RIEMANNIAN VELOCITY AND ACCELERATION TENSORS/VECTORS IN ROTATIONAL OBLATE SPHEROIDAL COORDINATES BASED UPON THE GREAT METRIC TENSOR

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Abstract

Since the time of Galileo (1564 - 1643), Euclidean geometry has been the foundation on which the theoretical formulations of all geometrical quantities in all orthogonal curvilinear coordinates in Physics and Mathematics were built. But with the discovery of the great metric tensor in spherical polar coordinates (r, θ, ϕ, x^0) in all gravitational fields in nature[5] has made Riemannian geometry to be opened up for exploration and exploitation and hence its application in Theoretical Physics and Mathematics. In this paper, we derive the Riemannian vector and acceleration tensor/vectors in Rotational Oblate Spheroidal coordinates for application in physics and other related fields.

Keywords: Riemannian geometry, great metric tensor, Riemannian velocity and acceleration and Rotational Oblate Spheroidal coordinates.

1:0 Introductions

Euclidean geometry has been the basic foundation on which Theoretical Physics and Mathematics are built upon because the geometry has a well-defined metric tensor for all orthogonal curvilinear coordinates. Base on this well-known Euclidean metric, we had derived the velocity and ac celebration in some of the orthogonal curvilinear coordinates [1, 2, 3, 4], added to the known velocity and acceleration in Cartesian, Cylindrical and Spherical Coordinates, for applications in Physics and other related fields.

Right from 1854, when George Friedrich Bernhard Riemann published his geometry for space-time known as Riemann geometry, it was assumed to be

more general than the Euclidean geometry, and it is generally accepted that Riemannian geometry has the potential of providing a more general foundation for Theoretical Physics and Mathematics [5]. However, the problem with the Riemannian geometry is that it was not founded on any metric tensor which makes its exploitation and possible application to Theoretical Physics and Mathematics eluded the whole world. Riemann, therefore, left behind the problem of finding the metric tensors for all gravitational fields in nature. Einstein tried to solve this problem in his contribution to classical mechanics known as Einstein's Geometrical Gravitational Field Equation [5]. In 1916, Karl Schwarzschild introduced a metric tensor for all gravitational fields due to the static homogeneous spherical distribution of mass. This metric tensor has been the basis for the development of Einstein's Geometrical Theory of Classical Mechanics in the gravitational field known as General Relativity. Despite the great result obtained from Einstein Geometrical Gravitational Field Equations, they cannot be applied to generate any natural metric tensor for the gravitational fields due to any distribution of mass in nature for all orthogonal curvilinear coordinates.

It is interesting to know that Professor S.X.K Howusu has developed a metric tensor called the great metric tensor for all gravitational fields in nature in 2009 [5]. This metric tensor is valid for all four coordinates of space-time and all regular geometries in nature and all regular distributions of mass in any coordinate. The first watershed of this metric tensor is the discovery of how the well-known tensorial Riemann's Geodesic equation of motion for particles of non-zero rest masses in gravitational field can be written equivalently in terms of (i) Riemann's linear momentum vector, (ii) Riemann's linear acceleration vector, (iii) Riemann's gravitational intensity, (iv) Riemann's gravitational force vector, (v) Riemman's Lagrangian, (vi) Riemann's kinetic energy and gravitational potential energy. All of which reduce to the corresponding pure Newtonian quantities in the limit of c^0, and in general contain post-Newton or pure Riemannian correction terms of all orders of c^{-2}. Therefore the well-known tensorial Riemann's Geodesic Equations of Motion for particles of non-zero rest masses in gravitational fields is one of the most convincing and compelling proofs that the gravitational phenomenon is dynamical precisely as Galileo and Newton and Cavendish discovered it.

The second watershed of this metric tensor is the discovery that precisely as there exists one and only one unique Euclidean metric tensor for all space-time without

gravitational field so also there exists precisely one and only one unique Riemannian metric tensor for all space time with gravitational field which (i) reduces to the corresponding Euclidean metric tensor in the limit of c^0 and (ii) in general contains post Euclid or pure Riemannian correction terms of all orders of c^{-2}, in perfect agreement with the principle of equivalence of physics and the principle of equivalence of mathematics.

We are now in a position to calculate all the theoretical predictions of Riemann's geometrical and physical concepts and principles and compare them with experimental physical evidence.

Following the introduction of this new metric tensor we had formulated some Riemannian geometrical quantities in Cartesian Coordinates [6] and some orthogonal curvilinear coordinates [7,8]. In this paper, we are out again to generate the Riemannian velocity and acceleration tensor/vector in Rotational Oblate Spheroidal coordinates for application in physics and mathematics. We choose this coordinate so that the results obtained here can be applied in un-manned weather satellite and rotational friction coefficient for sphereolds with the slipping boundary condition as suggested by Jack, B. Kuipers [11]

2:0THEORY

The Rotational Oblate Spheroidal Coordinates (u, v, w) can be expressed in terms of Cartesian coordinates (x, y, z) as [5]:

$$x = w(u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}} \tag{1}$$

$$y = (u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}\,(1 - w^2)^{\frac{1}{2}} \tag{2}$$

$$y = uv \tag{3}$$

The great metric tensor for all gravitational fields in nature in spherical polar coordinates (r, θ, ϕ, x^0) is given as [5]:

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right) \tag{4}$$

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1} \tag{5}$$

$$g_{22} = r^2 \tag{6}$$

$$g_{33} = r^2 \sin^2 \theta \tag{7}$$

$$g_{uv} = 0\,; Otherwise \tag{8}$$

The Rotational Oblate Spheroidal coordinates are related to the Spherical polar coordinates as:

$$r = [u^2 + d^2(1 - v^2)]^{\frac{1}{2}} \tag{9}$$

$$\theta = \cos^{-1}\left\{\frac{uv}{\left[u^2 + d^2(1 - v^2)^{\frac{1}{2}}\right]}\right\} \tag{10}$$

and

$$\phi = \tan^{-1}\left[\frac{(1 - w^2)^{\frac{1}{2}}}{w}\right] \tag{11}$$

From the well know transformation equation given by the covariant tensor [10] and consequently, upon transformation by using (4)-(11) we obtained the Riemannian metric tensor for all gravitational fields in Rotational Oblate Spheroidal coordinates as:

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right) \tag{12}$$

$$g_{11} = \frac{u^2 + v^2 d^2}{u^2 + d^2} + \frac{u^2}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{13}$$

$$g_{12} = \frac{-uvd^2}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{14}$$

$$g_{22} = \frac{u^2 + v^2 d^2}{(1 - v^2)} + \frac{v^2 d^4}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{15}$$

$$g_{33} = (u^2 + d^2)(1 - v^2) \tag{16}$$

$$g_{\mu\nu} = 0\,; Otherwise \tag{17}$$

It may be noted that the determinant of the metric tensor $g_{\mu\nu}$, denoted by g is obtained as:

$$g = -(u^2 + v^2 d^2)^2 \tag{18}$$

Also, the contravariant metric tensor for this Riemannian metric tensor denoted as $g^{\mu\upsilon}$ is given as:

$$g^{00} = -\left(1 + \frac{2}{c^2} f\right)^{-1} \tag{19}$$

$$g^{11} = \frac{u^2 + v^2 d^2}{u^2 + d^2}$$

$$\left\{1 + \frac{(1 - v^2) v^2 d^4}{(u^2 + v^2 d^2)[u^2 + d^2(1 - v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \tag{20}$$

$$g^{12} = \left\{\frac{uvd^2(1 - v^2)(u^2 + d^2)}{(u^2 + v^2 d^2)[u^2 + d^2(1 - v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \tag{21}$$

$$g^{22} = \frac{(1 - v^2)}{u^2 + v^2 d^2}$$

$$\left\{1 + \frac{u^2(u^2 + d^2)}{(u^2 + v^2 d^2)[u^2 + d^2(1 - v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \tag{22}$$

$$g^{33} = [(u^2 + d^2)(1 - v^2)]^{-1} \tag{23}$$

$$g^{\mu\upsilon} = 0;\ otherwise \tag{24}$$

These metric tensors define the Riemannian line element, Riemannian volume element, Riemannian gradient operator, Riemannian divergence, Riemannian curl

and Riemannian Laplacian in Rotational Oblate Spheroidal coordinates, according to the Theory of Tensor and Vector Analysis [9]. These quantities are necessary and sufficient for the derivation of fields in all Rotational Oblate Spheroidal distribution of mass, charge and current. Now for the derivation of the equation of motion for test particles in all gravitational fields, we shall derive the expression for Riemannian velocity and acceleration in Rotational Oblate Spheroidal coordinates.

2:1 Great Riemannian Velocity Tensor/Vector in Rotational Oblate Spheroidal Coordinates.

According to the theory of tensor analysis, the linear velocity in four-dimensional space-time, u^{α} is given in all gravitational fields in all orthogonal curvilinear coordinates x^{α} by [Spiegel, 1974]:

$$u^{\alpha} = \frac{d}{d\tau} x^{\alpha} = \dot{x}^{\alpha} \tag{25}$$

where τ is the proper time (time on a clock at rest outside gravitational field) is a contra-variant tensor of rank one which is called the linear velocity tensor and it is denoted by $\dot{x}^{\mu}$

Einstein Cartesian coordinate(x, y, z, x^0), u^0, u^1, $u^2 and\ u^3$ are given as:

$$u^0 = \dot{x}^0 = c\dot{t} \tag{26}$$

$$u^1 = \dot{x}^1 = \dot{u} \tag{27}$$

$$u^2 = \dot{x}^2 = \dot{v} \tag{28}$$

and

$$u^3 = \dot{x}^3 = \dot{w} \tag{29}$$

It may be noted that in Minkowski Cartesian coordinates, x^0 is given as:

$$u^0 = ic\dot{t} \tag{30}$$

The Great Riemannian Linear velocity tensor according to the theory of Tensor Analysis, the coordinates(u, v, w, x^0) is given as [9]:

$$\underline{U}_R = [U_u, U_v\, U_w, U_{x^0}] \tag{31}$$

where

$$\left(\underline{U}_R\right)_0 = -c\left(1 + \frac{2}{c^2} f\right)^{\frac{1}{2}} \dot{t} \tag{32}$$

$$\left(\underline{U}_R\right)_1 = \left[\frac{u^2 + v^2 d^2}{u^2 + d^2} + \frac{u^2}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n\right]^{\frac{1}{2}} \dot{u} \tag{33}$$

$$\left(\underline{U}_R\right)_2 = \left[\frac{u^2 + v^2 d^2}{(1 - v^2)} + \frac{v^2 d^4}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n\right]^{\frac{1}{2}} \dot{v} \tag{34}$$

and

$$\left(\underline{U}_R\right)_3 = [u^2 + d^2(1 - v^2)]^{\frac{1}{2}} \dot{w} \tag{35}$$

This is the great Riemannian velocity vector in Rotational Oblate Spheroidal coordinates.

It may be noted that the Riemannian velocity vector obtained here is also a tensor of rank one (i.e vector), but the magnitude is different from the Riemannian velocity tensor as derived from the transformation.

2:2 Great Riemannian Acceleration Tensor/Vector in Rotational Oblate Spheroidal Coordinates

Following the development of Great Riemannian velocity tensor/vectors, the Riemannian linear acceleration tensor in 4-dimensional space-time, $a_{R,}^{\alpha}$ in gravitational fields in nature and all orthogonal curvilinear coordinates x^{α} is given by the theory of tensor analysis as [9]:

$$a_R^{\alpha} = \ddot{x}^{\alpha} + \Gamma_{\mu\nu}^{\alpha} \dot{x}^{\mu} \dot{x}^{\nu} \tag{36}$$

Where a_R^{α} is the Riemannian acceleration tensor of the rank one and $\Gamma_{\mu\nu}^{\alpha}$ is the Christoffel symbol of the second kind (or Coefficient of affine connection) Pseudo

tensor, and a dot denotes one differentiation concerning proper timeτ. The non-zero results of $\Gamma^{\alpha}_{\mu\nu}$ based upon the great metric tensor in Rotational Oblates Spheroidal coordinates are given as:

$$\Gamma^{0}_{00} = \frac{1}{2} g^{00} g_{00,0} \tag{37}$$

$$\Gamma^{0}_{01} = \frac{1}{2} g^{00} g_{00,1} \tag{38}$$

$$\Gamma^{0}_{02} = \frac{1}{2} g^{00} g_{00,2} \tag{39}$$

$$\Gamma^{0}_{03} = \frac{1}{2} g^{00} g_{00,3} \tag{40}$$

$$\Gamma^{0}_{11} = -\frac{1}{2} g^{00} g_{11,0} \tag{41}$$

$$\Gamma^{0}_{12} = -\frac{1}{2} g^{00} g_{12,0} \tag{42}$$

$$\Gamma^{0}_{22} = -\frac{1}{2} g^{00} g_{22,0} \tag{43}$$

$$\Gamma^{0}_{33} = -\frac{1}{2} g^{00} g_{33,0} \tag{44}$$

$$\Gamma^{1}_{00} = -\frac{1}{2} g^{11} g_{00,1} - \frac{1}{2} g^{12} g_{00,2} \tag{45}$$

$$\Gamma^{1}_{01} = -\frac{1}{2} g^{11} g_{11,0} - \frac{1}{2} g^{12} g_{12,0} \tag{46}$$

$$\Gamma^{1}_{02} = -\frac{1}{2} g^{11} g_{21,0} + \frac{1}{2} g^{12} g_{22,0} \tag{47}$$

$$\Gamma^{1}_{11} = -\frac{1}{2} g^{11} g_{11,1} - \frac{1}{2} g^{12} g_{11,2} + g^{12} g_{12,1} \tag{48}$$

$$\Gamma^{1}_{12} = \frac{1}{2} g^{11} g_{11,2} + \frac{1}{2} g^{12} g_{22,1} \tag{49}$$

$$\Gamma^1_{13} = \frac{1}{2} g^{11} g_{11,3} + \frac{1}{2} g^{12} g_{12,3} \tag{50}$$

$$\Gamma^1_{22} = g^{11} g_{21,2} - \frac{1}{2} g^{11} g_{22,1} + \frac{1}{2} g^{12} g_{22,2} \tag{51}$$

$$\Gamma^1_{23} = \frac{1}{2} g^{11} g_{21,3} + \frac{1}{2} g^{12} g_{22,3} \tag{52}$$

$$\Gamma^1_{33} = -\frac{1}{2} g^{11} g_{33,1} - \frac{1}{2} g^{12} g_{33,2} \tag{53}$$

$$\Gamma^2_{00} = -\frac{1}{2} g^{21} g_{11,0} - \frac{1}{2} g^{22} g_{12,0} \tag{54}$$

$$\Gamma^2_{01} = \frac{1}{2} g^{21} g_{11,0} + \frac{1}{2} g^{22} g_{12,0} \tag{55}$$

$$\Gamma^2_{02} = \frac{1}{2} g^{21} g_{21,0} + \frac{1}{2} g^{22} g_{22,0} \tag{56}$$

$$\Gamma^2_{11} = \frac{1}{2} g^{21} g_{11,1} + g^{22} g_{12,1} - \frac{1}{2} g^{22} g_{11,2} \tag{57}$$

$$\Gamma^2_{12} = \frac{1}{2} g^{21} g_{11,2} + \frac{1}{2} g^{22} g_{22,1} \tag{58}$$

$$\Gamma^2_{13} = \frac{1}{2} g^{21} g_{11,3} + \frac{1}{2} g^{22} g_{12,3} \tag{59}$$

$$\Gamma^2_{22} = g^{21} g_{21,2} - \frac{1}{2} g^{21} g_{22,1} + \frac{1}{2} g^{22} g_{22,2} \tag{60}$$

$$\Gamma^2_{23} = \frac{1}{2} g^{21} g_{21,3} + \frac{1}{2} g^{22} g_{22,3} \tag{61}$$

$$\Gamma^2_{33} = -\frac{1}{2} g^{21} g_{33,1} - \frac{1}{2} g^{22}, g_{33,2} \tag{62}$$

$$\Gamma^3_{00} = -\frac{1}{2} g^{33} g_{00,3} \tag{63}$$

$$\Gamma^3_{11} = -\frac{1}{2} g^{33} g_{11,3} \tag{64}$$

$$\Gamma^3_{12} = -\frac{1}{2} g^{33} g_{12,3} \tag{65}$$

$$\Gamma^3_{13} = \frac{1}{2} g^{33} g_{33,1} \tag{66}$$

$$\Gamma^3_{22} = -\frac{1}{2} g^{33} g_{22,3} \tag{67}$$

$$\Gamma^3_{23} = \frac{1}{2} g^{33} g_{33,2} \tag{68}$$

$$\Gamma^3_{33} = \frac{1}{2} g^{33} g_{33,3} \tag{69}$$

$$\Gamma^\alpha_{\mu\nu} = 0; otherwise \tag{70}$$

It follows from equation (36) – (70) that:

$$a^0_R = c\ddot{t} + c^2\Gamma^0_{00}\dot{t}^2 + 2c\Gamma^0_{01}\dot{t}\dot{u} + 2c\Gamma^0_{02}\dot{t}\dot{u} + 2c\Gamma^0_{02}\dot{t}\dot{v} + 2c\Gamma^0_{03}\dot{t}\dot{w} + \Gamma^0_{11}\dot{u}^2 + 2\Gamma^0_{12}\dot{u}\dot{v} + \Gamma^0_{22}\dot{v}^2 + \Gamma^0_{33}\dot{w}^2 \tag{71}$$

and

$$a^1_R = \ddot{u} + c^2\Gamma^1_{00}\dot{t}^2 + 2c\Gamma^1_{01}\dot{t}\dot{u} + 2c\Gamma^1_{02}\dot{t}\dot{u} + \Gamma^1_{11}\dot{u}^2 + 2\Gamma^1_{12}\dot{u}\dot{v} + 2\Gamma^1_{13}\dot{u}\dot{w} + \Gamma^1_{22}\dot{v}^2 + 2\Gamma^1_{23}\dot{v}\dot{w} + \Gamma^1_{33}\dot{w}^2 \tag{72}$$

and

$$a^2_R = \ddot{v} + c^2\Gamma^2_{00}\dot{t}^2 + 2c\Gamma^2_{01}\dot{t}\dot{u} + 2c\Gamma^2_{02}\dot{t}\dot{v} + \Gamma^2_{11}\dot{u}^2 + 2\Gamma^2_{12}\dot{u}\dot{v} + 2\Gamma^2_{13}\dot{u}\dot{w} + \Gamma^2_{22}\dot{v}^2 + 2\Gamma^2_{23}\dot{v}\dot{w} + \Gamma^2_{33}\dot{w}^2 \tag{73}$$

and

$$a^3_R = \ddot{w} + c^2\Gamma^3_{00}\dot{t}^2 + 2c\Gamma^3_{03}\dot{t}\dot{w} + +\Gamma^3_{11}\dot{u}^2 + 2\Gamma^3_{12}\dot{u}\dot{v} + 2\Gamma^3_{13}\dot{u}\dot{w} + \Gamma^3_{22}\dot{v}^2 \tag{74}$$

Wherein Einstein coordinate coordinates of space-time in Rotational Oblate Spheroidal coordinates:

$$x^1 = u; x^2 = v;\ x^3 = u;\ x^o = ct \tag{75}$$

Equation (71) – (73) is called the Great Riemann Linear Acceleration Tensor in Rotational Oblate Spheroidal coordinates.

Hence, the Great Riemannian Acceleration Vector $\underline{a}_R$ is defined as:

$$\underline{a}_R = [(a_R)_u, (a_R)_v, (a_R)_w, (a_R)_{x^o}\ ] \tag{76}$$

where

$$(a_R)_{x^o} = (g_{00})^{\frac{1}{2}} a_R^0 \tag{77}$$

$$(a_R)_u = (g_{11})^{\frac{1}{2}} a_R^1 \tag{78}$$

$$(a_R)_v = (g_{22})^{\frac{1}{2}} a_R^2 \tag{79}$$

and

$$(a_R)_w = (g_{33})^{\frac{1}{2}} a_R^3 \tag{80}$$

Equation (77) – (80) is the Great Riemannian acceleration vector for all gravitational fields in nature in Rotational Oblate Spheroidal coordinates.

It may be noted that the Riemannian acceleration vector obtained is also a tensor of rank one but the magnitude is different from the Riemannian acceleration tensor as derived from the transformation.

It is now most interesting and instructive to note that the Riemann's acceleration vector obtained here can be used to the developed natural extension of the Riemann's Vectorial Geodesic Equation of Motion from the gravitational field to other interaction fields in nature given as:

$$m_0 \underline{a} = \underline{F}^{ng} \tag{81}$$

Or equivalently

$$m_0 \underline{a}_R = \underline{F}_R^g + \underline{F}^{ng} \tag{82}$$

Where $\underline{a}_R$ is the Riemann's acceleration vector in the gravitational field, $\underline{F}_R^g$ is the Riemann's gravitational force vector, and $\underline{F}^{ng}$ is the sum of all the non-gravitational vector forces acting on the particles. And this equation can be applied in all non-gravitational interaction fields in nature since every interaction field in nature has a corresponding vectorial force expression or representation in four-dimensional space-time. Consequently, the Riemann's Vectorial Geodesic Equation of Motion can be applied to every system in nature, without exception.

Therefore, it can be used to develop the following:

- Riemann's Dynamical Theory of the vertical free falling particle,
- Riemann's Dynamical Theory or the plane projective,
- Riemann's Dynamical Theory of the free fall in a viscous medium,
- The Gravitational Field of a Body with Rotational Symmetry in Einstein's Theory of Gravitation,
- The influence of Inertia on the Rotational Dynamics of Spheroidal particles suspended in shear flow,
- And so on.

3:0 Results and discussions

In this paper, we have derived the components of the Great Riemannian Linear velocity tensor/vector and the Great Riemannian linear acceleration tensor/vector in Rotational Oblate Spheroidal Coordinates as (71) – (73) and (77) – (80) respectively. These results obtained in this paper are necessary and sufficient for expressing all Riemannian mechanical quantities in all gravitational fields in nature (Riemannian Linear Momentum, Riemannian Kinetic Energy, Riemannian Lagragian and Riemannian Hamiltonian) regarding Rotational Oblate Spheroidal coordinates.

4:0 Conclusions

The Great Riemannian velocity vector (71) – (73) and the Great Riemannian Linear Acceleration vector (77) – (80) obtained in this paper pave a way for expressing all Riemannian Dynamical laws of motion (Newton's law, Lagrange's law, Hamilton's law, Einstein Special Relativistic law of motion and Schrödinger's law of quantum mechanics) entirely in terms of Rotational Oblate Spheroidal coordinates.

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