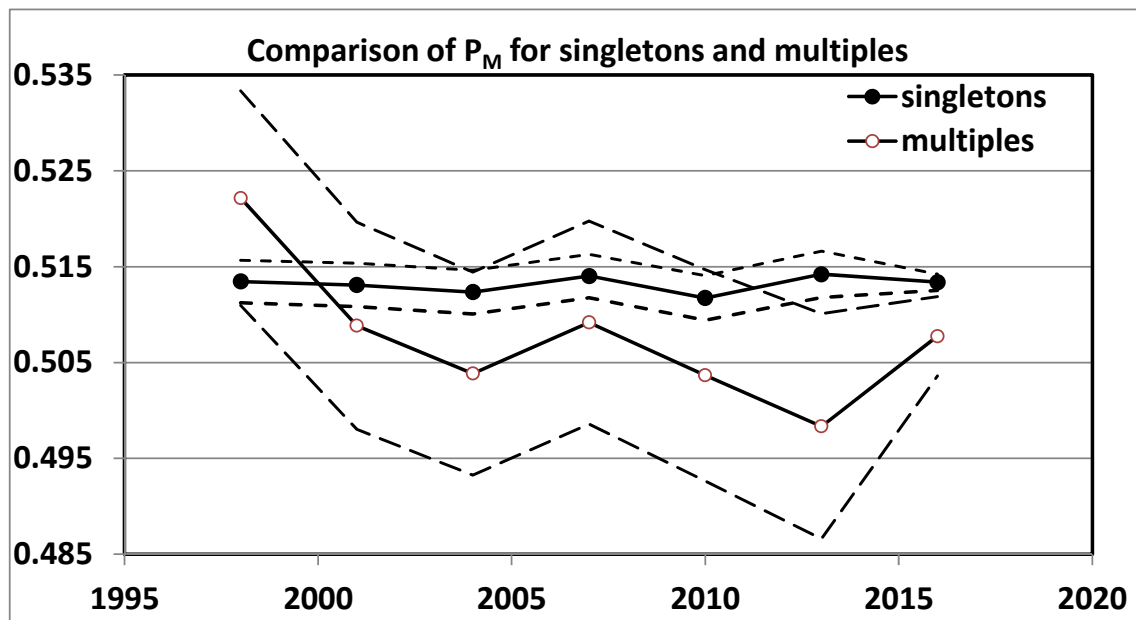


**Figure 3.** Temporal trends for  $P_M$  among singletons and multiples. The analysis is based on Danish data for 1997-2016. The figure includes 95% confidence bands.

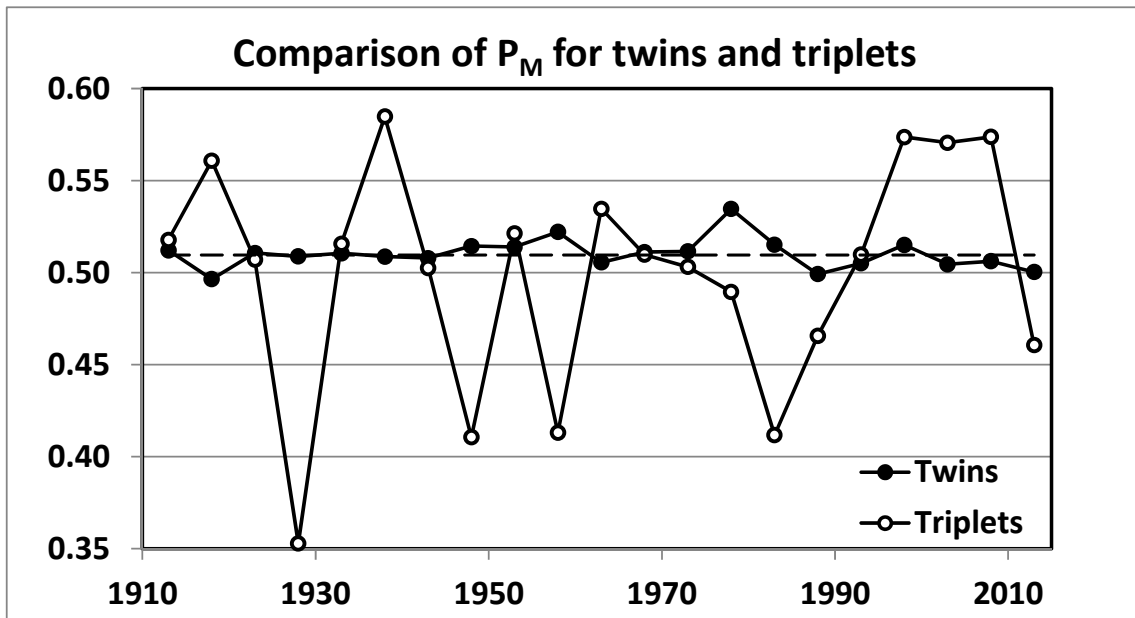


**Figure 4.** Temporal trends for  $P_M$  among twins and triplets. The analysis is based on Danish data for 1911-2016. The figure includes the total mean for twins. The results are discussed in the text.