riemannian velocity and acceleration tensors/vectors in rotational oblate spheroidal coordinates based upon the great metric tensor

Abstract

Since the time of Galileo (1564 - 1643), Euclidean geometry has been the foundation on which the theoretical formulations of all geometrical quantities in all orthogonal curvilinear coordinates in Physics and Mathematics were built. But with the discovery of the great metric tensor in spherical polar coordinates (r, θ, ϕ, x^0) in all gravitational fields in nature[5] has made Riemannian geometry to be opened up for exploration and exploitation and hence its application in theoretical physics and mathematics. In this paper, we derive the Riemannian vector and acceleration tensor/vectors in Rotational Oblate Spheroidal coordinates for application in physics and other related fields.

Keywords: Riemannian geometry, great metric tensor, Riemannian velocity and acceleration and Rotational Oblate Spheroidal coordinates.

1:0 Introductions

Before now, we had derived the velocity and acceleration in some orthogonal curvilinear coordinates [1, 2, 3, 4] based upon the old and well known Euclidean geometry. However, when the Riemannian geometry was discovered in 1854, it was accepted to be of higher potential and more powerful than the Euclidean counterpart; only that it was founded on an unknown metric tensor, so that it exploration and consequent exploitation were not possible. Now, with discovery of the great metric tensor [5], by Prof. S.X.K Howusu, Riemannian geometry is no longer without foundation on which it can be built upon. Following the introduction of this new metric tensor we had formulated some Riemannian geometrical quantities in Cartesian Coordinates [6] and some orthogonal curvilinear coordinates [7,8]. In this paper, we are out again to generate the Riemannian velocity and acceleration tensor/vector in Rotational Oblate Spheroidal coordinates for application in physics and mathematics.

2:0THEORY

The Rotational Oblate Spheriodal Coordinates (u, v, w) can be expressed in terms of Cartesian coordinates (x, y, z) as [5]:

$$x = w(u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}} \tag{1}$$

$$y = (u^2 + d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}\,(1 - w^2)^{\frac{1}{2}} \tag{2}$$

$$y = uv \tag{3}$$

The great metric tensor for all gravitational fields in nature in spherical polar coordinates (r, θ, ϕ, x^0) is given as [5]:

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right)$$

(4)

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1}$$

(5)

$$g_{22} = r^2$$

(6)

$$g_{33} = r^2 \sin^2 \theta$$

(7)

$$g_{uv} = 0\,; Otherwise$$

(8)

The Rotational Oblate Spheroidal coordinates are related to the Spherical polar coordinates as:

$$r = [u^2 + d^2(1 - v^2)]^{\frac{1}{2}} \tag{9}$$

$$\theta = \cos^{-1}\left\{\frac{uv}{\left[u^2 + d^2(1 - v^2)^{\frac{1}{2}}\right]}\right\} \tag{10}$$

and

$$\phi = \tan^{-1}\left[\frac{(1-w^2)^{\frac{1}{2}}}{w}\right] \quad (11)$$

From the well know transformation equation given by the covariant tensor [10] and consequently, upon transformation by using (4)-(11) we obtained the Riemannian metric tensor for all gravitational fields in Rotational Oblate Spheroidal coordinates as:

$$g_{00} = -\left(1+\frac{2}{c^2}f\right)$$

(12)

$$g_{11} = \frac{u^2+v^2d^2}{u^2+d^2} + \frac{u^2}{u^2+d^2(1-v^2)}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n \quad (13)$$

$$g_{12} = \frac{-uvd^2}{u^2+d^2(1-v^2)}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n \quad (14)$$

$$g_{22} = \frac{u^2+v^2d^2}{(1-v^2)} + \frac{v^2d^4}{u^2+d^2(1-v^2)}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n \quad (15)$$

$$g_{33} = (u^2+d^2)(1-v^2) \quad (16)$$

$$g_{\mu v} = 0\,;\, Otherwise \quad (17)$$

It may be noted that the determinant of the metric tensor $g_{\mu v}$, denoted by g is obtained as:

$$g = -(u^2 + v^2 d^2)^2 \quad (18)$$

Also, the contra variant metric tensor for this Riemannian metric tensor denoted as $g^{\mu v}$ is given as:

$$g^{00} = -\left(1 + \frac{2}{c^2} f\right)^{-1} \quad (19)$$

$$g^{11} = \frac{u^2 + v^2 d^2}{u^2 + d^2}$$

$$\left\{1 + \frac{(1 - v^2) v^2 d^4}{(u^2 + v^2 d^2)[u^2 + d^2(1 - v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \quad (20)$$

$$g^{12} = \left\{\frac{uvd^2(1 - v^2)(u^2 + d^2)}{(u^2 + v^2 d^2)[u^2 + d^2(1 - v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \quad (21)$$

$$g^{22} = \frac{(1 - v^2)}{u^2 + v^2 d^2}$$

$$\left\{1 + \frac{u^2(u^2 + d^2)}{(u^2 + v^2 d^2)[u^2 + d^2(1 - v^2)]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \right\} \left(1 + \frac{2}{c^2} f\right) \quad (22)$$

$$g^{33} = [(u^2 + d^2)(1 - v^2)]^{-1} \tag{23}$$

$$g^{\mu v} = 0; otherwise \tag{24}$$

These metric tensors define the Riemannian line element, Riemannian volume element, Riemannian gradient operator, Riemannian divergence, Riemannian curl and Riemannian Laplacian in Rotational Oblate Spheroidal coordinates, according to the Theory of Tensor and Vector Analysis [9]. These quantities are necessary and sufficient for derivation of fields in all Rotational Oblate Spheroidal distribution of mass, charge and current. Now for the derivation of the equation of motion for test particles in all gravitational fields, we shall derive the expression for Riemannian velocity and acceleration in Rotational Oblate Spheroidal coordinates.

2:1 Great Riemannian Velocity Tensor/Vector in Rotational Oblate Spheroidal Coordinates

According to the theory of tensor analysis, the linear velocity in four-dimensional space – time, u^α is given in all gravitational fields in all orthogonal curvilinear coordinates x^α by [Spiegel, 1974]:

$$u^\alpha = \frac{d}{d\tau} x^\alpha = \dot{x}^\alpha \tag{25}$$

Where τ is proper time and a dot denotes one differentiation with respect to time in Einstein Cartesian coordinate(x, y, z, x^0), u^0, $u^1, u^2 and\, u^3$ are given as:

$$u^0 = \dot{x}^0 = c\dot{t} \tag{26}$$

$$u^1 = \dot{x}^1 = \dot{u} \tag{27}$$

$$u^2 = \dot{x}^2 = \dot{v} \tag{28}$$

and

$$u^3 = \dot{x}^3 = \dot{w} \tag{29}$$

It may be noted that in Minkwoski Cartesian coordinates, x^0 is given as:

$$u^0 = ic\dot{t} \tag{30}$$

The Great Riemannian Linear velocity tensor according to the theory of Tensor Analysis, the coordinates(u, v, w, x^0) is given as [9]:

$$\underline{U_R} = [U_u, U_v\, U_w, U_{x^0}] \tag{31}$$

where

$$(\underline{U_R})_0 = -c\left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}}\dot{t} \tag{32}$$

$$(\underline{U_R})_1 = \left[\frac{u^2 + v^2d^2}{u^2 + d^2} + \frac{u^2}{u^2 + d^2(1 - v^2)}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n\right]^{\frac{1}{2}}\dot{u} \tag{33}$$

$$(\underline{U_R})_2 = \left[\frac{u^2 + v^2d^2}{(1 - v^2)} + \frac{v^2d^4}{u^2 + d^2(1 - v^2)}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n\right]^{\frac{1}{2}}\dot{v} \tag{34}$$

and

$$(\underline{U_R})_3 = [u^2 + d^2(1 - v^2)]^{\frac{1}{2}}\dot{w} \tag{35}$$

This is the great Riemannian velocity vector in Rotational Oblate Spheroidal coordinates.

2:2 Great Riemannian Acceleration Tensor/Vector in Rotational Oblate Spheroidal Coordinates

Following the development of Great Riemannian velocity tensor/vectors, the Riemannian linear acceleration tensor in 4-dimensional space – time, $a_{R,}^{\alpha}$ in gravitational fields in nature and all orthogonal curvilinear coordinates x^{α} is b y theory of tensor analysis as [9]:

$$a_R^{\alpha} = \ddot{x}^{\alpha} + \Gamma_{\mu\nu}^{\alpha}\dot{x}^{\mu}\dot{x}^{\nu} \quad (36)$$

Where $\Gamma_{\mu\nu}^{\alpha}$ is the Christoffel symbol of the second kind (or Coefficient of affine connection) Pseudo tensor and a dot denotes one differentiation with respect to proper timeτ. The non-zero results of $\Gamma_{\mu\nu}^{\alpha}$ based upon the great metric tensor in Rotational Oblates Spheroidal coordinates are given as:

$$\Gamma_{00}^{0} = \frac{1}{2}g^{00}g_{00,0} \quad (37)$$

$$\Gamma_{01}^{0} = \frac{1}{2}g^{00}g_{00,1} \quad (38)$$

$$\Gamma_{02}^{0} = \frac{1}{2}g^{00}g_{00,2} \quad (39)$$

$$\Gamma_{03}^{0} = \frac{1}{2}g^{00}g_{00,3} \quad (40)$$

$$\Gamma_{11}^{0} = -\frac{1}{2}g^{00}g_{11,0} \quad (41)$$

$$\Gamma_{12}^{0} = -\frac{1}{2}g^{00}g_{12,0} \quad (42)$$

$$\Gamma_{22}^{0} = -\frac{1}{2}g^{00}g_{22,0} \quad (43)$$

$$\Gamma_{33}^{0} = -\frac{1}{2}g^{00}g_{33,0} \quad (44)$$

$$\Gamma^1_{00} = -\frac{1}{2}g^{11}g_{00,1} - \frac{1}{2}g^{12}g_{00,2} \tag{45}$$

$$\Gamma^1_{01} = -\frac{1}{2}g^{11}g_{11,0} - \frac{1}{2}g^{12}g_{12,0} \tag{46}$$

$$\Gamma^1_{02} = -\frac{1}{2}g^{11}g_{21,0} + \frac{1}{2}g^{12}g_{22,0} \tag{47}$$

$$\Gamma^1_{11} = -\frac{1}{2}g^{11}g_{11,1} - \frac{1}{2}g^{12}g_{11,2} + g^{12}g_{12,1} \tag{48}$$

$$\Gamma^1_{12} = \frac{1}{2}g^{11}g_{11,2} + \frac{1}{2}g^{12}g_{22,1} \tag{49}$$

$$\Gamma^1_{13} = \frac{1}{2}g^{11}g_{11,3} + \frac{1}{2}g^{12}g_{12,3} \tag{50}$$

$$\Gamma^1_{22} = g^{11}g_{21,2} - \frac{1}{2}g^{11}g_{22,1} + \frac{1}{2}g^{12}g_{22,2} \tag{51}$$

$$\Gamma^1_{23} = \frac{1}{2}g^{11}g_{21,3} + \frac{1}{2}g^{12}g_{22,3} \tag{52}$$

$$\Gamma^1_{33} = -\frac{1}{2}g^{11}g_{33,1} - \frac{1}{2}g^{12}g_{33,2} \tag{53}$$

$$\Gamma^2_{00} = -\frac{1}{2}g^{21}g_{11,0} - \frac{1}{2}g^{22}g_{12,0} \tag{54}$$

$$\Gamma^2_{01} = \frac{1}{2}g^{21}g_{11,0} + \frac{1}{2}g^{22}g_{12,0} \tag{55}$$

$$\Gamma^2_{02} = \frac{1}{2}g^{21}g_{21,0} + \frac{1}{2}g^{22}g_{22,0} \tag{56}$$

$$\Gamma^2_{11} = \frac{1}{2}g^{21}g_{11,1} + g^{22}g_{12,1} - \frac{1}{2}g^{22}g_{11,2} \tag{57}$$

$$\Gamma^2_{12} = \frac{1}{2}g^{21}g_{11,2} + \frac{1}{2}g^{22}g_{22,1} \tag{58}$$

$$\Gamma^2_{13} = \frac{1}{2}g^{21}g_{11,3} + \frac{1}{2}g^{22}g_{12,3} \tag{59}$$

$$\Gamma^2_{22} = g^{21}g_{21,2} - \frac{1}{2}g^{21}g_{22,1} + \frac{1}{2}g^{22}g_{22,2} \tag{60}$$

$$\Gamma^2_{23} = \frac{1}{2}g^{21}g_{21,3} + \frac{1}{2}g^{22}g_{22,3} \tag{61}$$

$$\Gamma^2_{33} = -\frac{1}{2}g^{21}g_{33,1} - \frac{1}{2}g^{22}, g_{33,2} \tag{62}$$

$$\Gamma^3_{00} = -\frac{1}{2}g^{33}g_{00,3} \tag{63}$$

$$\Gamma^3_{11} = -\frac{1}{2}g^{33}g_{11,3} \tag{64}$$

$$\Gamma^3_{12} = -\frac{1}{2}g^{33}g_{12,3} \tag{65}$$

$$\Gamma^3_{13} = \frac{1}{2}g^{33}g_{33,1} \tag{66}$$

$$\Gamma^3_{22} = -\frac{1}{2}g^{33}g_{22,3} \tag{67}$$

$$\Gamma^3_{23} = \frac{1}{2}g^{33}g_{33,2} \tag{68}$$

$$\Gamma^3_{33} = \frac{1}{2}g^{33}g_{33,3} \tag{69}$$

$$\Gamma^\alpha_{\mu\nu} = 0; otherwise \tag{70}$$

It follows from equation (36) – (70) that:

$$a_R^0 = c\ddot{t} + c^2\Gamma_{00}^0\dot{t}^2 + 2c\Gamma_{01}^0\dot{t}\dot{u} + 2c\Gamma_{02}^0\dot{t}\dot{u} + 2c\Gamma_{02}^0\dot{t}\dot{v} + 2c\Gamma_{03}^0\dot{t}\dot{w} + \Gamma_{11}^0\dot{u}^2 + 2\Gamma_{12}^0\dot{u}\dot{v} + \Gamma_{22}^0\dot{v}^2 + \Gamma_{33}^0\dot{w}^2 \quad (71)$$

and

$$a_R^1 = \ddot{u} + c^2\Gamma_{00}^1\dot{t}^2 + 2c\Gamma_{01}^1\dot{t}\dot{u} + 2c\Gamma_{02}^1\dot{t}\dot{u} + \Gamma_{11}^1\dot{u}^2 + 2\Gamma_{12}^1\dot{u}\dot{v} + 2\Gamma_{13}^1\dot{u}\dot{w} + \Gamma_{22}^1\dot{v}^2 + 2\Gamma_{23}^1\dot{v}\dot{w} + \Gamma_{33}^1\dot{w}^2 \quad (72)$$

and

$$a_R^2 = \ddot{v} + c^2\Gamma_{00}^2\dot{t}^2 + 2c\Gamma_{01}^2\dot{t}\dot{u} + 2c\Gamma_{02}^2\dot{t}\dot{v} + \Gamma_{11}^2\dot{u}^2 + 2\Gamma_{12}^2\dot{u}\dot{v} + 2\Gamma_{13}^2\dot{u}\dot{w} + \Gamma_{22}^2\dot{v}^2 + 2\Gamma_{23}^2\dot{v}\dot{w} + \Gamma_{33}^2\dot{w}^2 \quad (73)$$

and

$$a_R^3 = \ddot{w} + c^2\Gamma_{00}^3\dot{t}^2 + 2c\Gamma_{03}^3\dot{t}\dot{w} + +\Gamma_{11}^3\dot{u}^2 + 2\Gamma_{12}^3\dot{u}\dot{v} + 2\Gamma_{13}^3\dot{u}\dot{w} + \Gamma_{22}^3\dot{v}^2 \quad (74)$$

Wherein Einstein coordinate coordinates of space-time in Rotational Oblate Spheroidal coordinates:

$$x^1 = u; x^2 = v;\ x^3 = u;\ x^o = ct \quad (75)$$

Equation (71) – (73) is called the Great Riemann Linear Acceleration Tensor in Rotational Oblate Spheroidal coordinates.

Hence, the Great Riemannian Acceleration Vector $\underline{a}_R$ is defined as:

$$\underline{a}_R = [(a_R)_u, (a_R)_v, (a_R)_w, (a_R)_{x^o}\ ] \quad (76)$$

where

$$(a_R)_{x^o} = (g_{00})^{\frac{1}{2}} a_R^0 \quad (77)$$

$$(a_R)_u = (g_{11})^{\frac{1}{2}} a_R^1 \quad (78)$$

$$(a_R)_v = (g_{22})^{\frac{1}{2}} a_R^2 \quad (79)$$

and

$$(a_R)_w = (g_{33})^{\frac{1}{2}} a_R^3 \qquad (80)$$

Equation (77) – (80) is the Great Riemannian acceleration vector for all gravitational fields in nature in Rotational Oblate Spheroidal coordinates.

3:0 Results and Discussions

In this paper, we have derived the components of the Great Riemannian Linear velocity tensor/vector and the Great Riemannian linear acceleration tensor/vector in Rotational Oblate Spheroidal Coordinates as (71) – (73) and (77) – (80) respectively. These results obtained in this paper are necessary and sufficient for expressing all Riemannian mechanical quantities in all gravitational fields in nature (Riemannian Linear Momentum, Riemannian Kinetic Energy, Riemannian Lagragian and Riemannian Hamiltonian) in terms of Rotational Oblate Spheroidal coordinates.

4:0 Conclusions

The Great Riemannian velocity vector (71) – (73) and the Great Riemannian Linear Acceleration vector (77) – (80) obtained in this paper pave a way for expressing all Riemannian Dynamical laws of motion (Newton's law, Lagrange's law, Hamilton's law, Einstein Special Relativistic law of motion and Schrödinger's law of quantum mechanics) entirely in terms of Rotational Oblate Spheroidal coordinates.

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