

COMPARISING OF GOODNESS OF FIT TESTS FOR NORMAL DISTRIBUTION.

ABSTRACT

Goodness of fit test is a test that have attracted researchers interest over the decades. This study is on goodness of fit test for normal distribution only. The Kolmogorov-Smirnov (K-St) and Pearson's Chi-square (χ^2 test) goodness of fit test were used to determine the normality of a given data. The result revealed that the data is normal under the two tests and that the Kolmogorov-Smirnov (K-S test) were preferred to Pearson's Chi-square (χ^2 test). The Kolmogorov-Smirnov (K-S) test of goodness of fit is the most suitable in terms of the p-value.

Introduction

The common objectives in goodness of fit test is to investigate (or fit) the closeness of the observed data to that of the expected data which is sometimes based on theoretical models. This addresses the question on whether the data follows any pattern or fit a specified or assumed probability distribution. Measure of goodness of fit typically summarizes the discrepancy between the observed values and the values expected under the model in question.

Such measures can be used in hypothesis testing, also to test for normality of residuals, to test whether outcomes of frequencies follow a specified distribution. Quite often the observed data represent the fit of the saturated, the most complex model possible with the given data. Thus, quite often the alternative hypothesis (H1) will represent the saturated model which fits perfectly because each observation has a separate parameter.

When talking about goodness of fit, more references are made towards Pearson's chi-square only. However, Pearson's Chi-square approximation which is commonly used is not adequate for small samples. Therefore, the study look into other forms of goodness of fit tests like the Kolmogorov-Smirnov (K-S) goodness of fit test.

Normal distribution

In probability theory, the normal (or Gaussian) distribution is a very common continuous probability distribution. Normal distributions are important in statistics and are often in the natural and social sciences to represent real valued random variables whose distributions are not known. The normal distribution is the only absolutely continuous distribution whose cummulants beyond the first two (i.e. other than the mean and variance)

are zero. The normal curve was developed mathematically in 1733 by DeMoivre as an approximation to the binomial distribution primarily from the fact that distributions of many natural occurrences are at least approximately normally distributed. The paper was discovered in 1924 by Karl Pearson. Laplace used the normal curve (Gaussian distribution) in 1783 to describe the distribution of errors. The normal distribution was first applied on the analysis of errors of measurement made in astronomical observations by Gauss in 1809.

Kolmogorov-Smirnov goodness of fit test

The Kolmogorov-Smirnov of goodness of fit is one of the nonparametric tests. The test can be used to verify that a sample comes from a population with some known distribution and also that two populations have the same distribution. It is commonly used as a test for normality to verify if a particular data or datasets is normally distributed. The K-S test can also be used to check the assumption of normality in ANOVA.

More specifically, the K-S test compares a known hypothetical probability distribution (e.g. normal distribution) to the distribution generated by your data the empirical distribution function in order to assess fit (Stephens, M. A., 1974). The goodness of fit test for a sample was introduced by Andrey Nikolaevich Kolmogorov (1933), he defined the empirical distribution function and find out how close this would be to the true distribution of $F(x)$ when this is continuous. The Kolmogorov-Smirnov test for two samples was introduced by Vladimir Ivanovich Smirnov (1939).

Chi-square goodness of fit test

The chi-square goodness of fit test is applied to binned data (i.e. data put into classes). This actually not a restriction since for non-binned data you can simply calculate a histogram or frequency table before generating the chi-square. Also, the chi-square goodness of fit test can be applied to discrete distributions such as the Binomial and the Poisson distribution while the Kolmogorov-Smirnov is restricted to continuous distributions. Pearson's paper of 1900 introduced what subsequently became known as chi-square goodness of fit to examine whether the observed data support a given specification. His work motivated research in testing of hypothesis and estimation unknown parameters and led to the development of statistics as a separated discipline. Goodness of fit test is highly used in business decision making. During the period 1899, Pearson statistical interest increased rapidly towards his system of frequency curve, which in return laid the foundation of multiple correlation and regression and obtain probable errors and correlation of estimated co-efficient in large samples.

Also in calculating goodness of fit, it is necessary to state the hypothesis (null, alternative), choose a significance level and to determine the critical value. Also in the analysis of variance, one of the components into which the variance is portioned may be a lack of fit sum of squares.

77 **Statement problem**

78 We tend to ask at times if a particular model follows or fit any assumed probability
79 distribution. Hence, this study analysed Kolmogorov-Smirnov goodness of fit test and chi-
80 square goodness of fit test for normal distribution. The result was compared and the most
81 suitable goodness of fit test is selected to test normality based on p-value.

82 **Result**

83 **KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST**

84 The Kolmogorov-Smirnov (K-S) test is based on the empirical distribution function
85 (ECDF). Given N ordered data points $Y_1, Y_2, \dots, Y_N$, the ECDF is defined as

$$86 \quad EN = n(i)/N \quad (1)$$

87 Where $n(i)$ = the number of points is less than Y_i and the Y_i are ordered from smallest to
88 largest value. This is a step function that increases by $1/N$ at the value of each ordered data
89 point.

90 The Kolmogorov-Smirnov statistic for a given cumulative distribution function $F(x)$ is

$$91 \quad D_n = \sup_x |F_n(x) - F(x)| \quad (2)$$

92 Where;

93 $\sup_x$ = the supremum of the set of distances.

94 $F_n(x)$ = the cdf of the hypothesized distribution.

95 $F(x)$ = the empirical distribution function of the observed

96 By the Glivenko-Cantelli theorem, if the sample comes from distribution $F(x)$, then
97 D_n converges to 0 almost surely in the limit when goes to infinity. For one-tailed test, omit
98 the absolute values from the formula. If D_n is greater than the critical value, the null
99 hypothesis is rejected. Critical values for D_n are found in the K-S Test P-Value Table. The
100 Kolmogorov-Smirnov test can be adapted for discrete variables. The form of the test statistic
101 remains the same as in the continuous case, but the calculation of its value is more subtle.

102 While the Kolmogorov-Smirnov test is usually used to test whether a given $F(x)$, is
103 the underlying probability distribution of $F_n(x)$, the procedure may be inverted to give
104 confidence limits on $F(x)$, itself. If one chooses a critical value of the test statistic D_α such
105 that $P(D_n > D_\alpha) = \alpha$, then a band of width $\pm D_\alpha$ around $F_n(x)$ will entirely contain $F(x)$ with
106 probability $1 - \alpha$.

107 When the p-value is greater than ($\alpha = 0.05$), we accept the null hypothesis and when
108 it is less than we reject the null hypothesis and draw conclusion.

CHI-SQUARE GOODNESS OF FIT TEST

$$\chi^2 = \sum_{i=1}^n \left[\frac{(O_i - E_i)^2}{E_i} \right] \quad (3)$$

Where, O_i = observed frequency

E_i = expected frequency

n= the sample size.

NORMAL DISTRIBUTION

The probability density function of the normal distribution is given by:

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x - \mu)^2 / 2\sigma^2} \quad (4)$$

Where, μ = the mean or expectation of the distribution

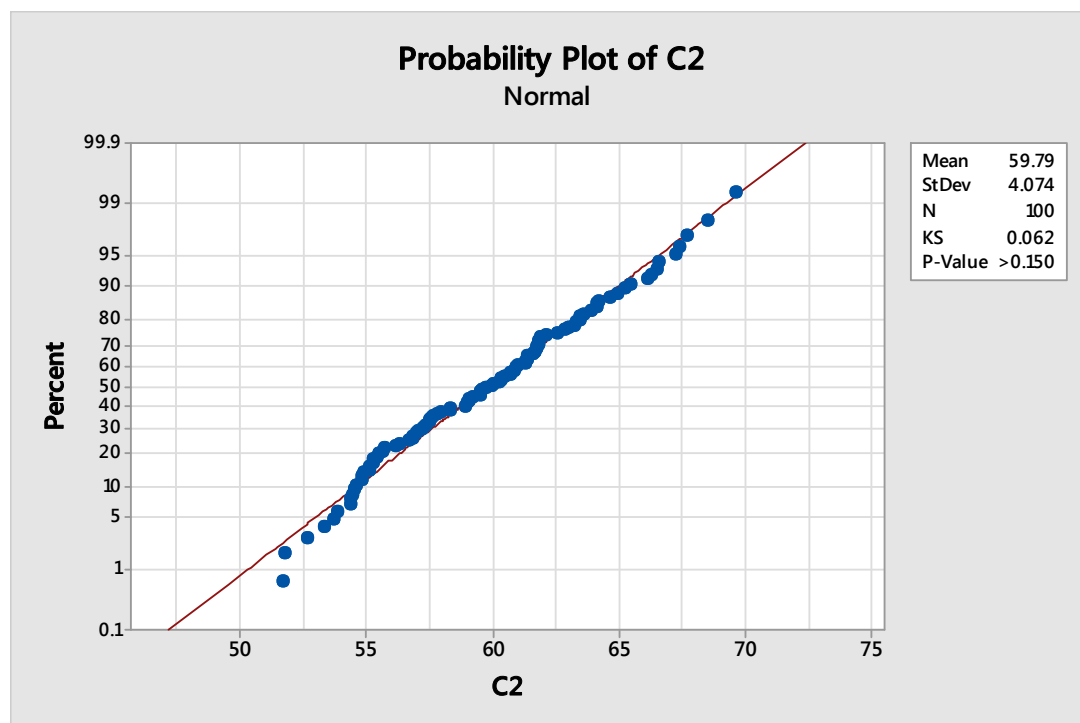
σ = is the standard deviation

σ^2 = is the variance.

This applies single variables to continuous data e.g. heights of plants, length of time etc. to calculate the value of the “standard score” or “z-score”

$$Z = \left(\frac{x - \mu}{\sigma} \right)$$

THE KOLMOGOROV-SMIRNOV TEST (100 sample sizes)



125 **Decision Rule:** Since the p-value is greater than 0.05, which is above our
 126 threshold of 5% (i.e. $0.150 \geq 0.05$) we accept the null hypothesis (H_0). Hence,
 127 we say that the distribution is normal (according to Kolmogorov-Smirnov test).

128 **CHI-SQUARE GOODNESS OF FIT TEST FOR (100 sample sizes)**

129 Test	Contribution				
130 Category	Observed	Proportion	Expected	to Chi-Sq	
131 1	59.4930	0.01	59.7851	0.00143	
132 2	62.8864	0.01	59.7851	0.16088	
133 3	55.6405	0.01	59.7851	0.28733	
134 4	57.5830	0.01	59.7851	0.08112	
135 5	67.7264	0.01	59.7851	1.05485	
136 6	59.4992	0.01	59.7851	0.00137	
137 7	54.9360	0.01	59.7851	0.39331	
138 8	66.2719	0.01	59.7851	0.70382	
139 9	58.9497	0.01	59.7851	0.01168	
140 10	60.3593	0.01	59.7851	0.00551	
141 11	64.2017	0.01	59.7851	0.32626	
142 12	59.5158	0.01	59.7851	0.00121	
143 13	57.0593	0.01	59.7851	0.12428	
144 14	68.5422	0.01	59.7851	1.28269	
145 15	60.0724	0.01	59.7851	0.00138	
146 16	66.1049	0.01	59.7851	0.66804	
147 17	55.2737	0.01	59.7851	0.34044	
148 18	57.3175	0.01	59.7851	0.10185	
149 19	53.3554	0.01	59.7851	0.69151	
150 20	64.6407	0.01	59.7851	0.39435	
151 21	63.6383	0.01	59.7851	0.24834	
152 22	66.5680	0.01	59.7851	0.76955	
153 23	59.7570	0.01	59.7851	0.00001	
154 24	57.6484	0.01	59.7851	0.07637	
155 25	63.4828	0.01	59.7851	0.22869	
156 26	61.7392	0.01	59.7851	0.06387	
157 27	59.5595	0.01	59.7851	0.00085	
158 28	58.3292	0.01	59.7851	0.03546	
159 29	58.9980	0.01	59.7851	0.01036	
160 30	57.2427	0.01	59.7851	0.10812	
161 31	61.3691	0.01	59.7851	0.04197	
162 32	55.7004	0.01	59.7851	0.27909	
163 33	63.2126	0.01	59.7851	0.19650	
164 34	57.5454	0.01	59.7851	0.08391	
165 35	57.3754	0.01	59.7851	0.09712	
166 36	59.0969	0.01	59.7851	0.00792	
167 37	65.4284	0.01	59.7851	0.53268	
168 38	54.6226	0.01	59.7851	0.44579	
169 39	57.5061	0.01	59.7851	0.08688	
170 40	64.0901	0.01	59.7851	0.30999	
171 41	60.7061	0.01	59.7851	0.01419	
172 42	55.1579	0.01	59.7851	0.35813	
173 43	60.3034	0.01	59.7851	0.00449	
174 44	54.5095	0.01	59.7851	0.46555	
175 45	61.7328	0.01	59.7851	0.06345	
176 46	61.9034	0.01	59.7851	0.07506	
177 47	58.3178	0.01	59.7851	0.03601	
178 48	63.4352	0.01	59.7851	0.22284	
179 49	51.6858	0.01	59.7851	1.09724	
180 50	57.9216	0.01	59.7851	0.05808	
181 51	64.1005	0.01	59.7851	0.31148	
182 52	51.7562	0.01	59.7851	1.07826	
183 53	61.0102	0.01	59.7851	0.02510	
184 54	59.9390	0.01	59.7851	0.00040	

185	55	61.3549	0.01	59.7851	0.04121
186	56	54.3843	0.01	59.7851	0.48789
187	57	61.6892	0.01	59.7851	0.06064
188	58	54.4691	0.01	59.7851	0.47269
189	59	67.4029	0.01	59.7851	0.97064
190	60	59.2042	0.01	59.7851	0.00564
191	61	61.2783	0.01	59.7851	0.03729
192	62	56.1507	0.01	59.7851	0.22094
193	63	52.6425	0.01	59.7851	0.85335
194	64	60.8505	0.01	59.7851	0.01898
195	65	55.2669	0.01	59.7851	0.34146
196	66	60.7334	0.01	59.7851	0.01504
197	67	61.8386	0.01	59.7851	0.07053
198	68	60.4728	0.01	59.7851	0.00791
199	69	55.5273	0.01	59.7851	0.30323
200	70	53.8824	0.01	59.7851	0.58280
201	71	65.2531	0.01	59.7851	0.50010
202	72	64.9194	0.01	59.7851	0.44092
203	73	61.3720	0.01	59.7851	0.04212
204	74	54.8447	0.01	59.7851	0.40827
205	75	62.5947	0.01	59.7851	0.13203
206	76	63.9012	0.01	59.7851	0.28338
207	77	60.9116	0.01	59.7851	0.02122
208	78	59.0900	0.01	59.7851	0.00808
209	79	61.2680	0.01	59.7851	0.03678
210	80	61.8430	0.01	59.7851	0.07083
211	81	63.0178	0.01	59.7851	0.17480
212	82	56.3016	0.01	59.7851	0.20298
213	83	69.6645	0.01	59.7851	1.63255
214	84	61.6288	0.01	59.7851	0.05686
215	85	57.8346	0.01	59.7851	0.06363
216	86	54.8108	0.01	59.7851	0.41389
217	87	56.8491	0.01	59.7851	0.14419
218	88	63.2730	0.01	59.7851	0.20348
219	89	56.8695	0.01	59.7851	0.14219
220	90	55.1001	0.01	59.7851	0.36714
221	91	60.2948	0.01	59.7851	0.00434
222	92	62.1054	0.01	59.7851	0.09005
223	93	56.9528	0.01	59.7851	0.13418
224	94	54.4178	0.01	59.7851	0.48187
225	95	55.4058	0.01	59.7851	0.32079
226	96	66.5318	0.01	59.7851	0.76134
227	97	61.8379	0.01	59.7851	0.07048
228	98	56.7078	0.01	59.7851	0.15840
229	99	53.6795	0.01	59.7851	0.62355
230	100	67.2664	0.01	59.7851	0.93618

231					
232					
233	N	DF	Chi-Sq	P-Value	
234	5978.51	99	27.4839	1.000	

235 **Decision Rule:** Since the p-value is 1.000 and is greater than 0.05 which is above threshold
 236 of 5% (i.e. $0.003 \geq 0.05$) we accept the null hypothesis (H_0). Hence, we say that the
 237 distribution is normal (according to Chi-square goodness of fit test).

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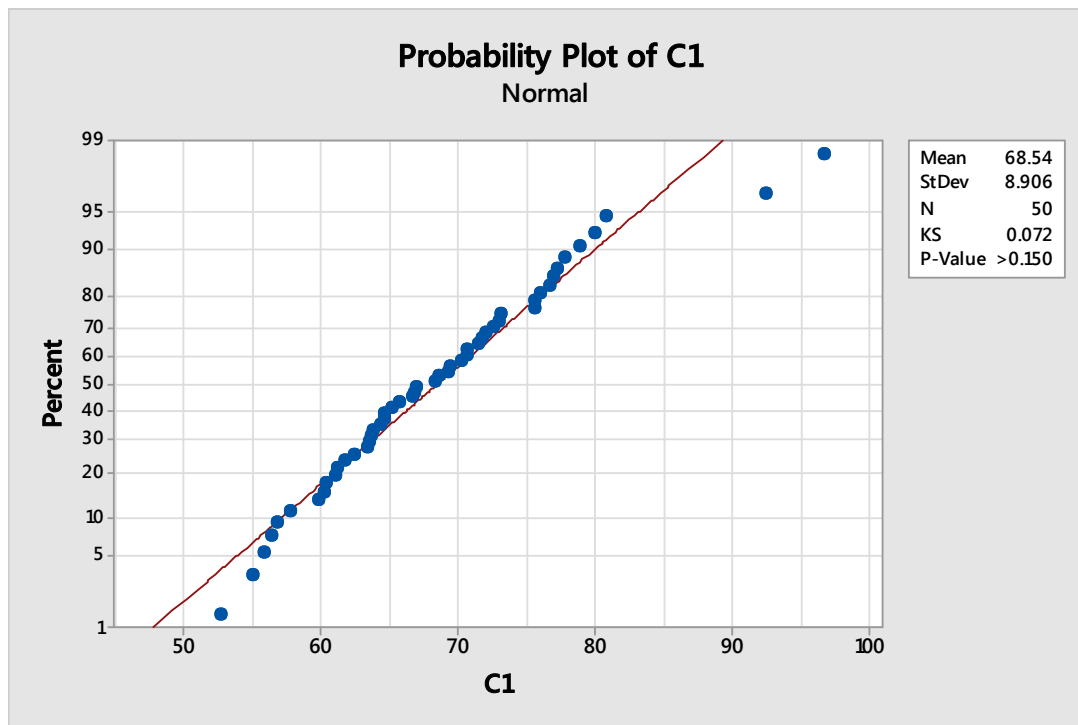
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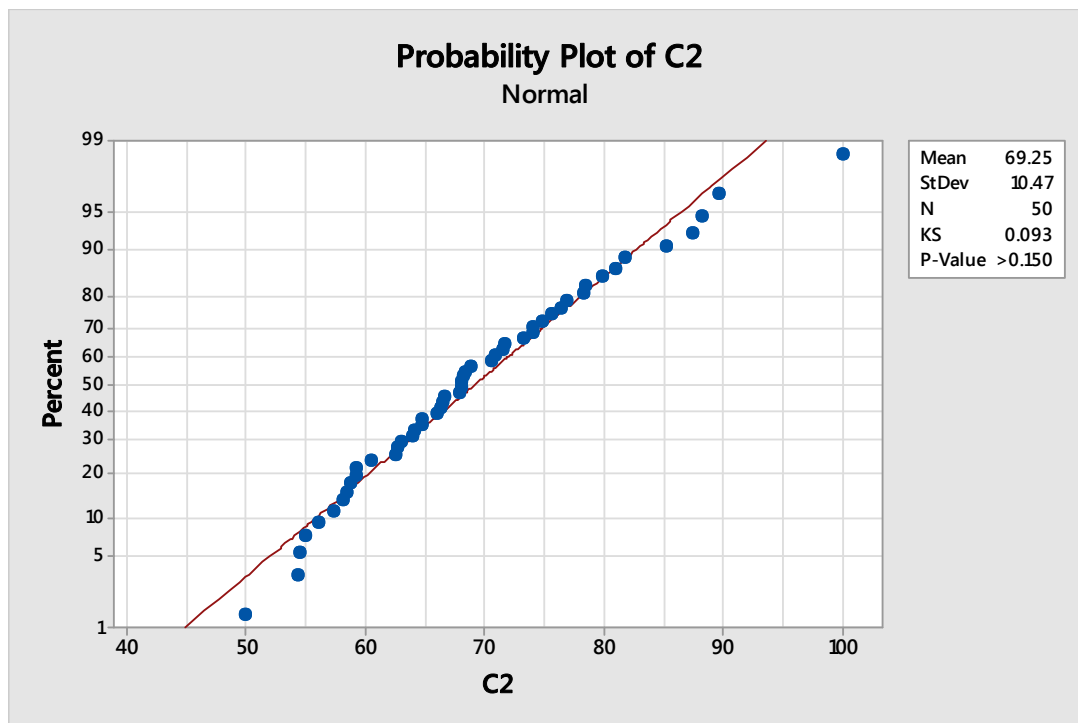
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243 THE KOLMOGOROV-SMIRNOV TEST (SIMULATED 15 DATA SET)

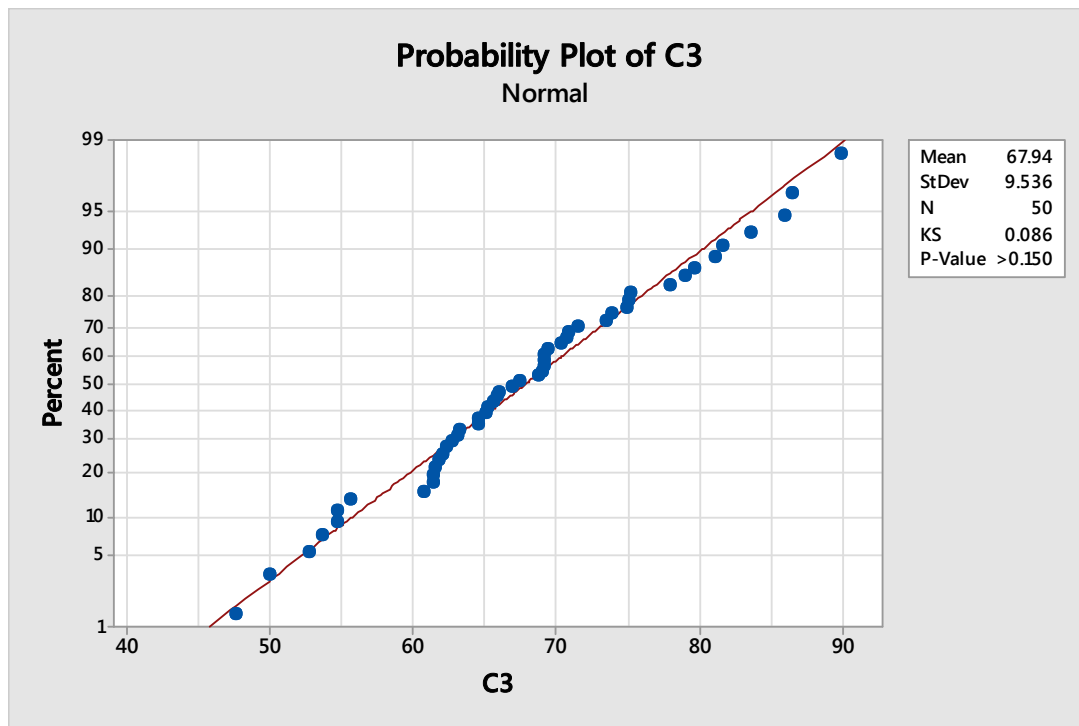


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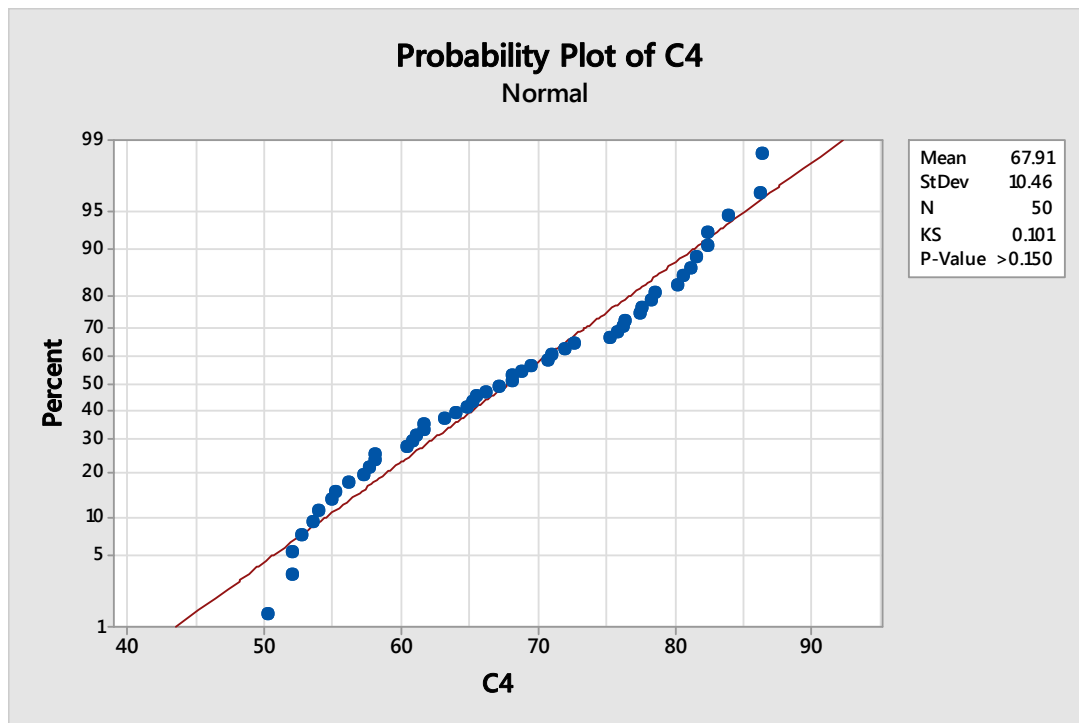
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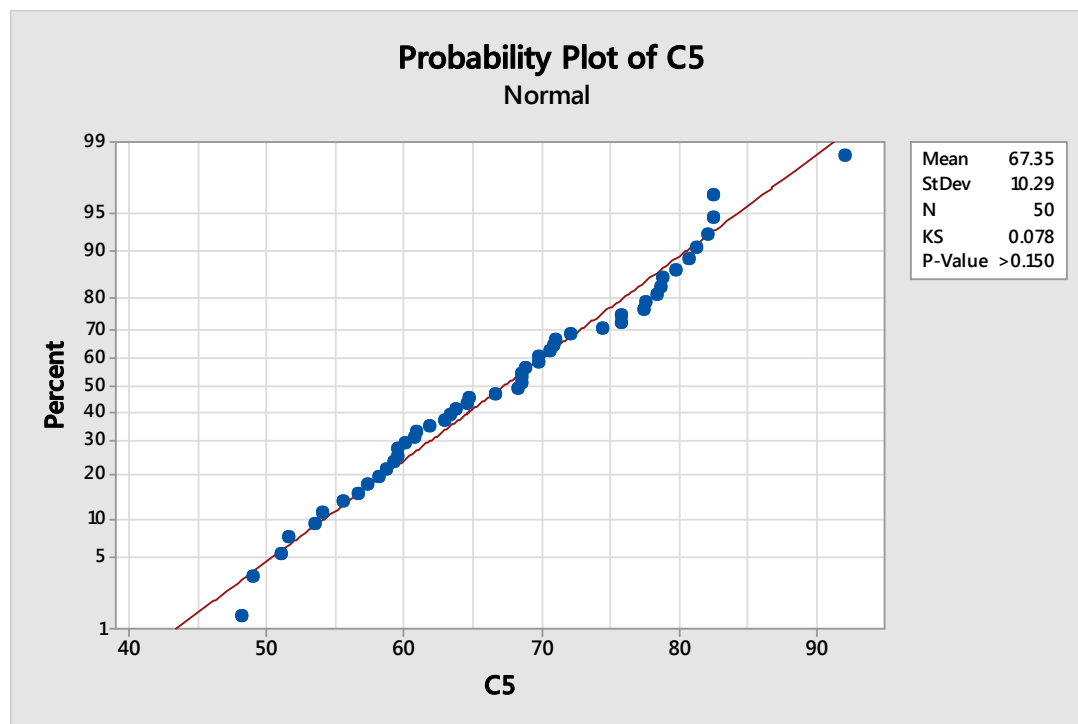
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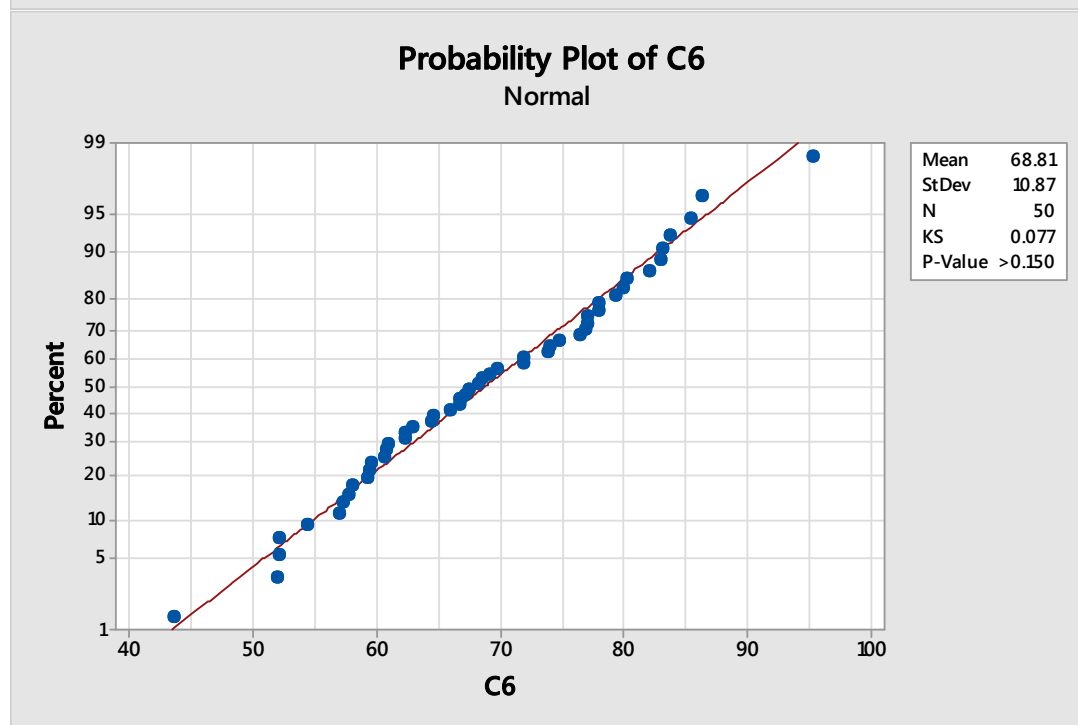
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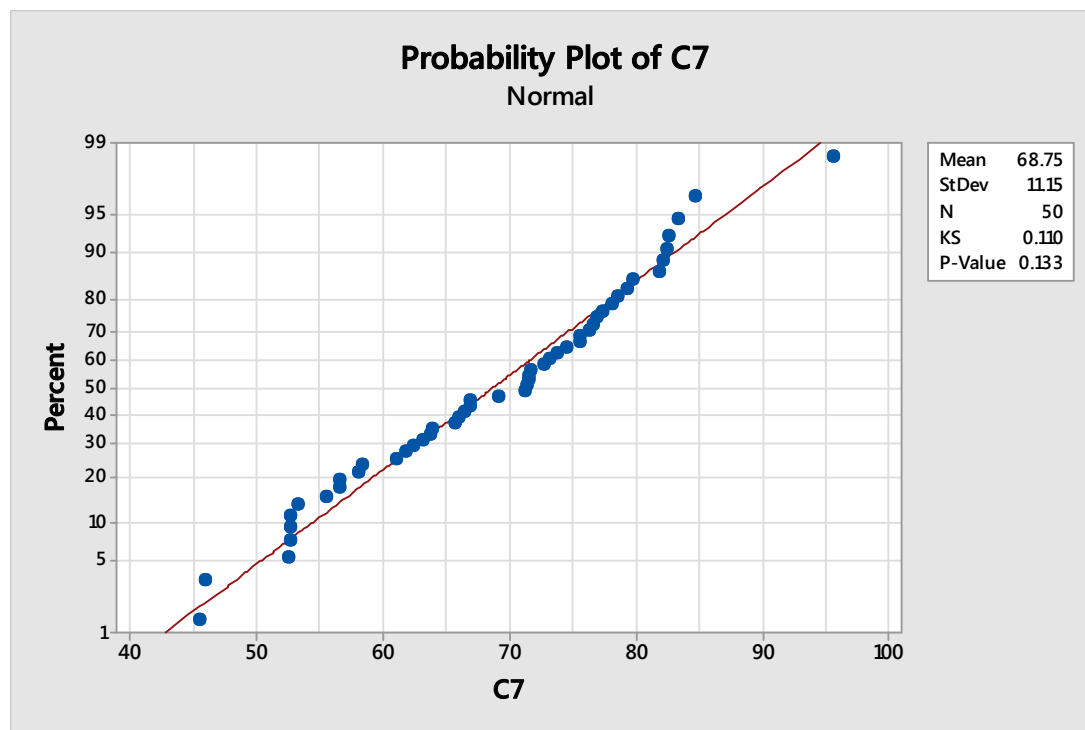
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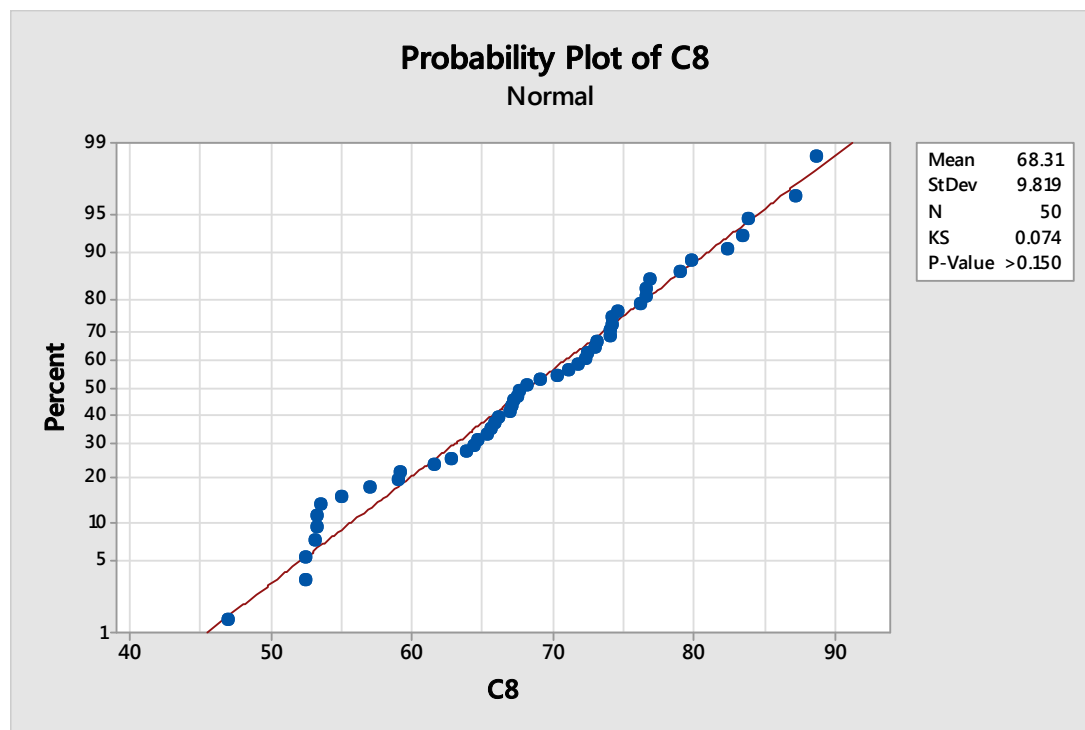


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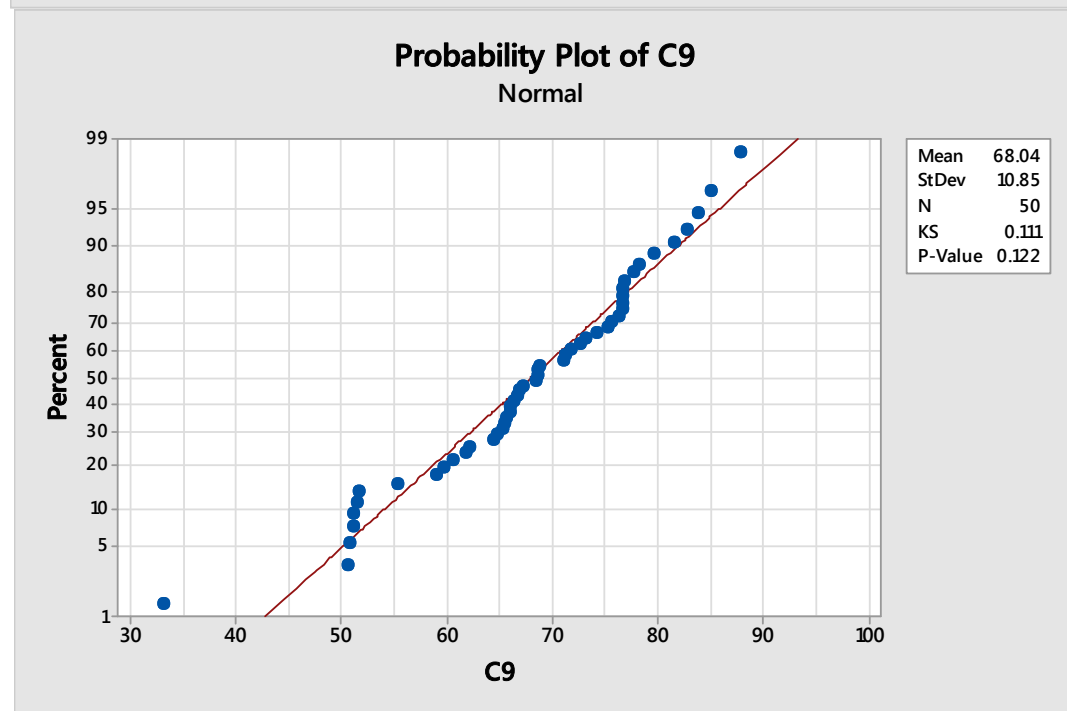


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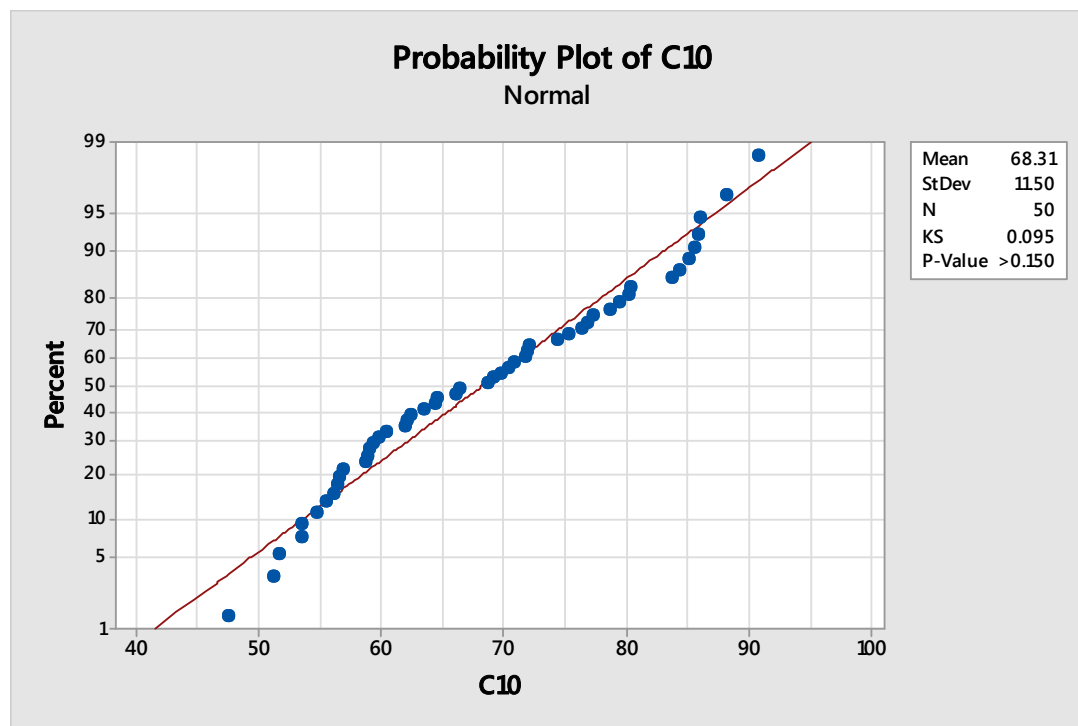




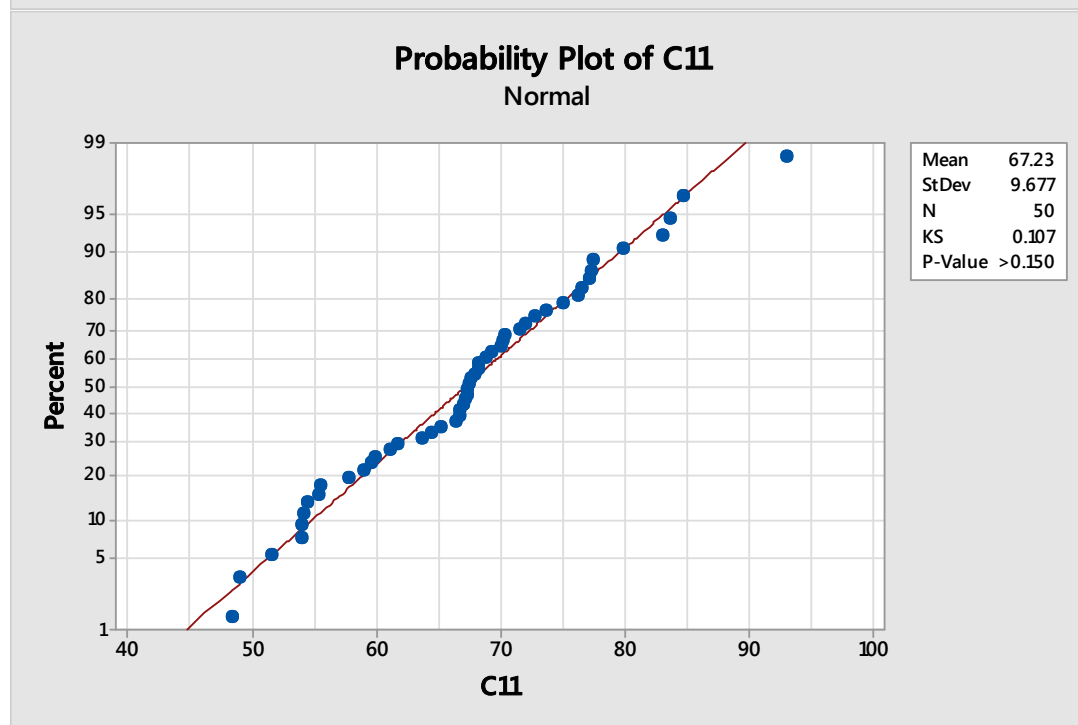
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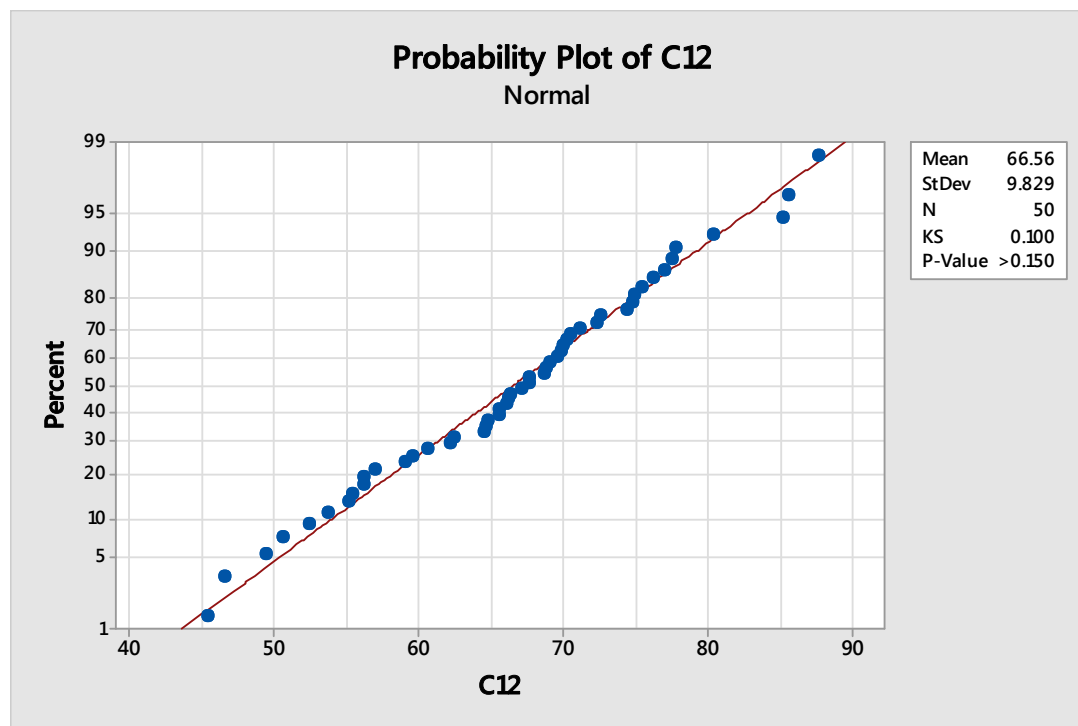
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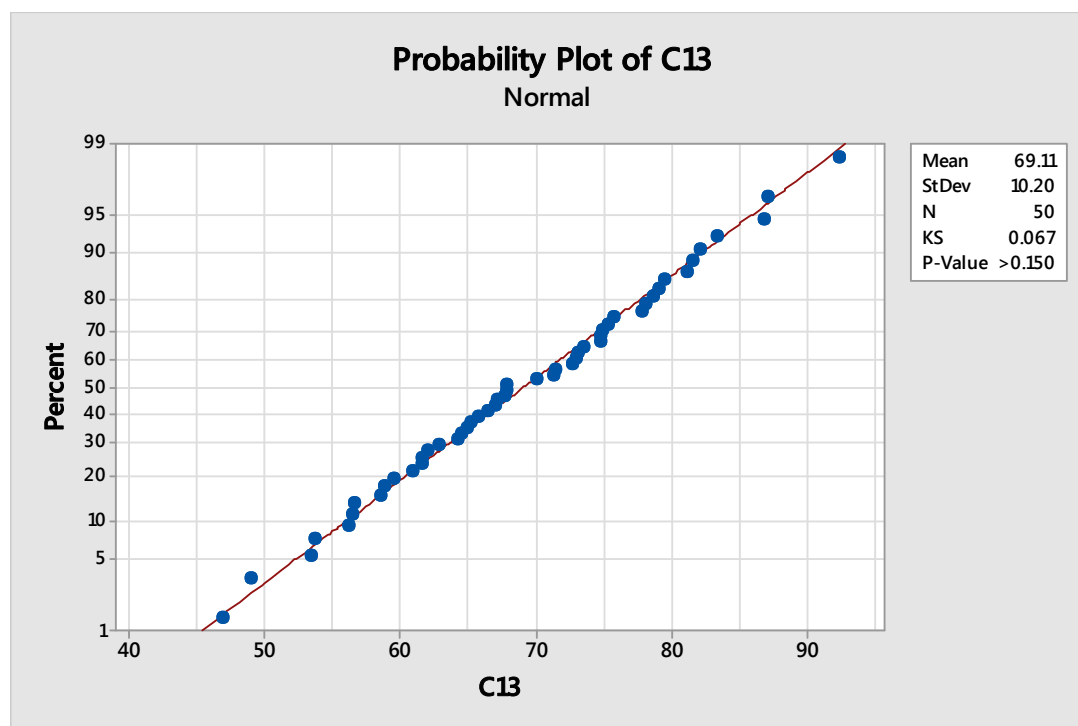
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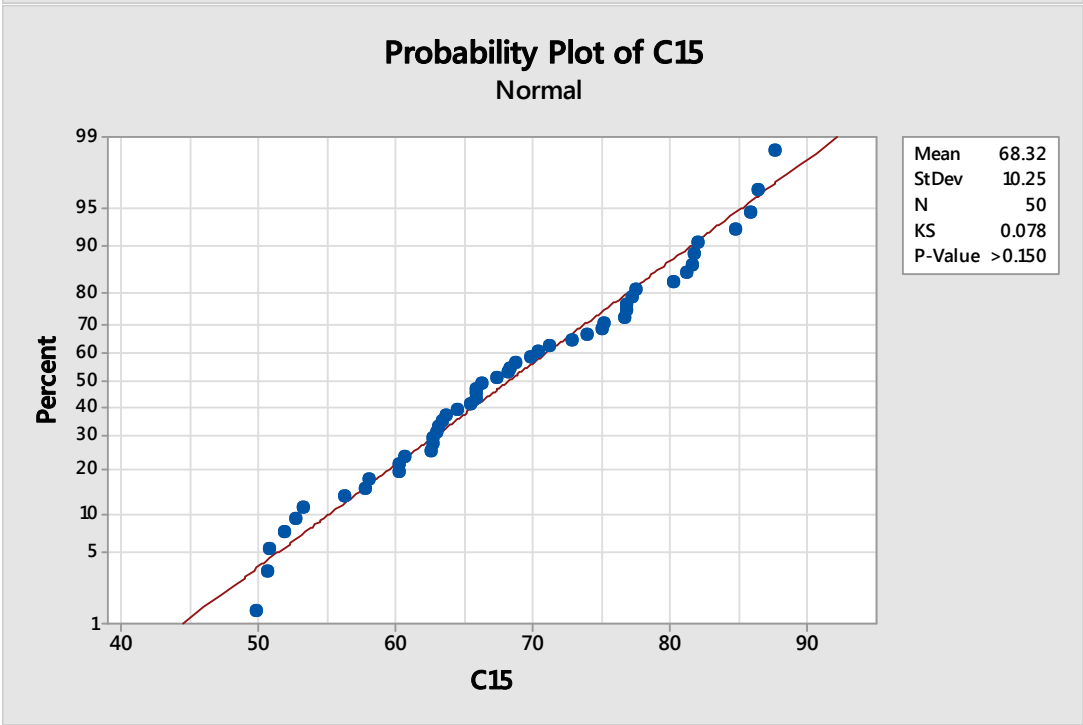
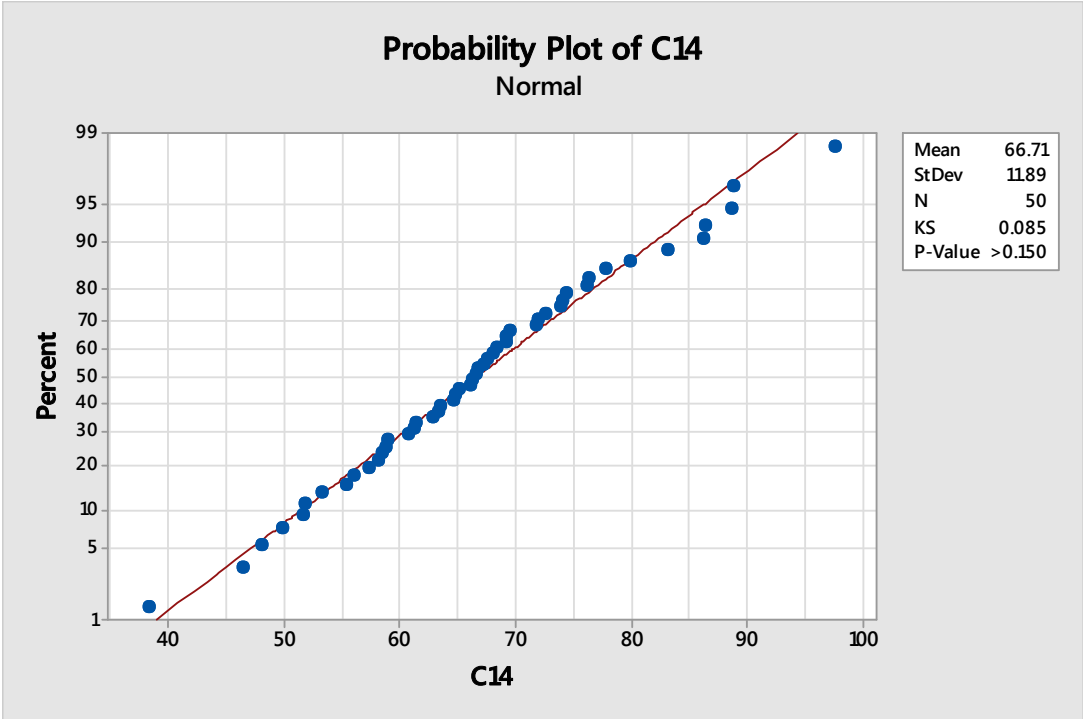
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CHI-SQUARE GOODNESS OF FIT TEST (15 DATA SET)

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C1

Test	Contribution
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267	Category	Observed	Proportion	Expected	to Chi-Sq
268	1	65.1546	0.02	68.5389	0.1671
269	2	75.6317	0.02	68.5389	0.7340
270	3	61.0160	0.02	68.5389	0.8257
271	4	80.7941	0.02	68.5389	2.1913
272	5	69.4623	0.02	68.5389	0.0124
273	6	66.7966	0.02	68.5389	0.0443
274	7	79.9395	0.02	68.5389	1.8963
275	8	77.2697	0.02	68.5389	1.1122
276	9	75.9614	0.02	68.5389	0.8038
277	10	71.5181	0.02	68.5389	0.1295
278	11	72.6195	0.02	68.5389	0.2430
279	12	75.5633	0.02	68.5389	0.7199
280	13	77.7378	0.02	68.5389	1.2346
281	14	64.6661	0.02	68.5389	0.2188
282	15	68.3449	0.02	68.5389	0.0005
283	16	52.7234	0.02	68.5389	3.6494
284	17	60.2750	0.02	68.5389	0.9964
285	18	62.4100	0.02	68.5389	0.5481
286	19	55.0689	0.02	68.5389	2.6473
287	20	72.9448	0.02	68.5389	0.2832
288	21	65.6683	0.02	68.5389	0.1202
289	22	73.1260	0.02	68.5389	0.3070
290	23	56.3987	0.02	68.5389	2.1504
291	24	61.2097	0.02	68.5389	0.7837
292	25	76.9885	0.02	68.5389	1.0417
293	26	71.7900	0.02	68.5389	0.1542
294	27	63.4845	0.02	68.5389	0.3727
295	28	63.6969	0.02	68.5389	0.3421
296	29	96.7918	0.02	68.5389	11.6463
297	30	57.8415	0.02	68.5389	1.6696
298	31	72.0244	0.02	68.5389	0.1773
299	32	66.6847	0.02	68.5389	0.0502
300	33	78.9152	0.02	68.5389	1.5709
301	34	63.3895	0.02	68.5389	0.3869
302	35	64.6001	0.02	68.5389	0.2264
303	36	59.8641	0.02	68.5389	1.0979
304	37	70.6655	0.02	68.5389	0.0660
305	38	64.3156	0.02	68.5389	0.2602
306	39	60.3518	0.02	68.5389	0.9780
307	40	68.5568	0.02	68.5389	0.0000
308	41	92.5052	0.02	68.5389	8.3804
309	42	76.6716	0.02	68.5389	0.9650
310	43	69.3454	0.02	68.5389	0.0095
311	44	70.2106	0.02	68.5389	0.0408
312	45	66.9196	0.02	68.5389	0.0383
313	46	70.7146	0.02	68.5389	0.0691
314	47	63.8778	0.02	68.5389	0.3170
315	48	55.8182	0.02	68.5389	2.3609
316	49	56.8241	0.02	68.5389	2.0023
317	50	61.7974	0.02	68.5389	0.6631

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320	N	DF	Chi-Sq	P-Value
321	3426.95	49	56.7060	0.210

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CHI-SQUARE GOODNESS-OF-FIT TEST FOR C2

Category	Observed	Test Proportion	Expected	Contribution to Chi-Sq
1	54.979	0.02	69.2502	2.9409
2	60.420	0.02	69.2502	1.1260
3	74.038	0.02	69.2502	0.3311
4	88.171	0.02	69.2502	5.1697
5	87.458	0.02	69.2502	4.7875
6	71.761	0.02	69.2502	0.0911
7	81.027	0.02	69.2502	2.0027
8	81.754	0.02	69.2502	2.2575
9	73.315	0.02	69.2502	0.2386
10	76.371	0.02	69.2502	0.7322
11	56.026	0.02	69.2502	2.5253
12	74.843	0.02	69.2502	0.4517
13	68.253	0.02	69.2502	0.0144
14	68.390	0.02	69.2502	0.0107
15	62.470	0.02	69.2502	0.6639
16	68.903	0.02	69.2502	0.0017
17	64.198	0.02	69.2502	0.3686
18	89.721	0.02	69.2502	6.0514
19	76.834	0.02	69.2502	0.8304
20	67.852	0.02	69.2502	0.0282
21	68.119	0.02	69.2502	0.0185
22	64.802	0.02	69.2502	0.2857
23	58.373	0.02	69.2502	1.7085
24	71.550	0.02	69.2502	0.0764
25	57.395	0.02	69.2502	2.0296
26	66.550	0.02	69.2502	0.1053
27	74.044	0.02	69.2502	0.3318
28	64.744	0.02	69.2502	0.2932
29	54.433	0.02	69.2502	3.1702
30	58.199	0.02	69.2502	1.7637
31	54.346	0.02	69.2502	3.2077
32	58.759	0.02	69.2502	1.5893
33	62.967	0.02	69.2502	0.5702
34	65.952	0.02	69.2502	0.1571
35	100.072	0.02	69.2502	13.7183
36	59.294	0.02	69.2502	1.4315
37	70.619	0.02	69.2502	0.0270
38	68.095	0.02	69.2502	0.0193
39	75.558	0.02	69.2502	0.5746
40	78.242	0.02	69.2502	1.1676
41	59.246	0.02	69.2502	1.4452
42	78.540	0.02	69.2502	1.2461
43	85.303	0.02	69.2502	3.7212
44	66.405	0.02	69.2502	0.1169
45	66.646	0.02	69.2502	0.0979
46	70.967	0.02	69.2502	0.0425
47	49.964	0.02	69.2502	5.3714
48	79.855	0.02	69.2502	1.6239
49	64.044	0.02	69.2502	0.3915
50	62.647	0.02	69.2502	0.6297

N	DF	Chi-Sq	P-Value
3462.51	49	77.5555	0.006

388 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C3**

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	Category	Observed	Proportion	Expected	to Chi-Sq
392	1	70.8227	0.02	67.9388	0.12242
393	2	71.5189	0.02	67.9388	0.18866
394	3	77.9724	0.02	67.9388	1.48184
395	4	73.8638	0.02	67.9388	0.51672
396	5	63.1454	0.02	67.9388	0.33819
397	6	69.4455	0.02	67.9388	0.03342
398	7	75.1361	0.02	67.9388	0.76248
399	8	70.7481	0.02	67.9388	0.11617
400	9	79.6661	0.02	67.9388	2.02434
401	10	50.0676	0.02	67.9388	4.70099
402	11	64.5797	0.02	67.9388	0.16608
403	12	62.3211	0.02	67.9388	0.46452
404	13	83.6186	0.02	67.9388	3.61882
405	14	69.1255	0.02	67.9388	0.02073
406	15	65.9991	0.02	67.9388	0.05538
407	16	68.9681	0.02	67.9388	0.01560
408	17	54.7138	0.02	67.9388	2.57437
409	18	65.2195	0.02	67.9388	0.10884
410	19	55.6536	0.02	67.9388	2.22147
411	20	70.2982	0.02	67.9388	0.08194
412	21	89.9244	0.02	67.9388	7.11478
413	22	47.6367	0.02	67.9388	6.06684
414	23	61.7723	0.02	67.9388	0.55971
415	24	63.3138	0.02	67.9388	0.31485
416	25	64.5781	0.02	67.9388	0.16624
417	26	52.7015	0.02	67.9388	3.41739
418	27	65.6691	0.02	67.9388	0.07582
419	28	65.0820	0.02	67.9388	0.12012
420	29	62.6935	0.02	67.9388	0.40497
421	30	66.8818	0.02	67.9388	0.01644
422	31	81.1064	0.02	67.9388	2.55212
423	32	62.0698	0.02	67.9388	0.50700
424	33	61.3664	0.02	67.9388	0.63580
425	34	69.1278	0.02	67.9388	0.02081
426	35	53.6567	0.02	67.9388	3.00238
427	36	68.7329	0.02	67.9388	0.00928
428	37	60.7577	0.02	67.9388	0.75903
429	38	61.5912	0.02	67.9388	0.59306
430	39	65.8544	0.02	67.9388	0.06395
431	40	75.1262	0.02	67.9388	0.76038
432	41	81.6331	0.02	67.9388	2.76036
433	42	61.4130	0.02	67.9388	0.62682
434	43	69.1910	0.02	67.9388	0.02308
435	44	79.0068	0.02	67.9388	1.80313
436	45	73.4903	0.02	67.9388	0.45363
437	46	85.9510	0.02	67.9388	4.77547
438	47	67.4218	0.02	67.9388	0.00393
439	48	54.7935	0.02	67.9388	2.54345
440	49	86.5256	0.02	67.9388	5.08504
441	50	74.9855	0.02	67.9388	0.73090

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455 N DF Chi-Sq P-Value
 456 3396.94 49 65.5798 0.057

457 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C4**

458			Test		
459	Category	Observed	Proportion	Expected	Contribution to Chi-Sq
460	1	81.5681	0.02	67.9121	2.74598
461	2	55.1698	0.02	67.9121	2.39082
462	3	58.0913	0.02	67.9121	1.42020
463	4	77.6479	0.02	67.9121	1.39571
464	5	72.7100	0.02	67.9121	0.33897
465	6	65.4607	0.02	67.9121	0.08849
466	7	56.2104	0.02	67.9121	2.01628
467	8	78.3371	0.02	67.9121	1.60030
468	9	60.8741	0.02	67.9121	0.72939
469	10	50.2737	0.02	67.9121	4.58113
470	11	86.2262	0.02	67.9121	4.93881
471	12	66.1375	0.02	67.9121	0.04637
472	13	64.8289	0.02	67.9121	0.13997
473	14	52.1107	0.02	67.9121	3.67658
474	15	60.4853	0.02	67.9121	0.81220
475	16	70.7940	0.02	67.9121	0.12230
476	17	54.0296	0.02	67.9121	2.83786
477	18	72.0210	0.02	67.9121	0.24860
478	19	53.5729	0.02	67.9121	3.02762
479	20	80.5690	0.02	67.9121	2.35890
480	21	63.1328	0.02	67.9121	0.33634
481	22	76.3505	0.02	67.9121	1.04850
482	23	80.1919	0.02	67.9121	2.22041
483	24	82.3596	0.02	67.9121	3.07351
484	25	68.1848	0.02	67.9121	0.00110
485	26	82.3855	0.02	67.9121	3.08454
486	27	77.4198	0.02	67.9121	1.33106
487	28	52.7840	0.02	67.9121	3.36995
488	29	61.0889	0.02	67.9121	0.68554
489	30	68.8381	0.02	67.9121	0.01263
490	31	69.5618	0.02	67.9121	0.04007
491	32	54.9396	0.02	67.9121	2.47801
492	33	83.8854	0.02	67.9121	3.75698
493	34	76.2092	0.02	67.9121	1.01370
494	35	78.6241	0.02	67.9121	1.68963
495	36	75.3292	0.02	67.9121	0.81006
496	37	63.9439	0.02	67.9121	0.23187
497	38	57.6663	0.02	67.9121	1.54579
498	39	57.2451	0.02	67.9121	1.67548
499	40	65.2012	0.02	67.9121	0.10821
500	41	61.7121	0.02	67.9121	0.56603
501	42	52.0600	0.02	67.9121	3.70023
502	43	61.6849	0.02	67.9121	0.57101
503	44	71.0712	0.02	67.9121	0.14696
504	45	58.0540	0.02	67.9121	1.43102
505	46	67.1626	0.02	67.9121	0.00827
506	47	86.3399	0.02	67.9121	5.00031
507	48	81.2195	0.02	67.9121	2.60756
508	49	75.7672	0.02	67.9121	0.90855
509	50	68.0749	0.02	67.9121	0.00039

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522 N DF Chi-Sq P-Value
 523 3395.61 49 78.9702 0.004

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C4

	Category	Observed	Test Proportion	Expected	Contribution to Chi-Sq
526	1	81.5681	0.02	67.9121	2.74598
527	2	55.1698	0.02	67.9121	2.39082
528	3	58.0913	0.02	67.9121	1.42020
529	4	77.6479	0.02	67.9121	1.39571
530	5	72.7100	0.02	67.9121	0.33897
531	6	65.4607	0.02	67.9121	0.08849
532	7	56.2104	0.02	67.9121	2.01628
533	8	78.3371	0.02	67.9121	1.60030
534	9	60.8741	0.02	67.9121	0.72939
535	10	50.2737	0.02	67.9121	4.58113
536	11	86.2262	0.02	67.9121	4.93881
537	12	66.1375	0.02	67.9121	0.04637
538	13	64.8289	0.02	67.9121	0.13997
539	14	52.1107	0.02	67.9121	3.67658
540	15	60.4853	0.02	67.9121	0.81220
541	16	70.7940	0.02	67.9121	0.12230
542	17	54.0296	0.02	67.9121	2.83786
543	18	72.0210	0.02	67.9121	0.24860
544	19	53.5729	0.02	67.9121	3.02762
545	20	80.5690	0.02	67.9121	2.35890
546	21	63.1328	0.02	67.9121	0.33634
547	22	76.3505	0.02	67.9121	1.04850
548	23	80.1919	0.02	67.9121	2.22041
549	24	82.3596	0.02	67.9121	3.07351
550	25	68.1848	0.02	67.9121	0.00110
551	26	82.3855	0.02	67.9121	3.08454
552	27	77.4198	0.02	67.9121	1.33106
553	28	52.7840	0.02	67.9121	3.36995
554	29	61.0889	0.02	67.9121	0.68554
555	30	68.8381	0.02	67.9121	0.01263
556	31	69.5618	0.02	67.9121	0.04007
557	32	54.9396	0.02	67.9121	2.47801
558	33	83.8854	0.02	67.9121	3.75698
559	34	76.2092	0.02	67.9121	1.01370
560	35	78.6241	0.02	67.9121	1.68963
561	36	75.3292	0.02	67.9121	0.81006
562	37	63.9439	0.02	67.9121	0.23187
563	38	57.6663	0.02	67.9121	1.54579
564	39	57.2451	0.02	67.9121	1.67548
565	40	65.2012	0.02	67.9121	0.10821
566	41	61.7121	0.02	67.9121	0.56603
567	42	52.0600	0.02	67.9121	3.70023
568	43	61.6849	0.02	67.9121	0.57101
569	44	71.0712	0.02	67.9121	0.14696
570	45	58.0540	0.02	67.9121	1.43102
571	46	67.1626	0.02	67.9121	0.00827
572	47	86.3399	0.02	67.9121	5.00031
573	48	81.2195	0.02	67.9121	2.60756
574	49	75.7672	0.02	67.9121	0.90855
575	50	68.0749	0.02	67.9121	0.00039

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589 N DF Chi-Sq P-Value
 590 3395.61 49 78.9702 0.004
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593 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C5**

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595			Test		Contribution
596	Category	Observed	Proportion	Expected	to Chi-Sq
597	1	80.7548	0.02	67.3542	2.66615
598	2	92.0513	0.02	67.3542	9.05578
599	3	82.0900	0.02	67.3542	3.22391
600	4	59.5873	0.02	67.3542	0.89564
601	5	62.9956	0.02	67.3542	0.28206
602	6	68.8397	0.02	67.3542	0.03276
603	7	69.8471	0.02	67.3542	0.09227
604	8	82.4777	0.02	67.3542	3.39579
605	9	78.6699	0.02	67.3542	1.90105
606	10	60.9375	0.02	67.3542	0.61131
607	11	59.5997	0.02	67.3542	0.89278
608	12	68.6101	0.02	67.3542	0.02342
609	13	81.3533	0.02	67.3542	2.90960
610	14	61.9140	0.02	67.3542	0.43941
611	15	77.4902	0.02	67.3542	1.52535
612	16	54.0418	0.02	67.3542	2.63118
613	17	51.0629	0.02	67.3542	3.94048
614	18	59.2114	0.02	67.3542	0.98442
615	19	70.9773	0.02	67.3542	0.19489
616	20	60.8392	0.02	67.3542	0.63019
617	21	58.7667	0.02	67.3542	1.09489
618	22	58.1324	0.02	67.3542	1.26262
619	23	57.4031	0.02	67.3542	1.47022
620	24	60.0559	0.02	67.3542	0.79082
621	25	70.6908	0.02	67.3542	0.16529
622	26	51.6022	0.02	67.3542	3.68392
623	27	68.5520	0.02	67.3542	0.02130
624	28	49.0712	0.02	67.3542	4.96286
625	29	56.7361	0.02	67.3542	1.67390
626	30	69.7586	0.02	67.3542	0.08583
627	31	64.5950	0.02	67.3542	0.11303
628	32	70.8621	0.02	67.3542	0.18270
629	33	63.7530	0.02	67.3542	0.19255
630	34	63.4375	0.02	67.3542	0.22776
631	35	55.5432	0.02	67.3542	2.07113
632	36	78.4558	0.02	67.3542	1.82982
633	37	68.5305	0.02	67.3542	0.02054
634	38	53.5405	0.02	67.3542	2.83308
635	39	48.2435	0.02	67.3542	5.42238
636	40	77.5587	0.02	67.3542	1.54603
637	41	82.5406	0.02	67.3542	3.42407
638	42	75.7857	0.02	67.3542	1.05547
639	43	64.7641	0.02	67.3542	0.09960
640	44	79.8253	0.02	67.3542	2.30912
641	45	78.7766	0.02	67.3542	1.93707
642	46	72.1496	0.02	67.3542	0.34141
643	47	68.2852	0.02	67.3542	0.01287
644	48	74.3916	0.02	67.3542	0.73528
645	49	75.8344	0.02	67.3542	1.06768
646	50	66.7185	0.02	67.3542	0.00600
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656 N DF Chi-Sq P-Value
657 3367.71 49 76.9676 0.007

658 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C6**

659			Test		Contribution
660	Category	Observed	Proportion	Expected	to Chi-Sq
661	1	86.3219	0.02	68.8085	4.4576
662	2	76.9007	0.02	68.8085	0.9517
663	3	67.1474	0.02	68.8085	0.0401
664	4	66.7692	0.02	68.8085	0.0604
665	5	59.6605	0.02	68.8085	1.2162
666	6	62.3847	0.02	68.8085	0.5997
667	7	77.9749	0.02	68.8085	1.2211
668	8	69.7618	0.02	68.8085	0.0132
669	9	95.2474	0.02	68.8085	10.1589
670	10	83.1864	0.02	68.8085	3.0043
671	11	80.1919	0.02	68.8085	1.8832
672	12	74.7413	0.02	68.8085	0.5115
673	13	57.3286	0.02	68.8085	1.9153
674	14	79.8755	0.02	68.8085	1.7800
675	15	73.9052	0.02	68.8085	0.3775
676	16	60.9518	0.02	68.8085	0.8971
677	17	85.3886	0.02	68.8085	3.9952
678	18	82.0308	0.02	68.8085	2.5408
679	19	74.0551	0.02	68.8085	0.4000
680	20	76.9919	0.02	68.8085	0.9733
681	21	77.0952	0.02	68.8085	0.9980
682	22	54.5006	0.02	68.8085	2.9751
683	23	59.3561	0.02	68.8085	1.2985
684	24	59.3836	0.02	68.8085	1.2909
685	25	78.0340	0.02	68.8085	1.2369
686	26	64.5772	0.02	68.8085	0.2602
687	27	43.6943	0.02	68.8085	9.1664
688	28	82.9828	0.02	68.8085	2.9198
689	29	68.2868	0.02	68.8085	0.0040
690	30	57.7834	0.02	68.8085	1.7665
691	31	58.1362	0.02	68.8085	1.6553
692	32	76.3968	0.02	68.8085	0.8368
693	33	62.2694	0.02	68.8085	0.6214
694	34	52.0600	0.02	68.8085	4.0767
695	35	67.4489	0.02	68.8085	0.0269
696	36	69.2350	0.02	68.8085	0.0026
697	37	60.6018	0.02	68.8085	0.9788
698	38	65.9027	0.02	68.8085	0.1227
699	39	57.0319	0.02	68.8085	2.0156
700	40	68.6221	0.02	68.8085	0.0005
701	41	71.8541	0.02	68.8085	0.1348
702	42	83.7519	0.02	68.8085	3.2453
703	43	66.7716	0.02	68.8085	0.0603
704	44	60.8638	0.02	68.8085	0.9173
705	45	71.8905	0.02	68.8085	0.1380
706	46	64.5203	0.02	68.8085	0.2672
707	47	62.9284	0.02	68.8085	0.5025
708	48	52.1107	0.02	68.8085	4.0521
709	49	79.2898	0.02	68.8085	1.5966
710	50	52.2294	0.02	68.8085	3.9947

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713 N DF Chi-Sq P-Value
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CHI-SQUARE GOODNESS-OF-FIT TEST FOR C7

		Test		Contribution	
Category	Observed	Proportion	Expected	to Chi-Sq	
1	45.5489	0.02	68.7456	7.8273	
2	56.5727	0.02	68.7456	2.1555	
3	75.5888	0.02	68.7456	0.6812	
4	63.9847	0.02	68.7456	0.3297	
5	75.5888	0.02	68.7456	0.6812	
6	69.1366	0.02	68.7456	0.0022	
7	84.6959	0.02	68.7456	3.7008	
8	71.7521	0.02	68.7456	0.1315	
9	73.8389	0.02	68.7456	0.3774	
10	79.8048	0.02	68.7456	1.7791	
11	71.5013	0.02	68.7456	0.1105	
12	52.6550	0.02	68.7456	3.7662	
13	63.2074	0.02	68.7456	0.4462	
14	52.7113	0.02	68.7456	3.7399	
15	65.6880	0.02	68.7456	0.1360	
16	61.0342	0.02	68.7456	0.8650	
17	71.3754	0.02	68.7456	0.1006	
18	58.3818	0.02	68.7456	1.5624	
19	46.0333	0.02	68.7456	7.5037	
20	66.0007	0.02	68.7456	0.1096	
21	63.7993	0.02	68.7456	0.3559	
22	66.4546	0.02	68.7456	0.0763	
23	66.9846	0.02	68.7456	0.0451	
24	71.2832	0.02	68.7456	0.0937	
25	95.6557	0.02	68.7456	10.5338	
26	66.8657	0.02	68.7456	0.0514	
27	52.6254	0.02	68.7456	3.7800	
28	76.6133	0.02	68.7456	0.9004	
29	53.3736	0.02	68.7456	3.4373	
30	62.4622	0.02	68.7456	0.5743	
31	74.5933	0.02	68.7456	0.4974	
32	82.6019	0.02	68.7456	2.7929	
33	55.5936	0.02	68.7456	2.5162	
34	71.5465	0.02	68.7456	0.1141	
35	83.3525	0.02	68.7456	3.1036	
36	56.6161	0.02	68.7456	2.1401	
37	78.1420	0.02	68.7456	1.2843	
38	78.5500	0.02	68.7456	1.3983	
39	81.8667	0.02	68.7456	2.5043	
40	79.2814	0.02	68.7456	1.6147	
41	61.8129	0.02	68.7456	0.6991	
42	58.1605	0.02	68.7456	1.6298	
43	76.3731	0.02	68.7456	0.8463	
44	82.4455	0.02	68.7456	2.7302	
45	82.1498	0.02	68.7456	2.6136	
46	77.3789	0.02	68.7456	1.0842	
47	73.1616	0.02	68.7456	0.2837	
48	72.7869	0.02	68.7456	0.2376	
49	76.9355	0.02	68.7456	0.9757	
50	52.7137	0.02	68.7456	3.7387	

N	DF	Chi-Sq	P-Value
3437.28	49	88.6589	0.003

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C8

		Test		Contribution	
Category	Observed	Proportion	Expected	to Chi-Sq	
1	65.8422	0.02	68.3078	0.08900	
2	83.8147	0.02	68.3078	3.52032	
3	71.8474	0.02	68.3078	0.18342	
4	78.9866	0.02	68.3078	1.66946	
5	74.0560	0.02	68.3078	0.48373	
6	74.1874	0.02	68.3078	0.50610	

786	7	66.9782	0.02	68.3078	0.02588
787	8	83.4256	0.02	68.3078	3.34586
788	9	65.5630	0.02	68.3078	0.11029
789	10	53.1505	0.02	68.3078	3.36335
790	11	72.4835	0.02	68.3078	0.25527
791	12	53.5231	0.02	68.3078	3.20002
792	13	64.3951	0.02	68.3078	0.22412
793	14	88.6733	0.02	68.3078	6.07189
794	15	74.2352	0.02	68.3078	0.51436
795	16	62.7458	0.02	68.3078	0.45288
796	17	76.2082	0.02	68.3078	0.91376
797	18	55.0063	0.02	68.3078	2.59018
798	19	74.0597	0.02	68.3078	0.48435
799	20	71.1001	0.02	68.3078	0.11415
800	21	76.6639	0.02	68.3078	1.02221
801	22	67.5080	0.02	68.3078	0.00936
802	23	87.1950	0.02	68.3078	5.22239
803	24	73.0906	0.02	68.3078	0.33489
804	25	53.3179	0.02	68.3078	3.28945
805	26	52.4708	0.02	68.3078	3.67176
806	27	73.0225	0.02	68.3078	0.32542
807	28	67.1978	0.02	68.3078	0.01804
808	29	68.2207	0.02	68.3078	0.00011
809	30	67.0408	0.02	68.3078	0.02350
810	31	69.0767	0.02	68.3078	0.00866
811	32	76.9085	0.02	68.3078	1.08294
812	33	74.5780	0.02	68.3078	0.57557
813	34	66.1074	0.02	68.3078	0.07088
814	35	63.9361	0.02	68.3078	0.27978
815	36	52.4834	0.02	68.3078	3.66592
816	37	79.8386	0.02	68.3078	1.94650
817	38	70.3518	0.02	68.3078	0.06116
818	39	72.3268	0.02	68.3078	0.23647
819	40	76.5586	0.02	68.3078	0.99661
820	41	53.2234	0.02	68.3078	3.33107
821	42	46.9829	0.02	68.3078	6.65739
822	43	65.4020	0.02	68.3078	0.12361
823	44	61.6430	0.02	68.3078	0.65028
824	45	59.1543	0.02	68.3078	1.22660
825	46	82.4294	0.02	68.3078	2.91947
826	47	67.5855	0.02	68.3078	0.00764
827	48	59.1156	0.02	68.3078	1.23698
828	49	64.6213	0.02	68.3078	0.19895
829	50	57.0547	0.02	68.3078	1.85383

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832	N	DF	Chi-Sq	P-Value
833	3415.39	49	69.1658	0.030

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838 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C9**

839	Test	Contribution			
840	Category	Observed	Proportion	Expected	to Chi-Sq
841	1	79.6792	0.02	68.0366	1.9923
842	2	65.4433	0.02	68.0366	0.0988
843	3	62.1656	0.02	68.0366	0.5066
844	4	50.7257	0.02	68.0366	4.4045
845	5	66.1147	0.02	68.0366	0.0543
846	6	61.9178	0.02	68.0366	0.5503
847	7	50.8206	0.02	68.0366	4.3563
848	8	65.6757	0.02	68.0366	0.0819

849	9	64.7388	0.02	68.0366	0.1599
850	10	77.7523	0.02	68.0366	1.3874
851	11	68.7321	0.02	68.0366	0.0071
852	12	68.6858	0.02	68.0366	0.0062
853	13	59.7075	0.02	68.0366	1.0197
854	14	76.6419	0.02	68.0366	1.0884
855	15	67.1994	0.02	68.0366	0.0103
856	16	51.7640	0.02	68.0366	3.8920
857	17	68.4532	0.02	68.0366	0.0026
858	18	76.8928	0.02	68.0366	1.1528
859	19	76.3893	0.02	68.0366	1.0254
860	20	71.3197	0.02	68.0366	0.1584
861	21	51.2325	0.02	68.0366	4.1504
862	22	66.6895	0.02	68.0366	0.0267
863	23	72.7467	0.02	68.0366	0.3261
864	24	78.2700	0.02	68.0366	1.5392
865	25	85.0973	0.02	68.0366	4.2781
866	26	66.4586	0.02	68.0366	0.0366
867	27	73.2114	0.02	68.0366	0.3936
868	28	68.7990	0.02	68.0366	0.0085
869	29	75.6163	0.02	68.0366	0.8444
870	30	74.2709	0.02	68.0366	0.5712
871	31	82.8522	0.02	68.0366	3.2262
872	32	76.7890	0.02	68.0366	1.1259
873	33	71.8220	0.02	68.0366	0.2106
874	34	71.1688	0.02	68.0366	0.1442
875	35	55.4382	0.02	68.0366	2.3329
876	36	59.0568	0.02	68.0366	1.1852
877	37	66.1115	0.02	68.0366	0.0545
878	38	81.7004	0.02	68.0366	2.7441
879	39	75.2892	0.02	68.0366	0.7731
880	40	76.6551	0.02	68.0366	1.0917
881	41	76.7567	0.02	68.0366	1.1176
882	42	33.1233	0.02	68.0366	17.9160
883	43	60.7132	0.02	68.0366	0.7883
884	44	64.4727	0.02	68.0366	0.1867
885	45	65.4318	0.02	68.0366	0.0997
886	46	51.1127	0.02	68.0366	4.2098
887	47	87.8685	0.02	68.0366	5.7807
888	48	66.9581	0.02	68.0366	0.0171
889	49	83.8196	0.02	68.0366	3.6613
890	50	51.4804	0.02	68.0366	4.0289

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893 N DF Chi-Sq P-Value

894 3401.83 49 84.8245 0.001

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899 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C10**

900	Test	Contribution			
901	Category	Observed	Proportion	Expected	to Chi-Sq
902	1	56.9674	0.02	68.3077	1.88268
903	2	53.4838	0.02	68.3077	3.21703
904	3	59.1074	0.02	68.3077	1.23917
905	4	77.3718	0.02	68.3077	1.20278
906	5	80.1996	0.02	68.3077	2.07029
907	6	58.7607	0.02	68.3077	1.33434
908	7	62.0002	0.02	68.3077	0.58244
909	8	71.7285	0.02	68.3077	0.17131

910	9	62.4696	0.02	68.3077	0.49897
911	10	70.4298	0.02	68.3077	0.06593
912	11	56.5502	0.02	68.3077	2.02377
913	12	75.2573	0.02	68.3077	0.70704
914	13	66.4659	0.02	68.3077	0.04966
915	14	51.7387	0.02	68.3077	4.01905
916	15	90.8331	0.02	68.3077	7.42803
917	16	59.2973	0.02	68.3077	1.18856
918	17	55.4977	0.02	68.3077	2.40231
919	18	76.7801	0.02	68.3077	1.05085
920	19	71.8922	0.02	68.3077	0.18810
921	20	60.3603	0.02	68.3077	0.92465
922	21	68.7608	0.02	68.3077	0.00301
923	22	47.5391	0.02	68.3077	6.31459
924	23	53.4773	0.02	68.3077	3.21987
925	24	85.1326	0.02	68.3077	4.14413
926	25	79.5195	0.02	68.3077	1.84028
927	26	72.1171	0.02	68.3077	0.21245
928	27	56.4353	0.02	68.3077	2.06351
929	28	85.9241	0.02	68.3077	4.54324
930	29	88.1467	0.02	68.3077	5.76193
931	30	78.7449	0.02	68.3077	1.59476
932	31	51.2051	0.02	68.3077	4.28206
933	32	84.3989	0.02	68.3077	3.79060
934	33	85.6011	0.02	68.3077	4.37817
935	34	74.4579	0.02	68.3077	0.55375
936	35	58.8690	0.02	68.3077	1.30423
937	36	63.4526	0.02	68.3077	0.34509
938	37	76.4065	0.02	68.3077	0.96023
939	38	83.7664	0.02	68.3077	3.49844
940	39	69.8318	0.02	68.3077	0.03401
941	40	80.3231	0.02	68.3077	2.11353
942	41	85.9982	0.02	68.3077	4.58154
943	42	62.1372	0.02	68.3077	0.55741
944	43	69.2102	0.02	68.3077	0.01192
945	44	64.4727	0.02	68.3077	0.21531
946	45	54.8284	0.02	68.3077	2.65989
947	46	64.6467	0.02	68.3077	0.19622
948	47	66.0610	0.02	68.3077	0.07390
949	48	56.1069	0.02	68.3077	2.17925
950	49	70.8301	0.02	68.3077	0.09315
951	50	59.7922	0.02	68.3077	1.06159

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954 N DF Chi-Sq P-Value

955 3415.38 49 94.8350 0.004

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957 CHI-SQUARE GOODNESS-OF-FIT TEST FOR C11

958 Test	Contribution				
959 Category	Observed	Proportion	Expected	to Chi-Sq	
960 1	61.0950	0.02	67.2314	0.56010	
961 2	63.7801	0.02	67.2314	0.17717	
962 3	67.1914	0.02	67.2314	0.00002	
963 4	65.1829	0.02	67.2314	0.06242	
964 5	67.6716	0.02	67.2314	0.00288	
965 6	70.2033	0.02	67.2314	0.13137	
966 7	83.0653	0.02	67.2314	3.72906	
967 8	61.7906	0.02	67.2314	0.44030	
968 9	70.3648	0.02	67.2314	0.14603	
969 10	68.0160	0.02	67.2314	0.00916	
970 11	83.6352	0.02	67.2314	4.00234	
971 12	49.0405	0.02	67.2314	4.92196	
972 13	77.5386	0.02	67.2314	1.58018	
973 14	72.7817	0.02	67.2314	0.45819	
974 15	59.9225	0.02	67.2314	0.79458	
975 16	68.2916	0.02	67.2314	0.01672	
976 17	57.8678	0.02	67.2314	1.30411	

977	18	66.7676	0.02	67.2314	0.00320
978	19	76.2743	0.02	67.2314	1.21631
979	20	77.2489	0.02	67.2314	1.49259
980	21	66.7716	0.02	67.2314	0.00314
981	22	48.4428	0.02	67.2314	5.25071
982	23	51.5531	0.02	67.2314	3.65617
983	24	68.8421	0.02	67.2314	0.03859
984	25	67.2762	0.02	67.2314	0.00003
985	26	72.0465	0.02	67.2314	0.34485
986	27	93.1191	0.02	67.2314	9.96810
987	28	76.5641	0.02	67.2314	1.29549
988	29	55.4570	0.02	67.2314	2.06210
989	30	71.5565	0.02	67.2314	0.27824
990	31	59.6470	0.02	67.2314	0.85561
991	32	66.4328	0.02	67.2314	0.00949
992	33	68.2342	0.02	67.2314	0.01496
993	34	70.1062	0.02	67.2314	0.12292
994	35	54.0559	0.02	67.2314	2.58203
995	36	54.4430	0.02	67.2314	2.43254
996	37	54.2282	0.02	67.2314	2.51495
997	38	66.9998	0.02	67.2314	0.00080
998	39	84.7125	0.02	67.2314	4.54530
999	40	64.4292	0.02	67.2314	0.11680
1000	41	77.4151	0.02	67.2314	1.54253
1001	42	75.0178	0.02	67.2314	0.90178
1002	43	55.5752	0.02	67.2314	2.02090
1003	44	79.8741	0.02	67.2314	2.37741
1004	45	59.0604	0.02	67.2314	0.99308
1005	46	69.3398	0.02	67.2314	0.06612
1006	47	54.0984	0.02	67.2314	2.56543
1007	48	67.2954	0.02	67.2314	0.00006
1008	49	67.4697	0.02	67.2314	0.00084
1009	50	73.7784	0.02	67.2314	0.63755

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1012 N DF Chi-Sq P-Value

1013 3361.57 49 68.2472 0.036

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1017 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C12**

1018	Test	Contribution			
1019	Category	Observed	Proportion	Expected	to Chi-Sq
1020	1	50.6826	0.02	66.5558	3.78567
1021	2	71.1572	0.02	66.5558	0.31812
1022	3	69.5642	0.02	66.5558	0.13598
1023	4	76.2487	0.02	66.5558	1.41163
1024	5	57.0176	0.02	66.5558	1.36692
1025	6	87.6484	0.02	66.5558	6.68457
1026	7	64.5730	0.02	66.5558	0.05907
1027	8	52.4708	0.02	66.5558	2.98076
1028	9	65.4995	0.02	66.5558	0.01676
1029	10	53.7796	0.02	66.5558	2.45255
1030	11	67.1281	0.02	66.5558	0.00492
1031	12	80.3293	0.02	66.5558	2.85040
1032	13	77.4867	0.02	66.5558	1.79525
1033	14	67.6421	0.02	66.5558	0.01773
1034	15	66.3429	0.02	66.5558	0.00068
1035	16	49.4104	0.02	66.5558	4.41681
1036	17	60.5819	0.02	66.5558	0.53620
1037	18	66.2034	0.02	66.5558	0.00187
1038	19	45.4085	0.02	66.5558	6.71927
1039	20	66.1123	0.02	66.5558	0.00296

1040	21	85.4846	0.02	66.5558	5.38347
1041	22	69.8069	0.02	66.5558	0.15881
1042	23	74.8858	0.02	66.5558	1.04258
1043	24	70.4746	0.02	66.5558	0.23074
1044	25	64.6737	0.02	66.5558	0.05322
1045	26	68.8549	0.02	66.5558	0.07942
1046	27	68.6356	0.02	66.5558	0.06499
1047	28	65.6050	0.02	66.5558	0.01358
1048	29	70.2139	0.02	66.5558	0.20106
1049	30	70.0020	0.02	66.5558	0.17844
1050	31	62.4296	0.02	66.5558	0.25581
1051	32	72.5866	0.02	66.5558	0.54646
1052	33	46.6356	0.02	66.5558	5.96214
1053	34	59.0249	0.02	66.5558	0.85212
1054	35	72.3276	0.02	66.5558	0.50054
1055	36	74.4465	0.02	66.5558	0.93551
1056	37	55.1416	0.02	66.5558	1.95750
1057	38	75.3944	0.02	66.5558	1.17378
1058	39	67.6437	0.02	66.5558	0.01778
1059	40	56.1823	0.02	66.5558	1.61683
1060	41	77.7679	0.02	66.5558	1.88883
1061	42	56.2617	0.02	66.5558	1.59216
1062	43	69.0584	0.02	66.5558	0.09410
1063	44	59.5602	0.02	66.5558	0.73530
1064	45	64.8121	0.02	66.5558	0.04568
1065	46	62.2477	0.02	66.5558	0.27885
1066	47	55.4451	0.02	66.5558	1.85481
1067	48	74.7500	0.02	66.5558	1.00886
1068	49	76.9423	0.02	66.5558	1.62088
1069	50	85.2068	0.02	66.5558	5.22662

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	N	DF	Chi-Sq	P-Value
1072	3327.79	49	71.1290	0.021

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1075 CHI-SQUARE GOODNESS-OF-FIT TEST FOR C13

1076	Test	Contribution			
1077	Category	Observed	Proportion	Expected	to Chi-Sq
1078	1	49.0537	0.02	69.1133	5.82218
1079	2	60.9058	0.02	69.1133	0.97467
1080	3	64.9959	0.02	69.1133	0.24529
1081	4	71.4613	0.02	69.1133	0.07977
1082	5	53.4991	0.02	69.1133	3.52760
1083	6	74.8732	0.02	69.1133	0.48002
1084	7	62.0831	0.02	69.1133	0.71512
1085	8	46.9244	0.02	69.1133	7.12376
1086	9	77.7631	0.02	69.1133	1.08255
1087	10	56.4763	0.02	69.1133	2.31062
1088	11	53.7922	0.02	69.1133	3.39642
1089	12	78.0699	0.02	69.1133	1.16071
1090	13	82.1324	0.02	69.1133	2.45244
1091	14	92.3526	0.02	69.1133	7.81415
1092	15	73.0083	0.02	69.1133	0.21951
1093	16	74.7384	0.02	69.1133	0.45783
1094	17	67.9013	0.02	69.1133	0.02126
1095	18	72.6621	0.02	69.1133	0.18222
1096	19	67.8750	0.02	69.1133	0.02219
1097	20	61.6342	0.02	69.1133	0.80936
1098	21	62.8456	0.02	69.1133	0.56841
1099	22	65.2170	0.02	69.1133	0.21966
1100	23	71.2716	0.02	69.1133	0.06740
1101	24	81.1833	0.02	69.1133	2.10792
1102	25	78.6830	0.02	69.1133	1.32505
1103	26	65.8078	0.02	69.1133	0.15810
1104	27	86.8527	0.02	69.1133	4.55317

1105	28	58.8894	0.02	69.1133	1.51243
1106	29	67.0961	0.02	69.1133	0.05888
1107	30	56.2633	0.02	69.1133	2.38917
1108	31	64.5305	0.02	69.1133	0.30388
1109	32	81.5376	0.02	69.1133	2.23346
1110	33	67.6533	0.02	69.1133	0.03084
1111	34	59.5760	0.02	69.1133	1.31611
1112	35	73.0711	0.02	69.1133	0.22664
1113	36	56.6235	0.02	69.1133	2.25709
1114	37	64.2907	0.02	69.1133	0.33651
1115	38	73.5581	0.02	69.1133	0.28585
1116	39	87.1496	0.02	69.1133	4.70686
1117	40	66.4684	0.02	69.1133	0.10122
1118	41	75.6757	0.02	69.1133	0.62310
1119	42	58.5321	0.02	69.1133	1.61997
1120	43	70.1086	0.02	69.1133	0.01433
1121	44	83.3389	0.02	69.1133	2.92804
1122	45	75.3332	0.02	69.1133	0.55976
1123	46	79.4953	0.02	69.1133	1.55955
1124	47	61.6596	0.02	69.1133	0.80388
1125	48	66.9878	0.02	69.1133	0.06537
1126	49	79.0530	0.02	69.1133	1.42950
1127	50	74.7114	0.02	69.1133	0.45344
1128					
1129					
1130	N	DF	Chi-Sq	P-Value	
1131	3455.67	49	73.7133	0.013	
1132					
1133					

1134	CHI-SQUARE GOODNESS-OF-FIT TEST FOR C14				
1135	Test	Contribution			
1136	Category	Observed	Proportion	Expected	to Chi-Sq
1137	1	58.1029	0.02	66.7141	1.1115
1138	2	74.3859	0.02	66.7141	0.8822
1139	3	74.1500	0.02	66.7141	0.8288
1140	4	64.8811	0.02	66.7141	0.0504
1141	5	86.2898	0.02	66.7141	5.7441
1142	6	49.8579	0.02	66.7141	4.2589
1143	7	58.5000	0.02	66.7141	1.0113
1144	8	58.8486	0.02	66.7141	0.9273
1145	9	71.7555	0.02	66.7141	0.3810
1146	10	73.9911	0.02	66.7141	0.7938
1147	11	57.2744	0.02	66.7141	1.3357
1148	12	63.5684	0.02	66.7141	0.1483
1149	13	61.4051	0.02	66.7141	0.4225
1150	14	69.2382	0.02	66.7141	0.0955
1151	15	46.4524	0.02	66.7141	6.1537
1152	16	61.3217	0.02	66.7141	0.4359
1153	17	66.7901	0.02	66.7141	0.0001
1154	18	48.0838	0.02	66.7141	5.2026
1155	19	69.2126	0.02	66.7141	0.0936
1156	20	83.1091	0.02	66.7141	4.0291
1157	21	88.9119	0.02	66.7141	7.3859
1158	22	38.3419	0.02	66.7141	12.0661
1159	23	66.1871	0.02	66.7141	0.0042
1160	24	66.5443	0.02	66.7141	0.0004
1161	25	66.3016	0.02	66.7141	0.0026
1162	26	60.8103	0.02	66.7141	0.5224
1163	27	97.5394	0.02	66.7141	14.2429
1164	28	72.0151	0.02	66.7141	0.4212
1165	29	76.1445	0.02	66.7141	1.3330
1166	30	53.2460	0.02	66.7141	2.7189
1167	31	77.8636	0.02	66.7141	1.8634
1168	32	76.3893	0.02	66.7141	1.4031
1169	33	51.6508	0.02	66.7141	3.4011

1170	34	51.8613	0.02	66.7141	3.3067
1171	35	62.8063	0.02	66.7141	0.2289
1172	36	55.4740	0.02	66.7141	1.8938
1173	37	86.3470	0.02	66.7141	5.7777
1174	38	69.5826	0.02	66.7141	0.1233
1175	39	63.3548	0.02	66.7141	0.1691
1176	40	68.4572	0.02	66.7141	0.0455
1177	41	67.5088	0.02	66.7141	0.0095
1178	42	64.6314	0.02	66.7141	0.0650
1179	43	79.8460	0.02	66.7141	2.5849
1180	44	55.9862	0.02	66.7141	1.7251
1181	45	68.0542	0.02	66.7141	0.0269
1182	46	65.1196	0.02	66.7141	0.0381
1183	47	72.6761	0.02	66.7141	0.5328
1184	48	88.6140	0.02	66.7141	7.1890
1185	49	67.3138	0.02	66.7141	0.0054
1186	50	58.9061	0.02	66.7141	0.9138
1187					
1188					
1189	N	DF	Chi-Sq	P-Value	
1190	3335.70	49	103.907	0.008	
1191					

1192

1193 **CHI-SQUARE GOODNESS-OF-FIT TEST FOR C15**

1194	Test	Contribution			
1195	Category	Observed	Proportion	Expected	to Chi-Sq
1196	1	60.2278	0.02	68.3234	0.95924
1197	2	65.8299	0.02	68.3234	0.09100
1198	3	65.5152	0.02	68.3234	0.11542
1199	4	73.8950	0.02	68.3234	0.45436
1200	5	62.9502	0.02	68.3234	0.42256
1201	6	65.9190	0.02	68.3234	0.08461
1202	7	50.6659	0.02	68.3234	4.56337
1203	8	80.2241	0.02	68.3234	2.07291
1204	9	64.4795	0.02	68.3234	0.21626
1205	10	75.2473	0.02	68.3234	0.70167
1206	11	87.6619	0.02	68.3234	5.47366
1207	12	81.1765	0.02	68.3234	2.41795
1208	13	52.7161	0.02	68.3234	3.56518
1209	14	65.9264	0.02	68.3234	0.08409
1210	15	81.7059	0.02	68.3234	2.62126
1211	16	77.4949	0.02	68.3234	1.23117
1212	17	68.2629	0.02	68.3234	0.00005
1213	18	62.6631	0.02	68.3234	0.46892
1214	19	68.7799	0.02	68.3234	0.00305
1215	20	57.8019	0.02	68.3234	1.62024
1216	21	60.2535	0.02	68.3234	0.95314
1217	22	63.7539	0.02	68.3234	0.30561
1218	23	69.9003	0.02	68.3234	0.03640
1219	24	50.8563	0.02	68.3234	4.46551
1220	25	66.2999	0.02	68.3234	0.05992
1221	26	71.2143	0.02	68.3234	0.12232
1222	27	86.3903	0.02	68.3234	4.77747
1223	28	63.4721	0.02	68.3234	0.34446
1224	29	51.9273	0.02	68.3234	3.93467
1225	30	68.3975	0.02	68.3234	0.00008
1226	31	85.8907	0.02	68.3234	4.51692
1227	32	72.8000	0.02	68.3234	0.29332
1228	33	84.8269	0.02	68.3234	3.98646
1229	34	76.7812	0.02	68.3234	1.04700
1230	35	60.6560	0.02	68.3234	0.86044
1231	36	75.0120	0.02	68.3234	0.65479
1232	37	82.0613	0.02	68.3234	2.76231

1233	38	76.6760	0.02	68.3234	1.02113
1234	39	49.9261	0.02	68.3234	4.95378
1235	40	62.6345	0.02	68.3234	0.47367
1236	41	81.6041	0.02	68.3234	2.58153
1237	42	67.4385	0.02	68.3234	0.01146
1238	43	63.2011	0.02	68.3234	0.38402
1239	44	62.7916	0.02	68.3234	0.44787
1240	45	56.3205	0.02	68.3234	2.10865
1241	46	70.4168	0.02	68.3234	0.06414
1242	47	77.2419	0.02	68.3234	1.16419
1243	48	53.3224	0.02	68.3234	3.29358
1244	49	58.1362	0.02	68.3234	1.51893
1245	50	76.8213	0.02	68.3234	1.05695

1246					
1247					
1248					
1249	N	DF	Chi-Sq	P-Value	
1250	3416.17	49	75.3677	0.009	

1251 The Mean and Standard Deviation of the K-S Test P-Values (15 Data Set)

1252		Mean	StDev.
1253	K-S	0.1470	0.00819

1254

1255 The Mean and Standard Deviation of the Chi-square Test P-Values (15 Data Set)

1256		Mean	StDev.
1257	CHI-SQUARE	0.0273	0.0529

1258

1259 Conclusion

1260 In other for the null hypothesis to be accepted when carrying out Kolmogorov-
1261 Smirnov and chi-square goodness of fit test, the P-value must be greater than $\alpha=0.05$ (i.e. P-
1262 value >0.05). From the mean and standard deviation of the simulated data p-values for the
1263 two goodness of fit tests, the Kolmogorov-Smirnov test has a higher mean value and standard
1264 deviation which is greater than that of the Chi-square test. Also for the simulated data, the
1265 analysis showed that Kolmogorov-Smirnov test is more precise with p-values greater than the
1266 Chi-square test.

1267 This study shows that in test for goodness of fit for normality (i.e. in the first 50
1268 simulated data), the Kolmogorov-Smirnov of fit test p-value is greater than the Chi-square
1269 goodness test p-value. But comparing the results to that of the simulated data, the
1270 Kolmogorov-Smirnov test tends to give a higher p-value than the Chi-square. This proves
1271 that chi-square goodness of fit test is not the most accurate of tests when taking goodness of
1272 fit test.

1273 Comparing the results of the 50 British boys heights and the 15 data set shows that
1274 Kolmogorov-Smirnov is more precise in comparison to the Chi-square test.

1275 Recommendation

1276 Based on the result of the study, the following recommendations made:

- 1277 i. When testing goodness of fit test for small samples, it is important that other types
1278 of goodness of fit test be used, rather than the more common Pearson Chi-square.
- 1279 ii. The test statistic is recommended when testing goodness of fit for a normal
1280 distribution than the Kolmogorov-Smirnov test statistic and the Pearson's Chi-
1281 square test statistic, as it has a precise p-value.

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