Dissipative heat transfer of micropolar hydromagnetic variable electric conductivity fluid past inclined plate with joule heating and non-uniform heat generation

Abstract
Computational simulations of hydromagnetic dissipative heat transfer of variable electric conductivity of micropolar fluid flow and non-uniform heat absorption or generation with joule heating have been studied in this work. The flow past an inclined surface with an unvarying heat flux. The transformed dimensionless equations of the governing model are solved by Runge-Kutta algorithm coupled with shooting scheme to depict the dimensionless temperature, microrotation and velocity distributions at the border layer. The substantial bodily quantities of the flow are conferred. The results depict that the significance of the coefficient of the skin friction and the Nusselt number increases for uneven electric conductivity and non- homogeneous heat sink or source at the plate.
Keywords: Hydromagnetic; Micropolar fluid; Dissipation; Joule heating; Heat flux

1 Introduction

A micropolar fluid consists of gyratory micro-components which make fluid to display non-Newtonian actions. The formulations of micropolar fluid flow was found helpful in examining body fluids, colloidal suspensions, lubricants, liquid crystals, fluid polymeric, flows in capillaries and microchannels, preservative suspensions and flow of shear turbulent. The presumption of micropolar fluids originated by Eringen [1] have been a lively research area for many years .

Ariman et al. [2] reviewed excellently the model and applications of micropolar fluid. Hoyt and Fabulan [3] carried out experimentally analysis to show that the present of minute Stabilizers polymeric in fluids can decrease the flow effect at the wall up to 25 to 30 percent. The decrease that was described by the hypothesis of micropolar fluids as reported by Power [4], the brain fluid that is an example of body fluids fluid can be sufficiently formulated as micropolar fluids.

Flow of convective fluids containing microstructure have several applications such as dilute polymer fluids solutions, liquid crystals and different kinds of suspensions. The fluid driven by buoyancy forces occur in wide-ranging of uniform fluid flow. Free convective flow of micropolar fluids past a curved or flat surfaces has fascinated the mind of scholars from the time when the flow model was devised. Several studies have accounted and analyzed outcomes on micropolar fluids. Hayat *et al.* [5,6] examined the effect of radiation on magnetohydrodynamics convective heat and mass transfer flow. Ahmadi [7] considered micropolar boundary layer fluid flow along semi-infinite surface using similarity solution to transform the models to ordinary different equations. Khilap and Manoj [8] verified the viscous dissipation influences on MHD micropolar flow with ohmic heating, heat generation and chemical reaction.

Many convective flow are caused by heat absorption or generation which may be as a result of the fluid chemical reaction. The occurrence of heat source or sink can affect the fluid heat distribution that alters the rate of deposition of particle in the structures such as semiconductor wafers, electronic chips, nuclear reactors etc. Heat absorption or generation has been assumed to be temperature dependent heat generation and surface dependent heat generation. Rahman *et al.* [9] examined the influence of non-homogenous heat absorption or generation and variable electric conductivity on micropolar fluid. It was noticed that the surface dependent heat generation is lower compared to temperature dependent heat generation. Mabood *et al.* [10,11] examined the effect of thermal-diffusion and non-homogenous heat sink/source on radiative micropolar hydromagnetic fluid past a permeable medium.

The influence of dissipation on magnetohydrodynamic fluid and energy transfer processes has become significant industrially. Several engineering practices happen at high temperature with viscous dissipation heat transfer. Such flows have been investigated by Siva and Shanshuddin [12] reported on viscous dissipation heat and mass transfer of hydromagnetic micropolar fluid with chemical reaction. Rawat *et al.* [13] studied magnetodyrodynamic micropolar fluid of heat and mass transport in a permeable medium broadening plate, chemical reaction, heat flux and variable micro inertia. Ziaul *et al.* [14] verified steady MHD Micropolar flow fluid with joule heating, viscous dissipation and constant mass and heat fluxes. it was observed that the flow field rises initially within $0 \leq \eta \leq 1$ as the microrotation parameter value increases. Later, the flow field gradually reduces for $\eta > 1$ as the microrotation parameter rises. Also, microrotation changes from negative to positive signs in the border layer.

Following the above cited litterature, the aim is to study the viscous dissipation hydromagnetic micropolar fluid behavior heat transfer over inclined surface in permeable media with heat fluxes and joule heating for high speed fluid in non-even heat absorption/generation. The flow equations are made up of partial differential equations that can be transformed by similarity solution to nonlinear ordinary coupled differential equations. The obtained equations are simulated by Runge-Kutta technique coupled with shooting scheme. Therefore, it is obligatory to study the flow distributions, temperature and microrotation crosswise the border layer in toting up to the surface wall effect and nusselt number.

2 The Flow Mathematical Formulation

Deem convective flow of two-dimensional magnetohydrodynamic viscous, laminar, micropolar fluid through a semi-finite flat surface inclined at an angle α to the vertical. The magnetic field differs in potency as function of x which is taken in y-direction as defined $\vec{B} = (0, B(x))$. The Reynolds number is small while the exterior electric field is taken as zero. Therefore, the applied external magnetic field is high contrasted to the stimulated magnetic field. The density (ρ) of the fluid is inert $(U_\infty = 0)$ with the buoyancy forces stimulated the convective motion. The fluid viscosity μ is taken to be unvarying while the body forces and the pressure gradient are neglected.

Follow from the assumption above, the steady convective micropolar fluid flow follow the Boussinesq approximation may be explained by the subsequent equations and geometry.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \left(\frac{\mu + r}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{r}{\rho}\frac{\partial m}{\partial y} + g_0\beta(T - T_\infty)cos\alpha - \frac{\sigma(B(x))^2}{\rho}u - \frac{\nu}{K}u \tag{2}$$

$$u\frac{\partial m}{\partial x} + v\frac{\partial m}{\partial y} = \frac{b}{\rho j}\frac{\partial^2 m}{\partial y^2} - \frac{r}{\rho j}\left(2m + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \left(\frac{\mu + r}{\rho c_p}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma(B(x))^2}{\rho c_p}u^2 + \frac{q^{'''}}{\rho c_p} \tag{4}$$

Subject to boundary conditions

$$\begin{aligned} &u = 0, v = 0, m = -a\frac{\partial u}{\partial y}, \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad at \quad y = 0 \\ &u = U_\infty = 0, m = 0, T = T_\infty \quad as \quad y \longrightarrow \infty \end{aligned} \tag{5}$$

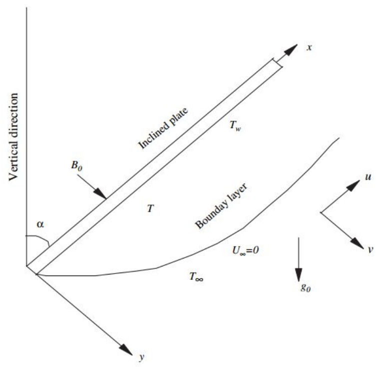

Figure 1 : Geometry of the flow

where u and v are the velocity in x and y coordinates respectively, K is the permeability of the porous medium, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, μ is the dynamic viscosity, ρ is the fluid density, m is the microrotaion in x and y components, $b = (\mu + \frac{r}{2})j$ is the micropolar viscosity, j is the micro-inertial per unit mass which is assumed to be constant, r is the microrotation coefficient, k is the thermal conductivity, T is the fluid temperature, c_p is the specific heat at constant pressure, g is the acceleration due to gravity and β is the thermal expansion coefficient.

A linear correlation involving the surface shear $\frac{\partial u}{\partial y}$ and microrotation function m is picked for studying the influence of vary surface circumstances for microroation. When the microroation term $a = 0$ implies $m = 0$, that is the microelement at the wall are not swiveling but when $a = 0.5$ implies varnishing of the anti-symmetric module of the stress tensor that stand for feeble concentration. This confirm that for a fine particle suspension at the wall, the particle swivel is the same as the fluid velocity but when $a = 1$ shows the turbulent boundary layer flows.
The non-uniform heat absorption or generation is represented as

$$q^{'''} = \frac{kU_0}{2\nu x}\left[\lambda(T - T_\infty) + \lambda^*(T_w - T_\infty)e^{-\eta}\right] \tag{6}$$

where λ^* and λ represent the heat sink/source temperature dependent and space coefficients respectively while T_∞ is the free stream temperature. Here, $\lambda > 0$ and $\lambda^* > 0$ stand for to heat source but $\lambda < 0$ and $\lambda^* < 0$ represent to heat sink.

In this study, it is assumed that the introduced magnetic field strength $B(x)$ is capricious and it is represented as $B(x) = \frac{B_0}{\sqrt{x}}$, where B_0 is constant. Also, the electrical conductivity σ depends on the fluid velocity and is defined as $\sigma = \sigma_0 u$, where σ_0 is constant see [15].
Introducing the following dimensionless variables

$$\eta = y\sqrt{\frac{U_0}{2\nu x}}, \psi = \sqrt{2\nu U_0 x} f(\eta), m = \sqrt{\frac{U_0^3}{2\nu x}} h(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

where U_0 is the reference velocity, ψ is the stream function and $T_w - T_\infty = \frac{q_w}{k\sqrt{\frac{2\nu x}{U_0}}}$

Since $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ then,

$$u = U_0 f'(\eta), \qquad v = -\sqrt{\frac{\nu U_0}{2x}}(f(\eta) - \eta f'(\eta)) \tag{8}$$

Using equations (7) and (8) along with the variable magnetic field, fluid velocity dependent electric conductivity and equation (6) on equation (1)-(5) to obtain,

$$(1+\delta)f''' + ff'' + \delta h' + G_r \theta cos\alpha - M f'^2 - \beta f' = 0 \tag{9}$$

$$\epsilon h'' - 2\delta(2h + f'')) + \epsilon(f'h + g'h) = 0 \tag{10}$$

$$\theta'' + (1+\delta)P_r E_c f''^2 + M P_r E_c f'^3 + P_r(f\theta' - \theta f') + (\lambda\theta + \lambda^* e^{-\eta}) = 0 \tag{11}$$

The boundary conditions becomes

$$\begin{aligned} &f = f' = 0, h = -af'', \theta' = -1 \quad at \quad \eta = 0 \\ &f' = 0, h = 0, \theta = 0 \quad as \quad \eta \longrightarrow \infty \end{aligned} \tag{12}$$

where $\beta = \frac{2xv}{U_0 K}$ is the permeability parameter, $\delta = \frac{r}{\mu}$ is the vortex viscosity term, $M = \frac{2\sigma_0 B_0^2}{\rho}$ is the magnetic field term, $\epsilon = \frac{jU_0}{\nu x}$ is the micro-inertia density term, $E_c = \frac{U_0^2}{(T_w - T_\infty)c_p}$ is the Eckert number, $G_r = \frac{2gx\beta(T_w - T_\infty)}{U_0^2}$ is the thermal Grasof number and $P_r = \frac{\mu c_p}{k}$ is the Prandtl number.

The substantial quantities of engineering concern for this flow are the local skin friction C_f and Nusselt number N_u given as:

$$C_f = \frac{2\tau_w}{\rho u_w^2}, \quad Nu = \frac{xq_w}{k(T_w - T_\infty)} \tag{13}$$

τ_w and q_w are respectively taken as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{14}$$

Hence,

$$C_f = (2Re_x^{-1})^{\frac{1}{2}} \left[1 + (1-a)\delta\right] f''(0), \qquad Nu_x = (2^{-1}Re_x)^{\frac{1}{2}} \frac{1}{\theta(0)} \tag{15}$$

The computational values for C_f and Nu_x are obtained from the equations (15)

Table 1: Comparison of $f''(0)$ for $\delta = \epsilon = G_r = \beta = E_c = 0$, various values of a

a	Cortell [16]	Hayat *et al.* [17]	Rahman *et al.* [9]	Present results
0.0	0.627547	0.627555	0.627498	0.623534
0.2	0.766758	0.766837	0.767066	0.766831
0.5	0.889477	0.889544	0.892366	0.891784
0.75	0.953786	0.953975	0.956365	0.954523
1.0	1.000000	1.000000	1.002125	1.001986

Table 2: Values of $f''(0)$ and $\theta'(0)$ for different values of M, E_c, λ, λ^*, β, and δ on (PP-Physical Parameters)

PP	values	$f''(0)$	$\theta'(0)$	PP	*values*	$f''(0)$	$\theta'(0)$
M	1	1.13057	6.51766	δ	0.1	1.22091	5.05342
	3	1.17476	7.95342		0.5	1.26975	5.56323
	5	1.22174	9.24167		1.5	1.01562	6.61431
	7	1.26775	10.44839		3.0	0.82392	7.90954
E_c	0.2	1.12160	6.11801	β	0.007	1.12076	5.83162
	0.5	1.18642	6.55818		0.1	1.12092	5.96995
	0.7	1.23454	6.89076		0.3	1.12274	6.26536
	1.0	1.31555	7.46070		0.5	1.12617	6.55794
λ	0.0	0.51587	2.76551	λ^*	0.0	1.02366	5.49423
	0.1	0.59847	3.17923		0.5	1.12160	6.11801
	0.3	0.81802	4.34817		1.0	1.20955	6.69142
	0.5	1.12160	6.11801		1.5	1.29016	7.22734

3 Results and Discussion

The computational results for the nonlinear coupled differential equation are obtainable for the dimensionless microrotation, temperature and velocity distributions. In this study, the following default parameter values are chosen: $a = 0.5, P_r = 0.73, G_r = 2.5, \beta = 0.2, \epsilon = 2, E_c = 0.2, M = 0.5, \alpha = 30^0, \delta = 1, \lambda = 0.5$ and $\lambda^* = 0.5$. The values are based on the choice of existing studies because of unavailability of investigational figures for vortex viscosity and micro-inertia density parameters, suitable figures are selected to verify the polar effect on flow properties.

Table 1 represents the computational results, that show the behavior of microrotation parameter a on the fluid flow aspects of the current outcome compared with the presented outcome. The comparison are found to be in an excellent agreement as shown in the tables.

Table 2 shows the simulated results, that illustrate the effect of some fluid properties on the heat and fluid flow aspect of the investigation. It is evidence from the table that a rise in the values of parameters M, E_c, λ, λ^* and β increases the skin friction and the temperature gradient at the wall due to an enhancement in the thermal and flow boundary layer thickness. Also, a rise in values of δ causes shrinking in the flow effect at the wall but enhances the thermal gradient at the wall.

Fig. 2-4 depict the velocity, microrotation and temperature distributions for distinction magnetic field term M values. It is noticed from Fig 2 that the flow decreases as the values of M increases due to the effect of Lorentz forces that retard the convective fluid motion. Fig 3, confirmed that as the values of the magnetic field term rises the microrotation near the plate increases but decreases as it moves distance from the plate. Fig 4 reveals the temperature field increases as the values of M rises, this is because the magnetic field liable to reduce the flow field which reduces heat transfer. The results show that the magnetic field may be use to manage the flow fluid and heat transport properties.

Fig 5-7 represent the consequent of different Eckert number E_c on the flow, microrotation and temperature. It is noticed that the flow velocity, microrotaion and temperature field increases with a rise in the values of E_c. This causes enhancement in the momentum, micropolar and temperature border layer thickness and thereby causes a rise in the profiles

Fig 8 and 9 illustrate the produces of the vortex viscosity δ on the microrotation and flow velocity field. It can be observed that a rise in the values of δ initial decreases the fluid velocity and microrotation profiles close to the plate at $\eta \leq 2$ and later increases as it moves distance away from the plate in the microrotation and velocity flow field.

Fig 10 and 11 illustrate the influence of porosity term β on the dimensionless velocity and microrotation distributions. It is evidenced that the flow and microrotation field decreases as the porosity parameter rises, this is as a result of the wall of the plate that gives an additional opposition to the flow mechanism by influencing the fluid to move at a decelerated rate.

Fig. 12-15 represent the microrotation and temperature distributions for vary values of temperature dependent and surface temperature heat absorption or generation terms λ and λ^*. Fig 12 and 14 show that a rise in the values of λ and λ^* reduces the microrotation close to the plate and increases as it moves far away from the plate. Also, in Fig 13 and 15 the temperature profile increases as the parameters values λ and λ^* increases. This is due to a rise in the temperature boundary layer.

Conclusions

The numerical simulations was carried out for dimensionless boundary layer equations of convective heat transfer in hydromagnetic joule heating, micropolar fluid past heated inclined surface with non-uniform heat sink/source and variable electric conductivity. It was observed from the study that, the magnetic field decreases the flow rate and angular velocity but increases the heat transfer phenomenon while the temperature dependent and surface temperature heat absorption or generation terms as well as viscous dissipation parameter increases the flow, angular velocity and temperature distributions. The vortex viscosity parameter decreases the flow and microrotation profiles near the wall but later increases the profiles distance away from the wall while porosity resist the flow and microrotation of the fluid by decreasing the profiles.

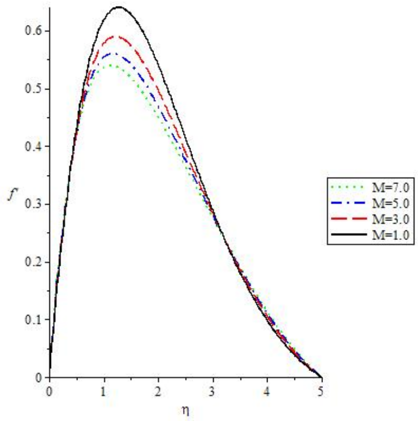

Figure 2: Velocity distributions for different values of M

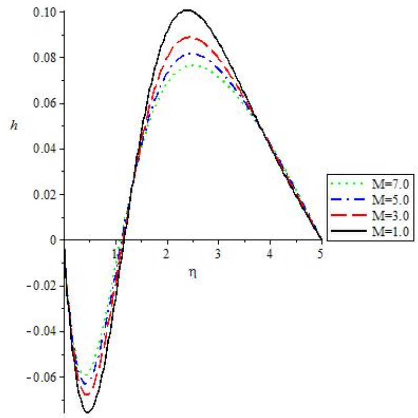

Figure 3: Microrotation distributions for different values of M

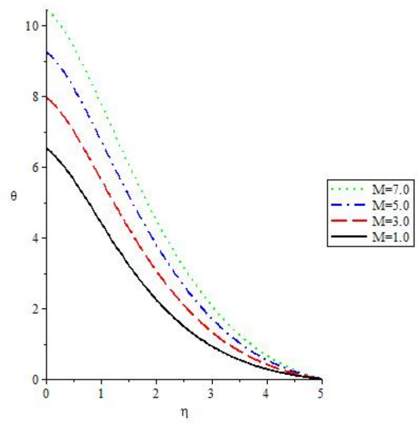

Figure 4: Temperature distributions for different values of M

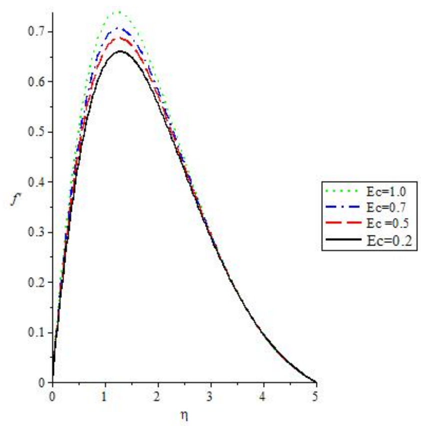

Figure 5: Velocity distributions for different values of E_c

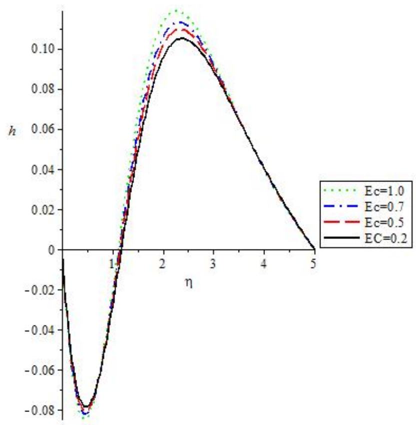

Figure 6: Microrotation distributions for different values of E_c

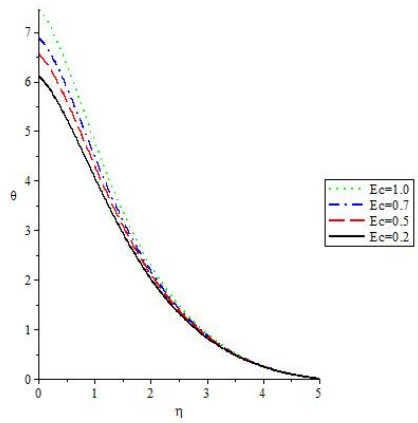

Figure 7: Temperature distributions for different values of E_c

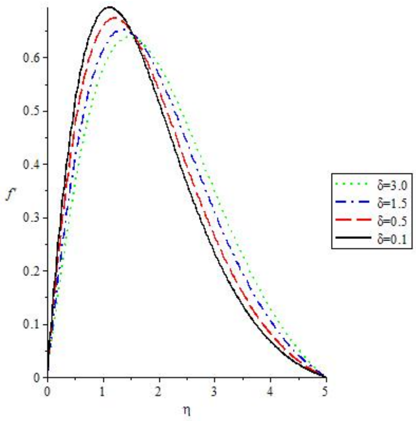

Figure 8: Velocity distributions for different values of δ

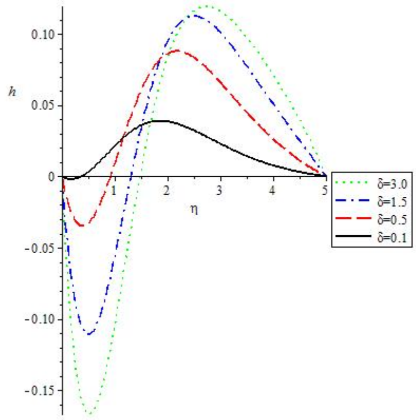

Figure 9: Microrotation distributions for different values of δ

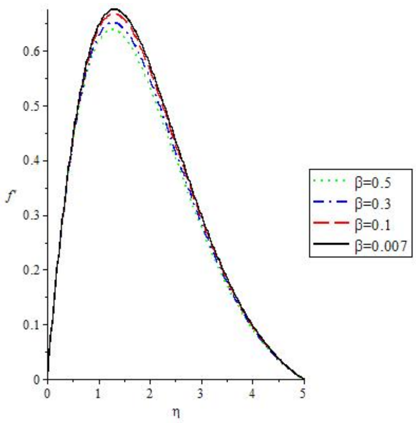

Figure 10: Velocity distributions for different values of β

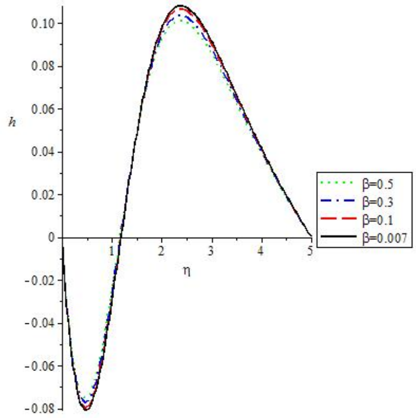

Figure 11: Microrotation distributions for different values of β

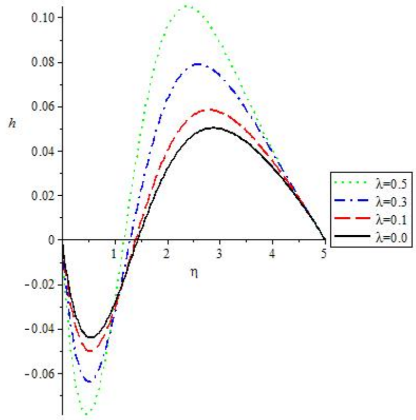

Figure 12: Microrotation distributions for different values of λ

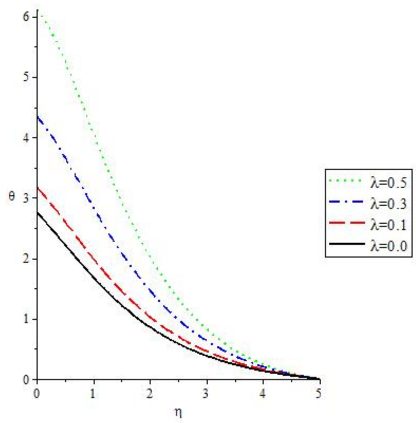

Figure 13: Temperature distributions for different values of λ

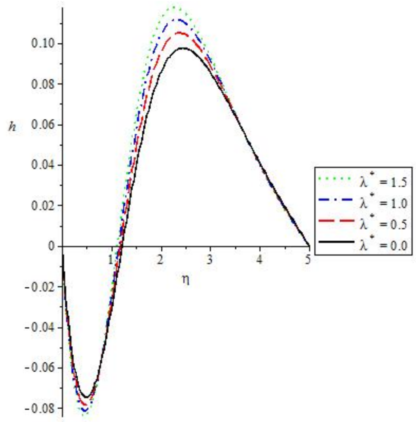

Figure 14: Microrotation distributions for different values of λ^*

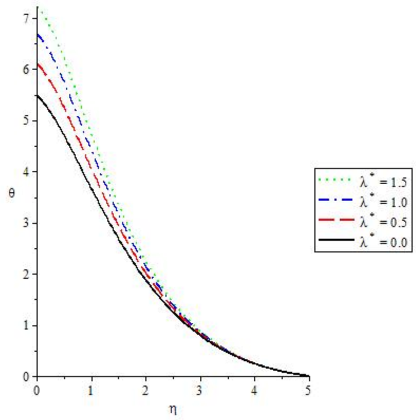

Figure 15: Temperature distributions for different values of λ^*

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