

The numerical study of effects of Soret, Dufour and viscous dissipation parameters on steady MHD Casson fluid flow through non-Darcy porous media

Abstract: In the present paper the numerical study of effects of Soret, Dufour and viscous dissipation parameters on steady MHD Casson fluid flow through non-Darcy porous medium is explored. By suitable similarity transformations, the governing boundary layer equations are transformed to ordinary differential equations. The method, the numerical computation with bvp4c, a MATLAB program, is applied to solve these equations. The effects of magnetic parameter, Soret parameter, Dufour parameter, Prandtl number, Schmidt number, Eckert number, and Casson parameter on velocity, heat transfer, and concentration profiles, Skin- frictions, local Nusselt number and local Sherwood number are computed and discussed numerically and presented through tables and graphs.

Keywords: Magnetohydrodynamics, Casson fluid, magnetic parameter, Soret number, Dufour number.

1. Introduction

Many natural, industrial as well as biological fluids (such as mud, condensed milk, glues, lubricating greases, paints, sugar solution, shampoos and tomato paste, polymers, liquid detergents, blood, fruit juices etc.) change their viscosity or flow behaviour under stress; and thus deviate from the classical Newton's law of viscosity. Different models of non-Newtonian fluids based on their diverse flow behaviours have been proposed by the researchers.

The rheological model was introduced originally by Casson [1] in his research on a flow equation for pigment oil-suspension of printing ink. Bird et al. [2] investigated the rheology and flow of viscoplastic materials. He reported that Casson model constitutes a plastic fluid model which exhibits shear thinning characteristics, yield stress, and high shear viscosity. Casson fluid behaves as solid when the shear stress is less than the yield stress and it starts to deform when shear stress becomes greater than the yield stress.

The fundamental analysis of the flow field of non-Newtonian fluids in a boundary layer adjacent to a stretching sheet or an extended surface is very important and is an essential part in the study of fluid dynamics, and heat and mass transfer.

Sakiadis [3] studied boundary layer behaviour on continuous solid surfaces: II. The boundary layer on continuous flat surface. Crane [4] studied the flow past a stretching plane. Nield et al.[5] studied convection in porous media.

Mukhopadhyay [6] investigated Casson fluid flow and heat transfer over a nonlinearly stretching surface. Mustafa et al. [7] studied Model for the flow of Casson nanofluid past a non-linearly stretching sheet, considering magnetic field effects. Medikare et al. [8] studied MHD stagnation point flow of a Casson fluid over a nonlinearly stretching sheet with viscous dissipation.

Pramanik [9] studied Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. Raju et al. [10] studied heat and mass transfer in magnetohydrodynamic Casson fluid over an exponentially permeable stretching surface. Saidulu et al. [11] studied slip effects on MHD flow of Casson fluid over an exponentially stretching sheet in presence of thermal radiation, heat source/sink and chemical reaction .

Sharada et al. [12] studied MHD mixed convection flow of a Casson fluid over an exponentially stretching surface with the effects of Soret, Dufour, thermal radiation and chemical reaction. Mukhopadhyay et al [13] studied exact solutions for the flow of Casson fluid over a stretching surface with transpiration and heat transfer effects. Hayat et al. [14] investigated Soret and Dufour effects on magnetohydrodynamic (MHD) flow of Casson fluid.

Mahdy [15] studied heat transfer and flow of a Casson fluid due to a stretching cylinder with the Soret and Dufour effects. Animasaun [16] studied effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with suction and nth order of chemical reaction .

Ullah et al. [17] investigated Effects of slip condition and Newtonian heating on MHD flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium.

Some recent studies concerning the flow, heat and mass transfer analysis of Casson fluid can be found in Refs. [18–58].

We consider (1) non-Darcy porous medium, (2) thermo –diffusion (Dufour term) term in energy equation, (3) mass equations, (4) diffusion -thermo term (Soret term) in the mass equation and (5) nonlinear surface (6), velocity slip factor, thermal slip factor, and mass slip factor in boundary conditions of velocity, temperature, and concentration respectively . In the study of references as mentioned above, these terms, simultaneously in one problem, are not investigated with the flow over nonlinear surface.

The present work is the extension of Ullah et al. [17] work, by considering above terms. It deals with the numerical study of effects of Soret , Dufour and viscous dissipation parameters on steady MHD Casson fluid flow through non-Darcy porous medium.

2. Mathematical Formulation of the Problem

In the formulation of the problem we consider following assumptions. Casson fluid is incompressible and electrically conducting. Flow is steady, laminar and two dimensional over a nonlinearly stretching sheet. Flow region is in non-Darcy porous medium. It is under the influence of transverse magnetic field B . The sheet is stretched nonlinearly along the x-axis (i.e. $y=0$) with velocity $u_w(x) = c x^n$; origin is taken as fixed and the fluid flow is confined to $y>0$. Here c is constant and $n(n \geq 0)$ is the nonlinear stretching sheet parameter; $n=1$ represents the linear sheet case and $n \neq 1$ is for nonlinear case. The magnetic Reynolds number of the flow is taken to be small enough so that induced magnetic field is assumed to be negligible in comparison with applied magnetic field so that $B = (0, B(x), 0)$, where $B(x)$ is the applied magnetic field acting normal to the plate and varies in strength as a function of x . The flow is assumed to be in the x -direction which is taken along the plate and y -axis is normal to it. There is a constant suction/injection velocity v_w normal to the plate.

Under these assumptions the rheological equation for incompressible flow of Casson fluid is given by (Sharada et al. [12], Mukhopadhyay et al. [13])

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y / \sqrt{2\pi}) e_{ij} & \pi > \pi_c, \\ 2(\mu_B + p_y / \sqrt{2\pi_c}) e_{ij} & \pi < \pi_c. \end{cases}$$

where $\pi = e_{ij} e_{ij}$ and e_{ij} is the (i, j) – th component of the deformation rate , π is the product of the components of deformation , π_c is critical value of the product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, and p_y is the yield stress of the fluid. The viscosity and thermal conductivity of the fluid are assumed to be constant. There is thermo-diffusion effect as well as diffusion-thermo effect. The pressure gradient, body forces and Joule heating are neglected compared with the effect of viscous dissipation. The temperature and

concentration of the stretching surface are always greater than their free stream values. The flow configuration and the coordinate system are shown in Figure 1.

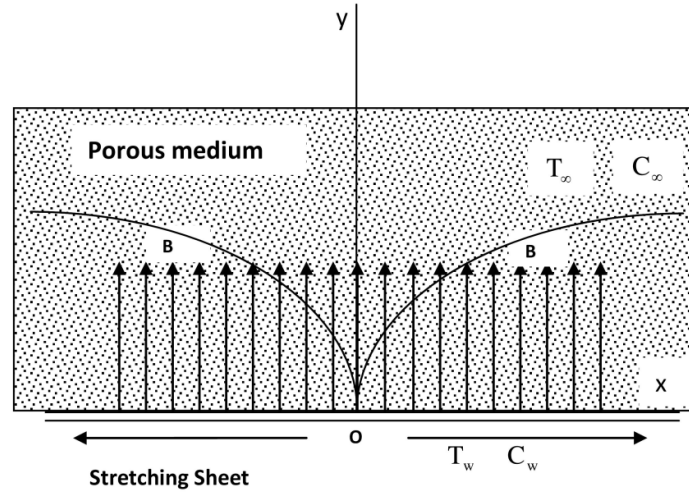


Figure 1 Physical model and coordinate system.

Under the above assumptions and using Boussinesq approximation, boundary layer equations for flow with heat and mass transfer in Casson fluid are given by the following.

The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

The momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty) - \frac{\sigma B^2(x)}{\rho} u - \frac{\nu}{K} u - \frac{b}{\sqrt{K}} u^2 \quad (2)$$

The energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \left(\frac{D_m K_T}{C_s C_p} \right) \frac{\partial^2 C}{\partial y^2} + \frac{\mu}{\rho C_p} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

The mass equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \left(\frac{D_m K_T}{T_M} \right) \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where u and v are velocity components along x and y axes, respectively, ρ is fluid density, ν is kinematic viscosity, μ is dynamic viscosity, $\beta = \mu_B \sqrt{2\pi} / p_y$ is the Casson fluid parameter, σ is the electrical conductivity of the fluid and is assumed to be constant, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, T_w is the temperature of the fluid at the stretching sheet, T is the temperature of the fluid within the boundary layer, T_∞ is the temperature of the fluid outside the boundary layer, k is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure p , C_w is the concentration of the fluid at the stretching sheet, C is the concentration of the fluid within the boundary layer, C_∞ is the concentration of the fluid outside the boundary layer, D_m is the chemical molecular diffusivity. Here, g is the acceleration due to gravity. The applied magnetic field is $B = B_0 x^{\frac{n-1}{2}}$ where B_0 is assumed to be constant.

Boundary conditions (Ullah et al. [17]):

$$\begin{aligned} \text{At } y = 0 : u &= cx^n + N_1 v \left(1 + \frac{1}{\beta} \right) \frac{\partial u}{\partial y}, \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_s (T - T_w), \quad \frac{\partial C}{\partial y} = -h_c (C - C_w) \\ \text{As } y \rightarrow \infty : u &\rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty. \end{aligned} \quad (5)$$

Here $N_1(x) = N x^{-\frac{n-1}{2}}$ denotes velocity of slip factor; it depends upon x , and $h_s(x) = h_{s0} c x^{\frac{n-1}{2}}$ represents the heat transfer parameter for Newtonian heating or temperature slip factor, and $h_c(x) = h_{c0} c x^{\frac{n-1}{2}}$ is concentration slip factor.

We consider following dimensionless variable to transform the system of equations (2), (3), (4) and (5) into a dimensionless form (Ullah et al. [17]):

135

$$\psi = \left(\frac{2\nu c}{n+1} \right)^{\frac{1}{2}} x^{\frac{n+1}{2}} f(\eta), \quad \eta = \left(\frac{c(n+1)}{2\nu} \right)^{\frac{1}{2}} x^{\frac{n-1}{2}} y, \quad \theta(\eta) = \frac{(T - T_{\infty})}{(T_w - T_{\infty})}, \quad \phi(\eta) = \frac{(C - C_{\infty})}{(C_w - C_{\infty})},$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad u = cx^n f'(\eta), \quad v = -\left(\frac{c\nu(n+1)}{2} \right)^{\frac{1}{2}} x^{\frac{n+1}{2}} \left(f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right)$$

Here η is the similarity variable. ψ is stream function. $c(c > 0)$ is a parameter related to the surface stretching speed, n is the power index related to the surface stretching speed.

Introducing these variables in the equations we get the following dimensionless forms of the equations:

$$(1 + \frac{1}{\beta}) f''' + f f'' + \frac{2n}{n+1} f'^2 + d_1 \theta + d_2 \phi - (M + (1/K_1)) f' - Fs (f')^2 = 0, \quad (7)$$

$$\frac{1}{Pr} \theta'' + f \theta' + \left(1 + \frac{1}{\beta} \right) Ec f'^2 = 0, \quad (8)$$

$$\frac{1}{Sc} \phi'' + f \phi' + Sr \theta'' = 0, \quad (9)$$

with parameters:

$$\left. \begin{aligned} M &= \frac{2\sigma B_0^2}{\rho c}, \quad K_1 = \frac{Kc x^{n-1}}{2\nu}, \quad Fs = \frac{2bx}{\sqrt{K}}, \quad Pr = \frac{\nu}{\alpha} = \frac{\rho \nu C_p}{k} = \frac{\mu C_p}{k}, \\ Gr_T &= \frac{2g\beta_T(T_w - T_{\infty})}{(n+1)c^2 x^{2n-1}}, \quad Gr_C = \frac{2g\beta_C(C_w - C_{\infty})}{(n+1)c^2 x^{2n-1}}, \quad u = cx^n \\ Ec &= \frac{u^2}{C_p(T_w - T_{\infty})} = \frac{c^2 x^{2n}}{C_p(T_w - T_{\infty})}, \quad \nu = \frac{\mu}{\rho}, \\ Sc &= \frac{\nu}{D_m}, \quad Du = \frac{D_m K_T (C_w - C_{\infty})}{\nu C_S C_p (T_w - T_{\infty})}, \quad Sr = \frac{D_m K_T (T_w - T_{\infty})}{\nu T_{\infty} (C_w - C_{\infty})}. \end{aligned} \right\} \quad (10)$$

Here M is magnetic parameter (Hartmann number); K_1 is Permeability parameter; Fs is Forchheimer parameter; Pr is Prandtl number; Gr_T is thermal Grashof number; Gr_C is concentration Grashof number; Ec is Eckert number; Du is Dufour number; Sc is Schmidt number; and Sr is Soret number. And corresponding boundary conditions are as follow:

$$\begin{aligned}
151 \quad & f(0) = 0, f'(0) = 1 + \delta \left(1 + \frac{1}{\beta}\right) f''(0), \theta'(0) = -\gamma_1 [1 + \theta(0)], \phi'(0) = -\gamma_2 [1 + \phi(0)] \\
& f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0
\end{aligned} \tag{11}$$

152

153 where

$$\begin{aligned}
154 \quad & N = N_1 x^{-\frac{n-1}{2}}, \delta = N \left(\frac{(n+1)c\nu}{2} \right)^{\frac{1}{2}}, h_T = h_{T0} c x^{\frac{n-1}{2}}, \gamma_1 = h_{T0} \left(\frac{2\nu}{c(n+1)} \right)^{\frac{1}{2}} \\
& h_C = h_{C0} c x^{\frac{n-1}{2}}, \gamma_1 = h_{C0} \left(\frac{2\nu}{c(n+1)} \right)^{\frac{1}{2}}
\end{aligned} \tag{12}$$

155 δ is called velocity slip parameter, γ_1 is called thermal slip parameter, and γ_2 is called concentration
156 slip parameter.

157 The physical quantities of Engineering interest are the **Skin-friction** coefficient (rate of shear stress),
158 the couple stress coefficient at the sheet, the **local Nusselt** number (rate of heat transfer), and the **local**
159 **Sherwood** number (rate of mass transfer).

160 The local Skin-friction C_f , local Nusselt Number Nu_x and local Sherwood Number Sh_x are defined
161 as follow:

$$\begin{aligned}
162 \quad & \left. \begin{aligned} C_f &= \frac{\tau_w}{\frac{\rho U_w^2}{2}} = \frac{\mu_B \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}} \Rightarrow C_f = \left(\frac{2\nu(n+1)}{c}\right)^{\frac{1}{2}} x^{-\frac{(n+1)}{2}} f''(0), \\ C_f &= \left(\frac{2(n+1)}{\text{Re}}\right)^{\frac{1}{2}} \left(1 + \frac{1}{\beta}\right) f''(0), \quad \text{Re} = \frac{cx^{(n+1)}}{\nu}. \end{aligned} \right\}
\end{aligned} \tag{13}$$

$$\begin{aligned}
163 \quad & \left. \begin{aligned} Nu &= -\frac{x \left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -\left(\frac{c(n+1)}{2\nu}\right)^{\frac{1}{2}} x^{\frac{(n+1)}{2}} \theta'(0), \\ Nu &= -\left(\frac{(n+1)c x^{(n+1)}}{2\nu}\right)^{\frac{1}{2}} \theta'(0) = -\left(\frac{(n+1)}{2} \text{Re}\right)^{\frac{1}{2}} \theta'(0). \end{aligned} \right\}
\end{aligned} \tag{14}$$

164 Here $-\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is a heat flux from the surface of the sheet.

$$\left. \begin{aligned} Sh &= -\frac{x \left(\frac{\partial C}{\partial y} \right)_{y=0}}{C_w - C_\infty} = -\left(\frac{c(n+1)}{2\nu} \right)^{\frac{1}{2}} x^{\frac{(n+1)}{2}} \phi'(0), \\ Sh &= -\left(\frac{(n+1)cx^{(n+1)}}{2\nu} \right)^{\frac{1}{2}} \phi'(0) = -\left(\frac{(n+1)}{2} \text{Re} \right)^{\frac{1}{2}} \phi'(0). \end{aligned} \right\} \quad (15)$$

Here $-\left(\frac{\partial C}{\partial y} \right)_{y=0}$ is a mass flow rate from the surface of the sheet and Re is the local Reynold

Number.

3. Method of Numerical Solution

The numerical solutions are obtained using the equations (7)-(9) and boundary conditions (11) for some values of the governing parameters, namely, the magnetic parameter M, Soret number Sr, Dufour number Du, Casson fluid parameter β , Prandtl number Pr, Schmidt number Sc, Eckert number Ec. Effects of M, Sr, Du, β , Pr, Sc, Ec on the steady boundary layer flow are discussed in detail. The numerical computation is done using the MATLAB in-built numerical solver bvp4c. In the computation we have taken $\eta_\infty = 10$ and axis according to the clear figure-visibility.

4. Result Analysis and discussion

In order to analyze the behaviour of non-dimensional linear velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ profiles of the physical problem, numerical calculations are carried out for various values of magnetic parameter M, Soret number Sr, Dufour number Du, Casson fluid parameter β , Prandtl number Pr, Schmidt number Sc, and Eckert number Ec. Also, the Skin-friction factor and local Nusselt and local Sherwood numbers are discussed. For illustration of the results, the numerical data are tabulated in tables 1-8 and plotted in Figs. 2–26.

The results for Skin-friction and local Nusselt number are compared with the previous published results, and are shown in Tables 1–2. It is observed that the obtained results are in good agreement with the published results.

Tables 1 and 2 present the values of Skin-friction coefficient and local Nusselt number for different values of nonlinear stretching parameter n1 and Prandtl number Pr, respectively. The present results are compared with the results of Cortell [18] and Ullah et al. [17]. It is also observed from Table 1

190 that magnitude of Skin-friction coefficient $\left(1 + \frac{1}{\beta}\right) f''(0)$, increases with the increase in n_1

191 whereas local Nusselt number decreases with the increase in n_1 and increases with increase in Pr
192 (Table 2).

193 For the convenience in calculation in Matlab following symbols are used for respective symbols used
194 in modelling equations and boundary conditions:

195 $a_1 = f''(0)$, $a_2 = \theta'(0)$, $a_3 = \phi'(0)$, $d = \delta$, $d_1 = Gr_T$, $d_2 = Gr_C$, $b = \beta$, $m_1 = \gamma_1$, $m_2 = \gamma_2$, $n_1 = n$.
196

197

198 **Table 1** Comparison of $-f''(0)$ for different values of n_1 with $F_s=0.0$, $d_1=0.0$, $d_2=0.0$, $d=0.0$,
199 $m_1=10^4$, $m_2=10^4$, $M=0$, $Pr=1.00$, $Du=0.0$, $Sc=0.22$, $a_1=0.0$, $a_2=1.0$, $a_3=1.0$, $b=10^8$, $Ec=0.0$, $Da=10^7$,
200 $Sr=0$.
201

$-f''(0)$			
n_1	Cortell[18]	Ullah et al.[17]	Present
0.0	0.627547	0.6276	0.627631963479766
0.2	0.766758	0.7668	0.766906263551595
0.5	0.889477	0.8896	0.889594172073448
1	1.0	1.0	1.000062567556568
3	1.148588	1.1486	1.148660394543063
10	1.234875	1.2349	1.234952969673218
100	1.276768	1.2768	1.276830449563257

202

203 **Table 2** Comparison of local Nusselt number $-\theta'(0)$ for various values of Pr and n_1 with $F_s=0.0$,
204 $d_1=0.0$, $d_2=0.0$, $d=0.0$, $m_1=10^4$, $m_2=10^4$, $M=0$, $Pr=1.00$, $Du=0.0$, $Sc=0.22$, $a_1=0.0$, $a_2=1.0$, $a_3=1.0$,
205 $b=10^8$, $Ec=0.0$, $Da=10^7$, $Sr=0$.
206

$-\theta'(0), Pr=1,$			
n_1	Cortell[18]	Ullah et al.[17]	Present
0.2	0.610262	0.6102	0.610277445039946
0.5	0.595277	0.5949	0.595283487517005
1.5	0.574537	0.5747	0.574829838650739
3.0	0.564472	0.5647	0.564775152915357

207

Figs. 2–26 are plotted to study the effects of various parameters (b , n_1 , M , Da , Fs , d , d_1 , d_2 , m_1 , m_2 , a_1 , a_2 , a_3 , Pr , Sc , Ec , Du and Sr) on the dimensionless velocity, temperature and concentration profiles.

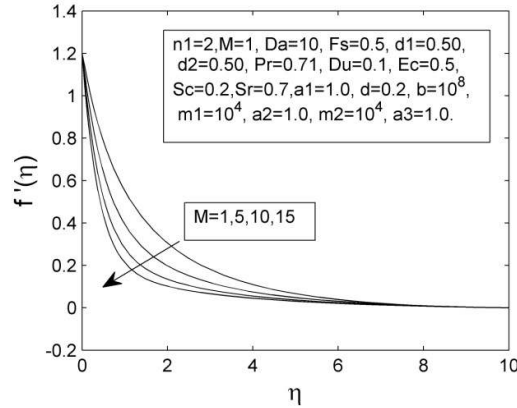


Figure 2 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Magnetic parameter M .

The effect of M is illustrated in Fig. 2. All the trajectories are of same family with the magnetic number in between 1 and 15. It is seen that fluid velocity reduces as M increases. It is due to the fact that drag force, also known as Lorentz force is produced when magnetic field is applied to the fluid. This force has the tendency to slow down the velocity of the fluid in the boundary layer region. The decreasing pattern of velocity field with the increase in similarity variable shows that transverse magnetic field opposes the transport phenomenon. Thus the momentum boundary layer thickness decreases as M increases.

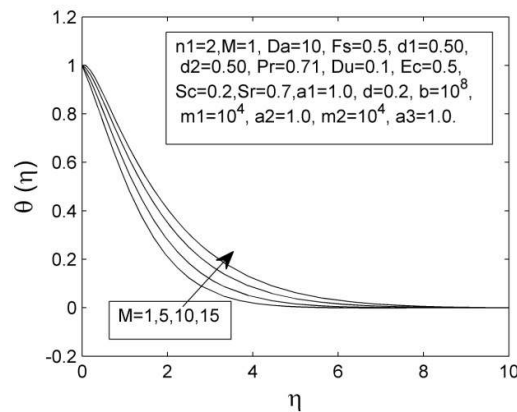


Figure 3 Temperature profile $\theta(\eta)$ with respect to η for some values of Magnetic parameter M .

Fig. 3 shows the behaviour of temperature profile for increasing values of Magnetic parameter M . The increasing values of M increase the fluid temperature as well as thermal boundary layer thickness. It is also observed that higher values of M increase the fluid temperature significantly and boundary layer expands away from the wall. The explanation of this phenomenon is that the increasing M leads to a increase in thermal diffusion and results in thickening of boundary layer region.

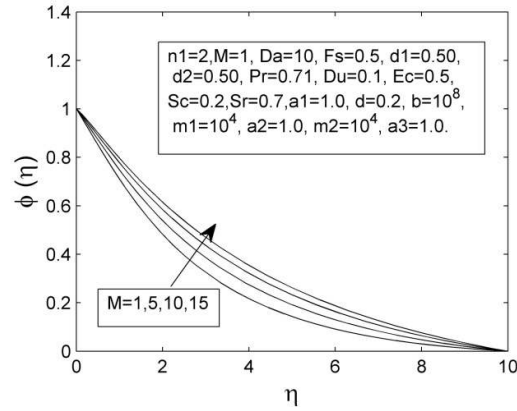


Figure 4 Concentration profile $\phi(\eta)$ with respect to η for some values of Magnetic parameter M .

Fig. 4 shows the behaviour of concentration profile for increasing values of Magnetic parameter M . The increasing values of M increase the fluid concentration as well as concentration boundary layer thickness. It is also observed that higher values of M increase the fluid concentration significantly and boundary layer expands away from the wall. The explanation of this phenomenon is that the increasing M leads to a increase in concentration diffusion and results in thickening boundary layer. Thus, the increasing values of M increase the fluid concentration as well as concentration boundary layer thickness.

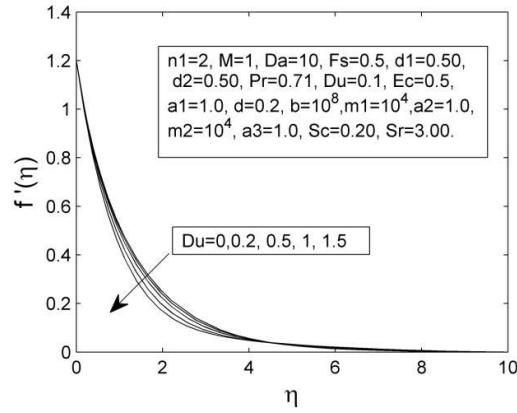


Figure 5 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Dufour parameter Du .

The effect of Dufour parameter is illustrated in Fig. 5. It is seen that fluid velocity profile reduces as Du increases and causes thinning of the corresponding boundary layer.

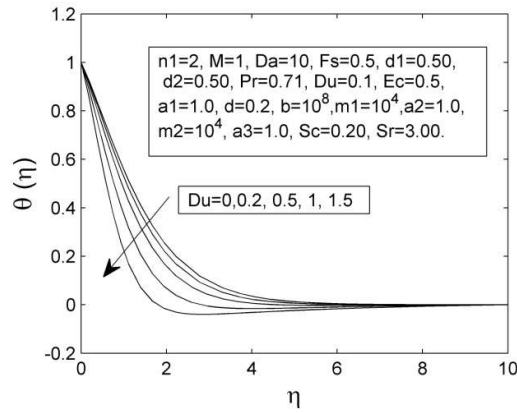


Figure 6 Temperature profile $\theta(\eta)$ with respect to η for some values of Dufour parameter Du .

It is shown in Fig. 6 that the temperature profile decreases with the increase in Dufour parameter and causes thinning of the corresponding boundary layer.

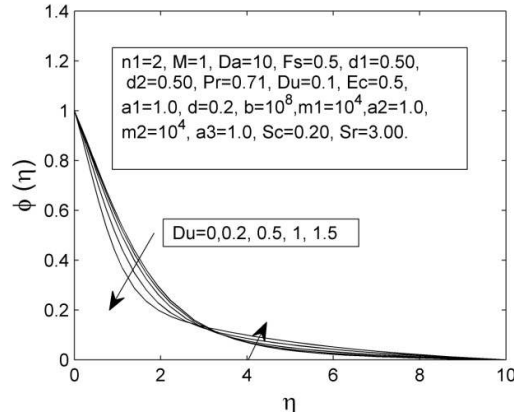


Figure 7 Concentration profile $\phi(\eta)$ with respect to η for some values of Dufour parameter Du .

It is shown in Fig. 7 that with the variation in the value of Dufour number Du , the concentration profiles show presence of point of intersection at some value of similarity variable in its (similarity variable's) range. Before the point of intersection concentration profiles decrease and after the point of intersection concentration profiles increase with the increase in Du and thus causes thinning before the point of intersection and thickening after the point of intersection of the corresponding boundary layers.

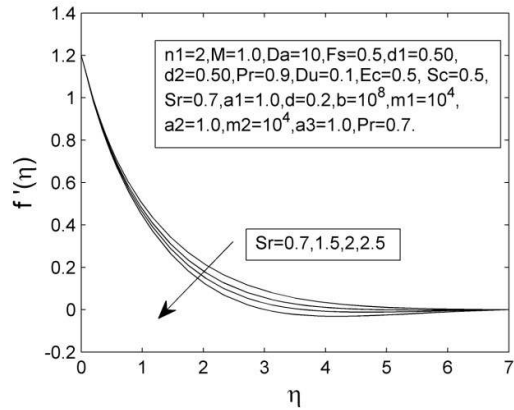
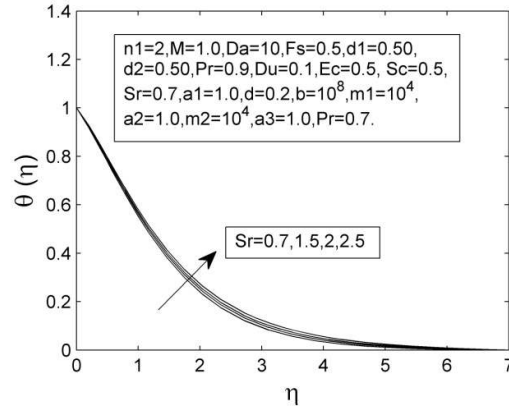


Figure 8 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Soret parameter Sr .

It is seen in Fig. 8 that velocity profile decreases with increasing Soret number and causes thinning of the corresponding boundary layer.

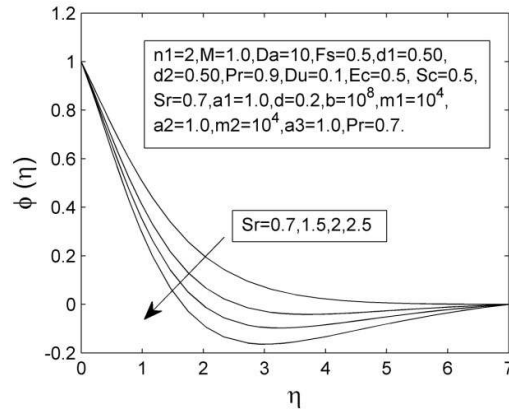
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271

272 **Figure 9 Temperature profile $\theta(\eta)$ with respect to η for some values of Soret parameter Sr .**

273 It is seen in Fig.9 that the temperature profile increases with increasing Soret number and causes
274 thickening of the corresponding boundary layer.



275

276 **Figure 10 Concentration profile $\phi(\eta)$ with respect to η for some values of Soret parameter Sr .**

277 It is seen in Fig. 10 that the concentration profile decreases with increasing Soret number and causes
278 thinning of the corresponding boundary layer.

279

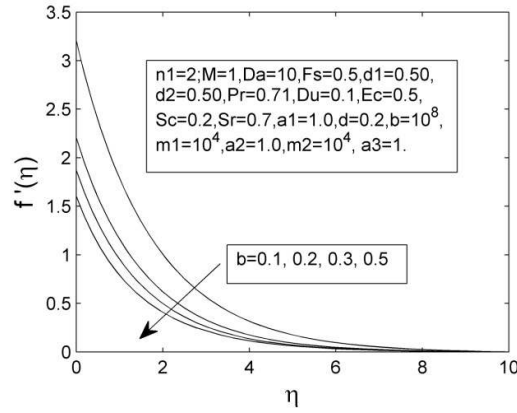


Figure 11 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Casson parameter b .

It is observed from Fig. 11 that velocity profile decreases with the increase in Casson parameter b . physically, with increase in b , the fluid becomes more viscous and in result the fluid velocity reduces. Also, the momentum boundary layer thickness decreases as b increases. Further, as $b \rightarrow \infty$ the present phenomenon reduces to Newtonian fluid.

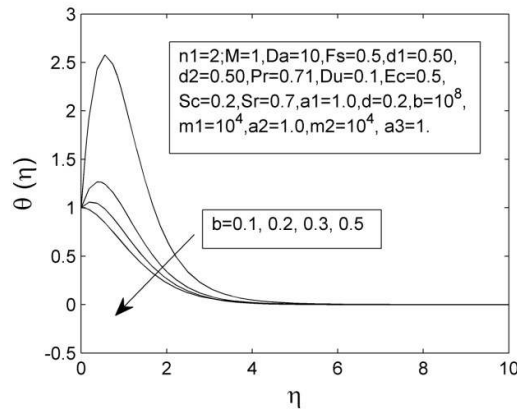


Figure 12 Temperature profile $\theta(\eta)$ with respect to η for some values of Casson parameter b .

It is observed from Fig. 12 that with the increase in the value of Casson parameter b , temperature profiles decrease. Also each curve shows having maxima and point of inflexion at some points in the range of similarity variable. And it causes thinning of the corresponding boundary layer before the point of inflexion.

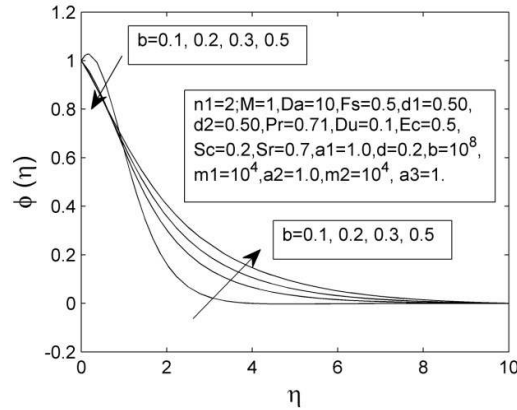


Figure 13 Concentration profile $\phi(\eta)$ with respect to η for some values of Casson parameter b .

It is observed from Fig. 13 that with the variation in the value of Casson parameter b , the concentration profiles shows presence of point of inflexion at some values of the similarity variable in its (similarity variable's) range. Before the point of inflexion concentration profile decreases and after the point of inflexion concentration profile increases with the increase in Casson parameter and thus causes thinning before the point of inflexion and thickening after the point of inflexion of the corresponding boundary layer.

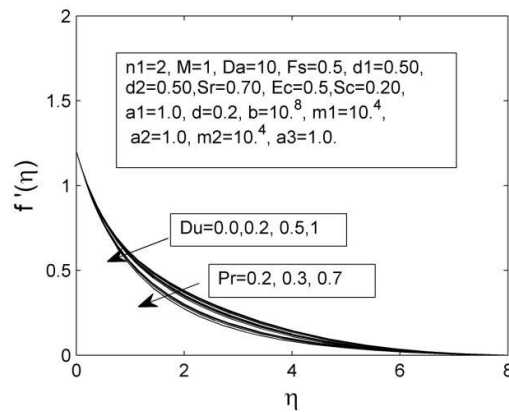


Figure 14 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Dufour parameter Du and Prandtl number Pr .

It is observed from Fig. 14 that velocity profiles decrease with the increase in Dufour parameter Du and Prandtl number Pr and thus cause thinning of the corresponding boundary layers.

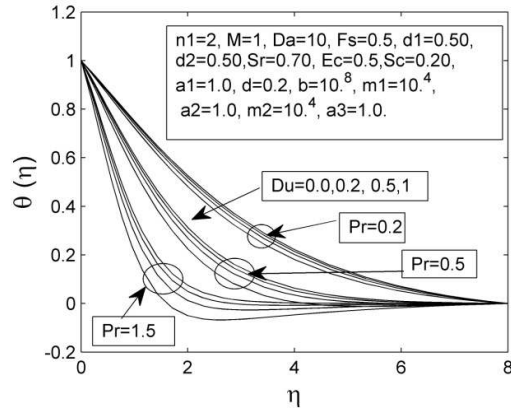


Figure 15 Temperature profile $\theta(\eta)$ with respect to η for some values of Dufour parameter Du and Prandtl number Pr .

It is observed from Fig. 15 that temperature profiles decrease with the increase in Dufour parameter Du and Prandtl number Pr and thus cause thinning of the corresponding boundary layers.

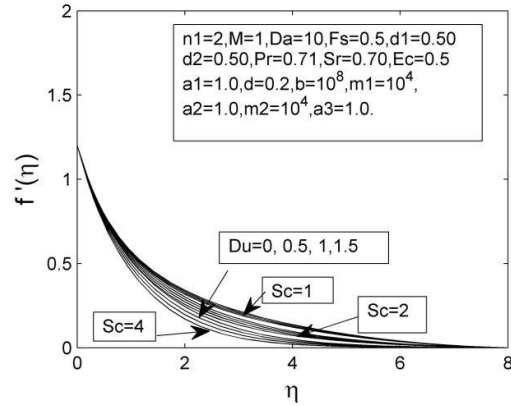


Figure 16 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Dufour parameter Du and Schmidt number Sc .

It is observed from Fig. 16 that velocity profiles decrease with the increase in Dufour parameter Du and Schmidt number Sc and thus cause thinning of the corresponding boundary layers.

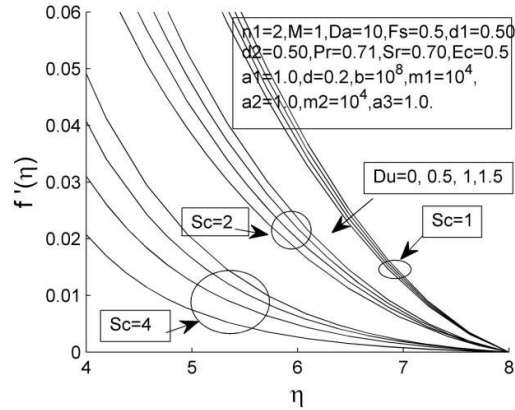


Figure 17 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Dufour parameter Du and Schmidt number Sc .

It is observed from Fig. 17 that velocity profiles decrease with the increase in Dufour parameter Du and Schmidt number Sc and thus cause thinning of the corresponding boundary layers.

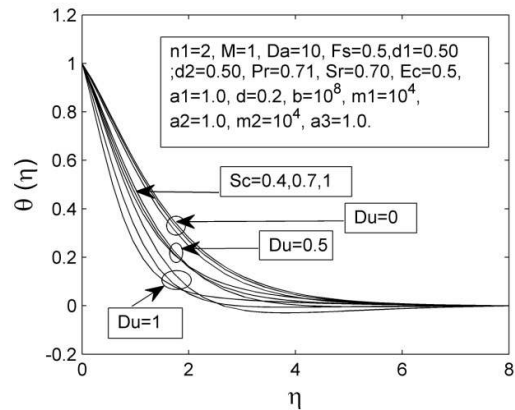


Figure 18 Temperature profile $\theta(\eta)$ with respect to η for some values of Dufour parameter Du and Schmidt number Sc .

It is shown in Fig. 18 that with the variation in the value of Dufour parameter Du and Schmidt number Sc , the temperature profiles show presence of point of intersection at some value of similarity variable in its (similarity variable's) range. Before the point of intersection temperature profiles decrease and after the point of intersection temperature profiles increase with the increase in Dufour parameter Du and Schmidt number Sc , and thus causes thinning before the point of intersection and thickening after the point of intersection of the corresponding boundary layers.

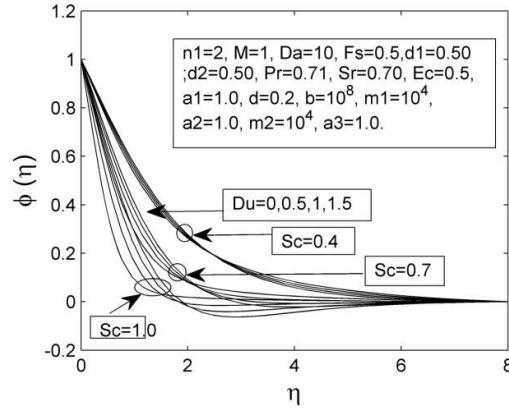


Figure 19 Concentration profile $\phi(\eta)$ with respect to η for some values of Dufour parameter Du and Schmidt parameter Sc.

It is shown in Fig. 19 that with the variation in the value of Dufour parameter Du and Schmidt number Sc, the concentration profiles show presence of point of intersection at some value of similarity variable in its (similarity variable's) range. Before the point of intersection concentration profiles decrease and after the point of intersection concentration profiles increase with the increase in Dufour parameter Du and Schmidt number Sc, and thus causes thinning before the point of intersection and thickening after the point of intersection of the corresponding boundary layers.

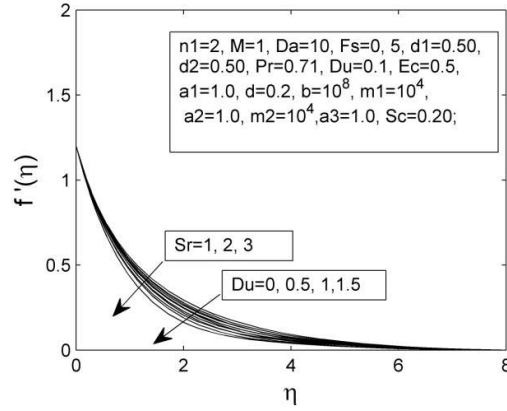


Figure 20 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Dufour parameter and Soret parameter Sr .

It is observed from Fig. 20 that velocity profiles decrease with the increase in Dufour parameter Du and Soret parameter Sr and thus cause thinning of the corresponding boundary layers.

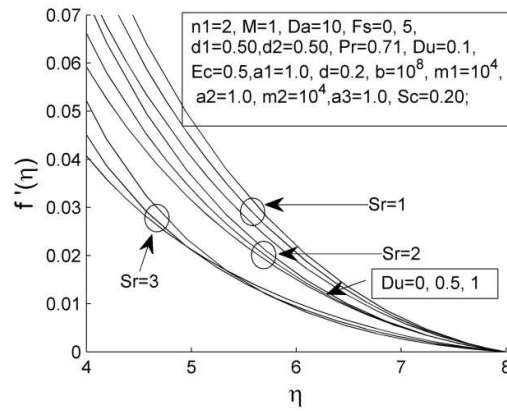


Figure 21 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Dufour parameter and Soret parameter Sr .

It is observed from Fig. 21 that velocity profiles decrease with the increase in Dufour parameter Du and Soret parameter Sr and thus cause thinning of the corresponding boundary layers.

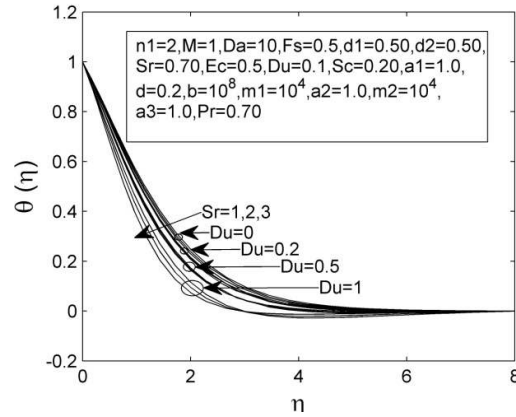


Figure 22 Temperature profile $\theta(\eta)$ with respect to η for some values of Dufour parameter Du and Soret parameter Sr .

It is shown in Fig. 22 that with the variation in the value of Dufour parameter Du and Soret parameter Sr , the temperature profiles show presence of point of intersection at some value of similarity variable in its (similarity variable's) range. Before the point of intersection temperature profiles decrease and after the point of intersection temperature profiles increase with the increase in Dufour parameter Du and Soret parameter Sr , and thus causes thinning before the point of intersection and thickening after the point of intersection of the corresponding boundary layers.

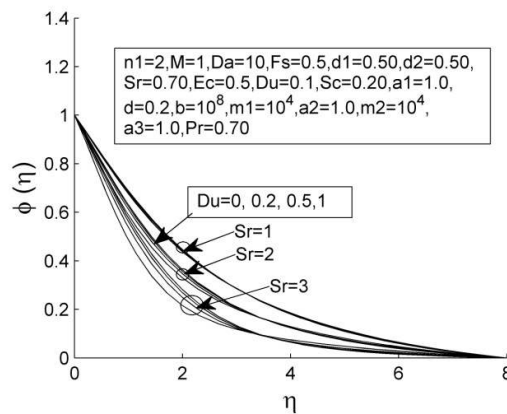


Figure 23 Concentration profile $\phi(\eta)$ with respect to η for some values of Dufour parameter Du and Soret parameter Sr .

It is shown in Fig. 23 that with the variation in the value of Dufour parameter Du and Soret parameter Sr , the concentration profiles show presence of point of intersection at some value of similarity variable in its (similarity variable's) range. Before the point of intersection concentration profiles decrease and after the point of intersection concentration profiles increase with the increase in Dufour parameter Du and Soret parameter Sr , and thus causes thinning before the point of intersection and thickening after the point of intersection of the corresponding boundary layers.

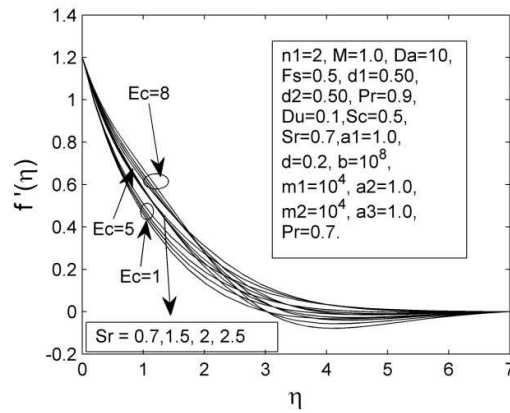


Figure 24 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Soret parameter Sr and Eckelt Parameter Ec .

It is observed from Fig. 24 that velocity profile decreases with the increase in Soret parameter Sr , thus cause thinning of the corresponding boundary layer and increases with the increase Eckert number Ec thus cause thickening of the corresponding boundary layer .

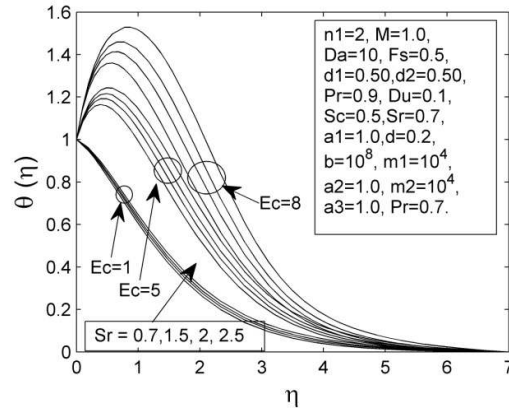


Figure 25 Temperature profile $\theta(\eta)$ with respect to η for some values of Soret parameter Sr and Eckert Parameter Ec .

It is observed from Fig. 25 that with the variation in the value of Soret parameter Sr , and Eckert number Ec , each of the temperature profiles shows maxima and point of inflexion in the range of similarity variable. With the each value of Soret parameter Sr , and Eckert number Ec , each curve shows having maxima before the point of inflexion. Also with the increase in the value of Soret parameter and Eckert number, temperature profiles show increase and thus cause increase in the corresponding boundary layer thickness.

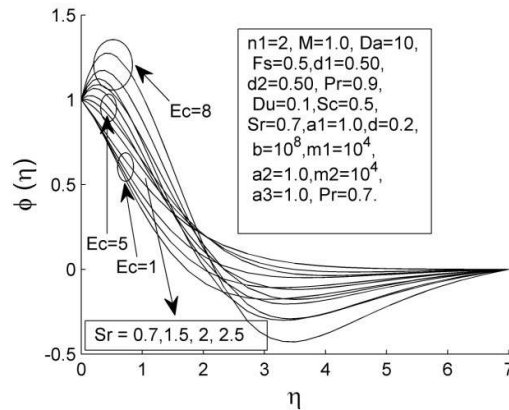


Figure 26 Concentration profile $\phi(\eta)$ with respect to η for some values of Soret parameter Sr and Eckert parameter Ec .

It is observed from Fig. 26 that with the variation in the value of Soret parameter Sr , and Eckert number Ec , each of the concentration profiles shows maxima and point of inflexion in the range of

similarity variable. With each value of Soret parameter Sr , and Eckert number Ec , each curve shows having maxima before the point of inflexion. Also with the increase in the value of Soret parameter Sr and Eckert number, concentration profiles show increase before the point of inflexion and decrease after the point of inflexion, and hence cause increase before the point of inflexion and decrease after the point of inflexion in the corresponding boundary layer thickness.

Table 3 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Magnetic parameter M .

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.71;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	-1.139886286400067	0.414856222735359	0.291371985956979
5	-1.707705031432430	0.240533363861078	0.236481214248188
10	-2.264381188870255	0.063274575662640	0.189093904953875
15	-2.724242857459311	-0.081367644844538	0.154636326522287

Table 4 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Casson parameter b .

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.70;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
b	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	-1.938133150664395	-6.295211331613203	-0.320484926788943
0.2	-1.473378060541108	-1.442823157565096	0.183465539708107
0.3	-1.324805467549381	-0.534950892963336	0.257221030686203
0.5	-1.214615842077180	-0.025337735376427	0.287249701545771

Table 5 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Darcy parameter Da.

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.71;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Da	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-1.428193623699829	0.328040454192572	0.262762395741959
1	-1.281374388721462	0.372846473212556	0.277173948582174
2	-1.203957866571336	0.395996971875209	0.284911124661897
3	-1.177501050005703	0.403819967527664	0.287573408826969

Table 6 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Forchheimer parameter Fs.

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.71;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Fs	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-1.139886286400067	0.414856222735359	0.291371985956979
1	-1.208612809561646	0.401628776298048	0.287495526076845
2	-1.338383349016651	0.376015619156389	0.280167649510050
3	-1.459425519349771	0.351486715493245	0.273342292473521

441

442 **Table 7 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local**
 443 **Sherwood number $-\phi'(0)$ for various values of Prandtl number Pr.**

444

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50; Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Pr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.7	-1.139171110747207	0.412286376828377	0.291199312367934
1	-1.157231825306829	0.478869265424159	0.296181869262054
2	-1.191414911346804	0.611623320537683	0.308360091315121
3	-1.209804887356637	0.679960625037383	0.315203633463110

445

446

447

448 **Table 8 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local**
 449 **Sherwood number $-\phi'(0)$ for various values of Eckert number Ec.**

450

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.70;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Ec	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	-1.147988244422926	0.540232249116676	0.307582454320623
0.2	-1.145767434719725	0.507907740910780	0.303440467248767
0.3	-1.143558180413784	0.475809857492194	0.299329499846729
0.5	-1.139171110747207	0.412286376828377	0.291199312367934

451

452

Table 9 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Dufour number Du .

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.70;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Du	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	-1.139341254219237	0.400032497066902	0.296468803292375
0.2	-1.143544759569682	0.417496237457631	0.297833579239204
0.5	-1.150011965516246	0.444375863555847	0.299947517761085
1	-1.161253400554637	0.491133315364581	0.303662254401080

Table 10 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Schmidt number Sc .

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.71;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7; a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Sc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.2	-1.139886286400067	0.414856222735359	0.291371985956979
0.4	-1.176935210977778	0.386151355496659	0.443770131850051
0.7	-1.215081398526230	0.360817373104762	0.628942510557584
1.2	-1.255405680773496	0.343770968980801	0.873476192982244

Table 11 Comparison of Skin-friction $f''(0)$, local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$ for various values of Soret number Sr .

n1=2;M=1;Da=10;Fs=0.5;d1=0.50;d2=0.50;Pr=0.70;Du=0.1;Ec=0.5;Sc=0.2;Sr=0.7;			
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a1=1.0;d=0.2;b=10 ⁸ ;m1=10 ⁴ ;a2=1.0;m2=10 ⁴ ;a3=1.0.			
Sr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.7	-1.143632365431753	0.405620505160762	0.302985470126012
1.5	-1.154724386800227	0.397577058525297	0.340314127119831
2	-1.161673814489628	0.392217170730794	0.362464009525076
2.5	-1.168628603664313	0.386577782183745	0.383611994611814

467

468 Table 3 shows with the increase in Magnetic parameter Skin-friction, local Nusselt number and local
 469 Sherwood number decreases.

470 Table 4 shows with the increase in Casson parameter Skin-friction, local Nusselt number and local
 471 Sherwood number increases.

472 Table5 shows with the increase in Darcy parameter Skin-friction, local Nusselt number and local
 473 Sherwood number increases.

474 Table 6 shows with the increase in Forchheimer parameter Skin-friction, local Nusselt number and
 475 local Sherwood number decreases.

476 Table 7 shows with the increase in Prandtl number Skin-friction decreases and local Nusselt number
 477 and local Sherwood number increases.

478 Table 8 shows with the increase in Eckert number Skin-friction increases and local Nusselt number
 479 and local Sherwood number decrease.

480 Table 9 shows with the increase in Dufour number Skin-friction decreases and local Nusselt number
 481 and local Sherwood number increases.

482 Table 10-11 shows with the increase in Schmidt number and Soret parameter, Skin-friction, and local
 483 Nusselt number decreases and local Sherwood number increases.

484 5. Conclusion

485 In the present paper the numerical study of effects of Soret and Dufour and viscous dissipation
 486 parameters on steady magnetohydrodynamic Casson fluid flow through non-Darcy porous medium is
 487 explored. By suitable similarity transformations, the governing boundary layer equations are
 488 transformed to ordinary differential equations. The method, the numerical computation with bvp4c, a
 489 MATLAB program, is applied to solve these equations. The effects of magnetic Parameter, Soret
 490 parameter, Dufour parameter, Prandtl number, Schmidt number, Eckert number, and Casson
 491 parameter on velocity, heat transfer, and concentration profiles, Skin- frictions, local Nusselt number

492 and local Sherwood number are computed and discussed numerically and presented through tables
493 and graphs.

494 From the above work following results are concluded.

495 Velocity profiles decrease and thus cause thinning of the corresponding boundary layers with the
496 increase in Magnetic parameter, Dufour parameter, Soret parameter, Casson parameter, Prandtl
497 number, and Schmidt number.

498 Temperature profiles decrease and thus cause thinning of the corresponding boundary layers with the
499 increase in Dufour parameter, Casson parameter, Prandtl number, and Schmidt number.

500 Concentration profiles decrease and thus cause thinning of the corresponding boundary layers with the
501 increase in Soret parameter, and Schmidt number.

502 Before the point of inflexion concentration profile decreases and after the point of inflexion
503 concentration profile increases and thus causes thinning before the point of inflexion and thickening
504 after the point of inflexion of the corresponding boundary layer, with the increase in Dufour
505 parameter, and Casson parameter.

506 Velocity profiles increase and thus cause thickening of the corresponding boundary layers with the
507 increase in Eckert number.

508 Temperature profiles increase and thus cause thickening of the corresponding boundary layers with
509 the increase in magnetic parameter, Soret parameter, Eckert number.

510 Concentration profiles increase and thus cause thickening of the corresponding boundary layers with
511 the increase in magnetic parameter.

512 Concentration profiles show increase before the point of inflexion and decrease after the point of
513 inflexion, and hence cause increase before the point of inflexion and decrease after the point of
514 inflexion in the corresponding boundary layer thickness, with the increase in the value of Soret
515 parameter and Eckert number.

516 Skin-friction increases with the increase in Casson parameter, Darcy parameter, Prandtl number, and
517 Dufour parameter.

518 Local Nusselt number increases with the increase in Casson parameter, Darcy parameter, Eckert
519 number, Schmidt number, and Soret parameter.

520 Local Sherwood number increases with the increase in Casson parameter, Darcy parameter, Eckert
521 number, Schmidt number.

Skin friction decreases with the increase in Magnetic parameter, Forchheimer parameter, Eckert number, , Schmid number , and Soret parameter.

Local Nusselt number decreases with the increase in Magnetic parameter, Forchheimer parameter, Prandtl number, and Dufour parameter.

Local Sherwood number decreases Magnetic parameter, Forchheimer parameter, Prandtl number, Dufour parameter , Schmid number , and Soret parameter.

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Nomenclature and SI units

μ	Dynamic viscosity($\text{kgm}^{-2}\text{s}^{-1}$)
ν	Kinematic viscosity(m^2s^{-1})
B	Magnetic field($\text{N}/(\text{mA})$)
h	Convective heat transfer coefficient($\text{W}/\text{m}^2\text{K}$)
c	Specific heat (J/kgK)
D	Mass diffusivity(m^2s^{-1})
G	Acceleration due to gravity (ms^{-2})
k	Thermal conductivity($\text{Wm}^{-1}\text{K}^{-1}$)
m	Mass kg
V	Volume m^3
ρ	Density kg/m^3
t	Time(s)

786	u	Horizontal component of velocity(m/s)
787	v	Vertical component of velocity(m/s)
788	T	Temperature of fluid (K)
789	β	Thermal expansion coefficient(K ⁻¹)
790		
791		
792	M	Magnetic parameter
793	n	Stretching index parameter
794	Nu	Local Nusselt number
795	Pr	Prandtl number Prandtl number
796	Re	Local Reynold number
797	Sc	Smidth number
798	Sh	Local Sherwood number
799	C	Concentration of fluid
800	Gr_T	Thermal Grashof number
801	C_p	Specific heat at constant pressure
802		
803		
804	Gr_c	Mass(concentration) Grashof number
805	x	Distance along the plate distance along the plate
806	α	Thermal diffusivity
807	η	Similarity variable
808	x, y	Cartesian coordianates
809	θ	Dimensionless temperature
810	ψ	Dimentiosless stream function
811		
812		
813		