RIEMANNIAN VELOCITY AND ACCELERATION TENSORS/VECTORS IN ROTATIONAL OBLATE SPHEROIDAL COORDINATES BASED UPON THE GREAT METRIC TENSOR

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Abstract

Since the time of Galileo (1564 - 1643), Euclidean geometry has been the foundation on which the theoretical formulations of all geometrical quantities in all orthogonal curvilinear coordinates in Physics and Mathematics were built. But with the discovery of the great metric tensor in spherical polar coordinates (r, θ, ϕ, x^0) in all gravitational fields in nature[5] has made Riemannian geometry to be opened up for exploration and exploitation and hence its application in Theoretical Physics and Mathematics. In this paper, we derive the Riemannian vector and acceleration tensor/vectors in Rotational Oblate Spheroidal coordinates for application in physics and other related fields.

Keywords: Riemannian geometry, great metric tensor, Riemannian velocity and acceleration and Rotational Oblate Spheroidal coordinates.

1:0 Introductions

Euclidean geometry has been the basic foundation on which Theoretical Physics and Mathematics are built upon because the geometry has a well-defined metric tensor for all orthogonal curvilinear coordinates. Base on this well-known Euclidean metric, we had derived the velocity and ac celebration in some of the orthogonal curvilinear coordinates [1, 2, 3, 4], added to the known velocity and acceleration in Cartesian, Cylindrical and Spherical Coordinates, for applications in Physics and other related fields.

Right from 1854, when George Friedrich Bernhard Riemann published his geometry for space-time known as Riemann geometry, it was assumed to be more general than the Euclidean geometry and it is generally accepted that Riemannian geometry has the potential of providing a more general foundation for Theoretical Physics and Mathematics [5]. However, the problem with the Riemannian geometry is that it was not founded on any metric tensor which makes its exploitation and possible application to Theoretical Physics and Mathematics eluded the whole world. Riemann therefore left behind the problem of finding the metric tensors for all gravitational fields in nature. Einstein tried to solve this problem in his contribution to classical mechanics known as Einstein's Geometrical Gravitational Field Equation [5]. In 1916, Karl Schwarzschild introduced a metric tensor for all gravitational fields due to static homogeneous spherical distribution of mass. This metric tensor has been the basic for the development of Einstein's Geometrical Theory of Classical Mechanics in the gravitational field known as General Relativity. Despite the great result obtained from Einstein Geometrical Gravitational Field Equations, they cannot be applied to generate any natural metric tensor for the gravitational fields due to any distribution of mass in nature for all orthogonal curvilinear coordinates.

It is interesting to know that a metric tensor called the great metric tensor for all gravitational fields in nature has been developed by Professor S.X.K Howusu in 2009 [5]. This metric tensor is valid for all four coordinates of space-time and for all regular geometries in nature and for all regular distributions of mass in any coordinate. In the limit of c^0, it reduces to the well-known Euclidean metric tensor for all space-time in gravitational fields in nature, in perfect agreement with the principle of equivalence of Physics and the principle of equivalence of Mathematics. We are now in position to calculate all the theoretical predictions of Riemann's geometrical and physical concepts and principles and compare them with experimental physical evidence.

Following the introduction of this new metric tensor we had formulated some Riemannian geometrical quantities in Cartesian Coordinates [6] and some orthogonal curvilinear coordinates [7,8]. In this paper, we are out again to generate the Riemannian velocity and acceleration tensor/vector in Rotational Oblate Spheroidal coordinates for application in physics and mathematics.

2:0THEORY

The Rotational Oblate Spheriodal Coordinates (u, v, w) can be expressed in terms of Cartesian coordinates (x, y, z) as [5]:

$$x = w(u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}} \tag{1}$$

$$y = (u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}\,(1 - w^2)^{\frac{1}{2}} \tag{2}$$

$$y = uv \tag{3}$$

The great metric tensor for all gravitational fields in nature in spherical polar coordinates (r, θ, ϕ, x^0) is given as [5]:

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right) \tag{4}$$

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1} \tag{5}$$

$$g_{22} = r^2 \tag{6}$$

$$g_{33} = r^2 \sin^2 \theta \tag{7}$$

$$g_{uv} = 0\,;\, Otherwise \tag{8}$$

The Rotational Oblate Spheroidal coordinates are related to the Spherical polar coordinates as:

$$r = [u^2 + d^2(1 - v^2)]^{\frac{1}{2}} \tag{9}$$

$$\theta = \cos^{-1}\left\{\frac{uv}{\left[u^2 + d^2(1 - v^2)^{\frac{1}{2}}\right]}\right\} \tag{10}$$

and

$$\phi = \tan^{-1}\left[\frac{(1 - w^2)^{\frac{1}{2}}}{w}\right] \tag{11}$$

From the well know transformation equation given by the covariant tensor [10] and consequently, upon transformation by using (4)-(11) we obtained the

Riemannian metric tensor for all gravitational fields in Rotational Oblate Spheroidal coordinates as:

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right) \tag{12}$$

$$g_{11} = \frac{u^2 + v^2 d^2}{u^2 + d^2} + \frac{u^2}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{13}$$

$$g_{12} = \frac{-uvd^2}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{14}$$

$$g_{22} = \frac{u^2 + v^2 d^2}{(1 - v^2)} + \frac{v^2 d^4}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{15}$$

$$g_{33} = (u^2 + d^2)(1 - v^2) \tag{16}$$

$$g_{\mu v} = 0\,;\, Otherwise \tag{17}$$

It may be noted that the determinant of the metric tensor $g_{\mu v}$, denoted by g is obtained as:

$$g = -(u^2 + v^2 d^2)^2 \tag{18}$$

Also, the contra variant metric tensor for this Riemannian metric tensor denoted as $g^{\mu v}$ is given as:

$$g^{00} = -\left(1 + \frac{2}{c^2} f\right)^{-1} \tag{19}$$

$$g^{11} = \frac{u^2 + v^2 d^2}{u^2 + d^2}$$

$$\left\{1 + \frac{(1-v^2)v^2 d^4}{(u^2 + v^2 d^2)[u^2 + d^2(1-v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \qquad (20)$$

$$g^{12} = \left\{\frac{uvd^2(1-v^2)(u^2+d^2)}{(u^2 + v^2 d^2)[u^2 + d^2(1-v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \quad (21)$$

$$g^{22} = \frac{(1-v^2)}{u^2 + v^2 d^2}$$

$$\left\{1 + \frac{u^2(u^2+d^2)}{(u^2 + v^2 d^2)[u^2 + d^2(1-v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \qquad (22)$$

$$g^{33} = [(u^2 + d^2)(1-v^2)]^{-1} \qquad (23)$$

$$g^{\mu v} = 0; otherwise \qquad (24)$$

These metric tensors define the Riemannian line element, Riemannian volume element, Riemannian gradient operator, Riemannian divergence, Riemannian curl and Riemannian Laplacian in Rotational Oblate Spheroidal coordinates, according to the Theory of Tensor and Vector Analysis [9]. These quantities are necessary and sufficient for derivation of fields in all Rotational Oblate Spheroidal distribution of mass, charge and current. Now for the derivation of the equation of motion for test particles in all gravitational fields, we shall derive the expression for Riemannian velocity and acceleration in Rotational Oblate Spheroidal coordinates.

2:1 Great Riemannian Velocity Tensor/Vector in Rotational Oblate Spheroidal Coordinates.

According to the theory of tensor analysis, the linear velocity in four-dimensional space – time, u^{α} is given in all gravitational fields in all orthogonal curvilinear coordinates x^{α} by [Spiegel, 1974]:

$$u^{\alpha} = \frac{d}{d\tau} x^{\alpha} = \dot{x}^{\alpha} \tag{25}$$

Where τ is proper time and a dot denotes one differentiation with respect to time in Einstein Cartesian coordinate(x, y, z, x^0), u^0, u^1, $u^2 and\ u^3$ are given as:

$$u^0 = \dot{x}^0 = c\dot{t} \tag{26}$$

$$u^1 = \dot{x}^1 = \dot{u} \tag{27}$$

$$u^2 = \dot{x}^2 = \dot{v} \tag{28}$$

and

$$u^3 = \dot{x}^3 = \dot{w} \tag{29}$$

It may be noted that in Minkwoski Cartesian coordinates, x^0 is given as:

$$u^0 = ic\dot{t} \tag{30}$$

The Great Riemannian Linear velocity tensor according to the theory of Tensor Analysis, the coordinates(u, v, w, x^0) is given as [9]:

$$\underline{U}_R = [U_u, U_v\, U_w, U_{x^0}] \tag{31}$$

where

$$\left(\underline{U}_R\right)_0 = -c\left(1 + \frac{2}{c^2} f\right)^{\frac{1}{2}} \dot{t} \tag{32}$$

$$\left(\underline{U}_R\right)_1 = \left[\frac{u^2 + v^2 d^2}{u^2 + d^2} + \frac{u^2}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n\right]^{\frac{1}{2}} \dot{u} \tag{33}$$

$$\left(\underline{U}_R\right)_2 = \left[\frac{u^2 + v^2 d^2}{(1 - v^2)} + \frac{v^2 d^4}{u^2 + d^2(1 - v^2)} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n\right]^{\frac{1}{2}} \dot{v} \tag{34}$$

and

$$\left(\underline{U}_R\right)_3 = [u^2 + d^2(1 - v^2)]^{\frac{1}{2}}\dot{w} \qquad (35)$$

This is the great Riemannian velocity vector in Rotational Oblate Spheroidal coordinates.

2:2 Great Riemannian Acceleration Tensor/Vector in Rotational Oblate Spheroidal Coordinates

Following the development of Great Riemannian velocity tensor/vectors, the Riemannian linear acceleration tensor in 4-dimensional space – time, $a_{R,}^{\alpha}$ in gravitational fields in nature and all orthogonal curvilinear coordinates x^{α} is b y theory of tensor analysis as [9]:

$$a_R^{\alpha} = \ddot{x}^{\alpha} + \Gamma_{\mu\nu}^{\alpha}\dot{x}^{\mu}\dot{x}^{\nu} \qquad (36)$$

Where $\Gamma_{\mu\nu}^{\alpha}$ is the Christoffel symbol of the second kind (or Coefficient of affine connection) Pseudo tensor and a dot denotes one differentiation with respect to proper timeτ. The non-zero results of $\Gamma_{\mu\nu}^{\alpha}$ based upon the great metric tensor in Rotational Oblates Spheroidal coordinates are given as:

$$\Gamma_{00}^{0} = \frac{1}{2}g^{00}g_{00,0} \qquad (37)$$

$$\Gamma_{01}^{0} = \frac{1}{2}g^{00}g_{00,1} \qquad (38)$$

$$\Gamma_{02}^{0} = \frac{1}{2}g^{00}g_{00,2} \qquad (39)$$

$$\Gamma_{03}^{0} = \frac{1}{2}g^{00}g_{00,3} \qquad (40)$$

$$\Gamma_{11}^{0} = -\frac{1}{2}g^{00}g_{11,0} \qquad (41)$$

$$\Gamma_{12}^{0} = -\frac{1}{2}g^{00}g_{12,0} \qquad (42)$$

$$\Gamma^0_{22} = -\frac{1}{2} g^{00} g_{22,0} \qquad (43)$$

$$\Gamma^0_{33} = -\frac{1}{2} g^{00} g_{33,0} \qquad (44)$$

$$\Gamma^1_{00} = -\frac{1}{2} g^{11} g_{00,1} - \frac{1}{2} g^{12} g_{00,2} \qquad (45)$$

$$\Gamma^1_{01} = -\frac{1}{2} g^{11} g_{11,0} - \frac{1}{2} g^{12} g_{12,0} \qquad (46)$$

$$\Gamma^1_{02} = -\frac{1}{2} g^{11} g_{21,0} + \frac{1}{2} g^{12} g_{22,0} \qquad (47)$$

$$\Gamma^1_{11} = -\frac{1}{2} g^{11} g_{11,1} - \frac{1}{2} g^{12} g_{11,2} + g^{12} g_{12,1} \qquad (48)$$

$$\Gamma^1_{12} = \frac{1}{2} g^{11} g_{11,2} + \frac{1}{2} g^{12} g_{22,1} \qquad (49)$$

$$\Gamma^1_{13} = \frac{1}{2} g^{11} g_{11,3} + \frac{1}{2} g^{12} g_{12,3} \qquad (50)$$

$$\Gamma^1_{22} = g^{11} g_{21,2} - \frac{1}{2} g^{11} g_{22,1} + \frac{1}{2} g^{12} g_{22,2} \qquad (51)$$

$$\Gamma^1_{23} = \frac{1}{2} g^{11} g_{21,3} + \frac{1}{2} g^{12} g_{22,3} \qquad (52)$$

$$\Gamma^1_{33} = -\frac{1}{2} g^{11} g_{33,1} - \frac{1}{2} g^{12} g_{33,2} \qquad (53)$$

$$\Gamma^2_{00} = -\frac{1}{2} g^{21} g_{11,0} - \frac{1}{2} g^{22} g_{12,0} \qquad (54)$$

$$\Gamma^2_{01} = \frac{1}{2} g^{21} g_{11,0} + \frac{1}{2} g^{22} g_{12,0} \qquad (55)$$

$$\Gamma^2_{02} = \frac{1}{2} g^{21} g_{21,0} + \frac{1}{2} g^{22} g_{22,0} \qquad (56)$$

$$\Gamma^2_{11} = \frac{1}{2} g^{21} g_{11,1} + g^{22} g_{12,1} - \frac{1}{2} g^{22} g_{11,2} \qquad (57)$$

$$\Gamma^2_{12} = \frac{1}{2} g^{21} g_{11,2} + \frac{1}{2} g^{22} g_{22,1} \quad (58)$$

$$\Gamma^2_{13} = \frac{1}{2} g^{21} g_{11,3} + \frac{1}{2} g^{22} g_{12,3} \quad (59)$$

$$\Gamma^2_{22} = g^{21} g_{21,2} - \frac{1}{2} g^{21} g_{22,1} + \frac{1}{2} g^{22} g_{22,2} \quad (60)$$

$$\Gamma^2_{23} = \frac{1}{2} g^{21} g_{21,3} + \frac{1}{2} g^{22} g_{22,3} \quad (61)$$

$$\Gamma^2_{33} = -\frac{1}{2} g^{21} g_{33,1} - \frac{1}{2} g^{22}, g_{33,2} \quad (62)$$

$$\Gamma^3_{00} = -\frac{1}{2} g^{33} g_{00,3} \quad (63)$$

$$\Gamma^3_{11} = -\frac{1}{2} g^{33} g_{11,3} \quad (64)$$

$$\Gamma^3_{12} = -\frac{1}{2} g^{33} g_{12,3} \quad (65)$$

$$\Gamma^3_{13} = \frac{1}{2} g^{33} g_{33,1} \quad (66)$$

$$\Gamma^3_{22} = -\frac{1}{2} g^{33} g_{22,3} \quad (67)$$

$$\Gamma^3_{23} = \frac{1}{2} g^{33} g_{33,2} \quad (68)$$

$$\Gamma^3_{33} = \frac{1}{2} g^{33} g_{33,3} \quad (69)$$

$$\Gamma^\alpha_{\mu\nu} = 0; otherwise \quad (70)$$

It follows from equation (36) – (70) that:

$$a^0_R = c\ddot{t} + c^2\Gamma^0_{00}\dot{t}^2 + 2c\Gamma^0_{01}\dot{t}\dot{u} + 2c\Gamma^0_{02}\dot{t}\dot{u} + 2c\Gamma^0_{02}\dot{t}\dot{v} + 2c\Gamma^0_{03}\dot{t}\dot{w} + \Gamma^0_{11}\dot{u}^2 + 2\Gamma^0_{12}\dot{u}\dot{v} + \Gamma^0_{22}\dot{v}^2 + \Gamma^0_{33}\dot{w}^2 \quad (71)$$

and

$$a_R^1 = \ddot{u} + c^2\Gamma_{00}^1\dot{t}^2 + 2c\Gamma_{01}^1\dot{t}\dot{u} + 2c\Gamma_{02}^1\dot{t}\dot{u} + \Gamma_{11}^1\dot{u}^2 + 2\Gamma_{12}^1\dot{u}\dot{v} + 2\Gamma_{13}^1\dot{u}\dot{w} + \Gamma_{22}^1\dot{v}^2 + 2\Gamma_{23}^1\dot{v}\dot{w} + \Gamma_{33}^1\dot{w}^2 \quad (72)$$

and

$$a_R^2 = \ddot{v} + c^2\Gamma_{00}^2\dot{t}^2 + 2c\Gamma_{01}^2\dot{t}\dot{u} + 2c\Gamma_{02}^2\dot{t}\dot{v} + \Gamma_{11}^2\dot{u}^2 + 2\Gamma_{12}^2\dot{u}\dot{v} + 2\Gamma_{13}^2\dot{u}\dot{w} + \Gamma_{22}^2\dot{v}^2 + 2\Gamma_{23}^2\dot{v}\dot{w} + \Gamma_{33}^2\dot{w}^2 \quad (73)$$

and

$$a_R^3 = \ddot{w} + c^2\Gamma_{00}^3\dot{t}^2 + 2c\Gamma_{03}^3\dot{t}\dot{w} + +\Gamma_{11}^3\dot{u}^2 + 2\Gamma_{12}^3\dot{u}\dot{v} + 2\Gamma_{13}^3\dot{u}\dot{w} + \Gamma_{22}^3\dot{v}^2 \quad (74)$$

Wherein Einstein coordinate coordinates of space-time in Rotational Oblate Spheroidal coordinates:

$$x^1 = u; x^2 = v;\ x^3 = u;\ x^o = ct \quad (75)$$

Equation (71) – (73) is called the Great Riemann Linear Acceleration Tensor in Rotational Oblate Spheroidal coordinates.

Hence, the Great Riemannian Acceleration Vector $\underline{a}_R$ is defined as:

$$\underline{a}_R = [(a_R)_u, (a_R)_v, (a_R)_w, (a_R)_{x^o}\ ] \quad (76)$$

where

$$(a_R)_{x^o} = (g_{00})^{\frac{1}{2}} a_R^0 \quad (77)$$

$$(a_R)_u = (g_{11})^{\frac{1}{2}} a_R^1 \quad (78)$$

$$(a_R)_v = (g_{22})^{\frac{1}{2}} a_R^2 \quad (79)$$

and

$$(a_R)_w = (g_{33})^{\frac{1}{2}} a_R^3 \quad (80)$$

Equation (77) – (80) is the Great Riemannian acceleration vector for all gravitational fields in nature in Rotational Oblate Spheroidal coordinates.

3:0 Results and discussions

In this paper, we have derived the components of the Great Riemannian Linear velocity tensor/vector and the Great Riemannian linear acceleration tensor/vector in Rotational Oblate Spheroidal Coordinates as (71) – (73) and (77) – (80) respectively. These results obtained in this paper are necessary and sufficient for expressing all Riemannian mechanical quantities in all gravitational fields in nature (Riemannian Linear Momentum, Riemannian Kinetic Energy, Riemannian Lagragian and Riemannian Hamiltonian) in terms of Rotational Oblate Spheroidal coordinates.

4:0 Conclusions

The Great Riemannian velocity vector (71) – (73) and the Great Riemannian Linear Acceleration vector (77) – (80) obtained in this paper pave a way for expressing all Riemannian Dynamical laws of motion (Newton's law, Lagrange's law, Hamilton's law, Einstein Special Relativistic law of motion and Schrödinger's law of quantum mechanics) entirely in terms of Rotational Oblate Spheroidal coordinates.

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