

Original Research Article

The Effect of Chemical reaction, and Radiation on magnetohydrodynamic Casson fluid flow over non-linearly stretching surface with suction

Abstract: In the present paper, we investigate numerically the effect of chemical reaction, and radiation on magnetohydrodynamic Casson fluid flow over non-linearly stretching surface with fluid suction. By suitable similarity transformations, the governing boundary layer equations are transformed to ordinary differential equations and to solve these equations the method applied is numerical computation with bvp4c, a MATLAB program. The effects of Magnetic, Casson, Stretching index, Suction, Thermal index, Concentration index, chemical reaction and Radiation parameters, and Prandtl number and Schmidt number, on velocity, heat transfer, and concentration profiles, Skin-frictions, Nusselt Number and Sherwood Number are computed and discussed numerically and presented through tables and graphs.

Keywords: Magnetohydrodynamic fluid flow, Casson fluid , Radiation parameter, Chemical reaction parameter, Suction parameter.

1. Introduction

Many natural, industrial as well as biological fluids such as mud, condensed milk, glues, lubricating greases, multi grade oils, gypsum pastes, emulsions, paints, sugar solution, shampoos and tomato paste, ceramics, polymers, liquid detergents, blood, fruit juices etc. change their viscosity or flow behaviour under stress and thus deviate from the classical Newton's law of viscosity. Different models of non-Newtonian fluids based on their diverse flow behaviours have been proposed by the researchers.

The rheological model was introduced originally by Casson [1] in his research on a flow equation for pigment oil-suspensions of printing ink.

Bird et al. [2] investigated the rheology and flow of visco-plastic materials and reported that Casson model constitutes a plastic fluid model which exhibits shear thinning characteristics, yield stress, and high shear viscosity. Casson fluid behaves as solid when the shear stress is less than the yield stress and it starts to deform when shear stress becomes greater than the yield stress.

35 The fundamental analysis of the flow field of non-Newtonian fluids in a boundary layer adjacent to a
 36 stretching sheet or an extended surface is very important and is an essential part in the study of fluid
 37 dynamics and heat and mass transfer.

38

39 Sakiadis, B.C.[3]studied boundary layer behaviour on continuous solid surfaces: II. The boundary
 40 layer on continuous flat surface. Crane L.J.[4] studied the Flow past a stretching plane. Nield, D.A.et
 41 al.[5] studied Convection in porous media.

42

43 Mukhopadhyay, S. et al. investigated Casson fluid flow and heat transfer over a nonlinearly stretching
 44 surface. Mustafa, M. et al. [7] studied Model for flow of Casson nanofluid past a non-linearly
 45 stretching sheet considering magnetic field effects. Medikare, M., et al.[8] studied MHD Stagnation
 46 Point Flow of a Casson Fluid over a Nonlinearly Stretching Sheet with Viscous Dissipation.

47

48 Pramanik, S. [9] studied Casson fluid flow and heat transfer past an exponentially porous stretching
 49 surface in presence of thermal radiation. Raju, C.S.K. et al.[10] studied Heat and mass transfer in
 50 magnetohydrodynamic Casson fluid over an exponentially permeable stretching surface. Saidulu, N.
 51 et al. [11] studied Slip Effects on MHD Flow of Casson Fluid over an Exponentially Stretching Sheet
 52 in Presence of Thermal Radiation, Heat Source/Sink and Chemical Reaction .

53

54 Sharada, K. et al. [12] studied MHD Mixed Convection Flow of a Casson Fluid over an
 55 Exponentially Stretching Surface with the Effects of Soret, Dufour, Thermal Radiation and Chemical
 56 Reaction. Mukhopadhyay, Swati et al [13] studied Exact solutions for the flow of Casson fluid over a
 57 stretching surface with transpiration and heat transfer effects. Hayat et al. [14] investigated Soret and
 58 Dufour effects on magnetohydrodynamic (MHD) flow of Casson fluid. Mahdy, A.[15] studied heat
 59 transfer and mass transfer of a Casson fluid due to a stretching cylinder with the Soret and Dufour effects.
 60 Animasaun, I.L. [16] studied Effects of thermophoresis, variable viscosity and thermalconductivity on
 61 free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with
 62 suction and nth order of chemical reaction .

63

64 Ullah, I., Sharidan et al. [17] investigated Effects of slip condition and Newtonian heating on MHD
 65 flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium.

66

67 Reza et al. [29] solved numerically the, boundary layered equations in the work ‘Radiation effect on
 68 MHD Casson fluid flow over a power-law stretching sheet with chemical reaction’ using finite
 69 difference method using Thomas algorithm.

70

71 Some recent studies concerning the flow, heat and mass transfer analysis of Casson fluid can be found
72 in Refs. 18–29.

73

74 In the study of the reference, **none of the work concerns with** using the MATLAB in-built
75 Numerical Solver **bvp4c**, **to solve** boundary layered equations of the work: **the effect of chemical**
76 **reaction, and radiation on magneto hydrodynamic Casson fluid flow over non linearly**
77 **stretching surface with fluid suction** [29].

78 **The present work is based** on ‘the numerical computation done, using the MATLAB in-built
79 Numerical Solver **bvp4c**’ of boundary layered equations of the work [29]

80 The accuracy **the results are compare with** the works of Cortell (2007), Wang (1989), Gorla
81 and Sidawi (1994), Khan and Pop (2010), Ullah et al. (2016) [17].

82

83 2. Mathematical Formulation of the Problem

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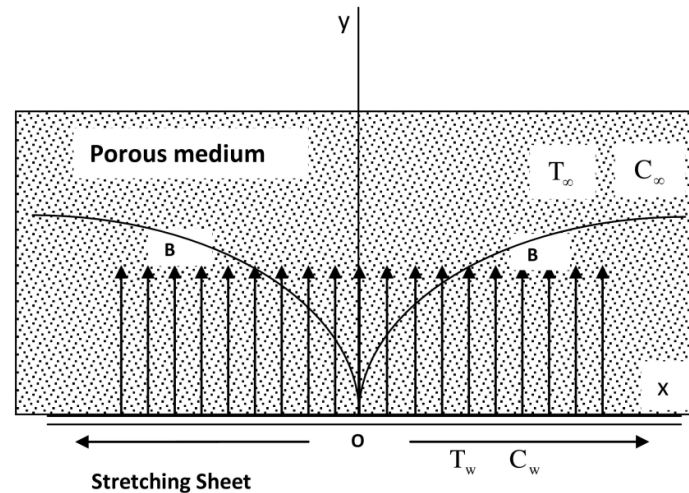
85 In the formulation of the problem we consider following assumptions. Casson fluid is incompressible
86 and electrically conducting. Flow is steady, laminar and two dimensional over a nonlinearly stretching
87 sheet. Flow region is under the influence of transverse magnetic field B . The sheet is stretched
88 nonlinearly along the x-axis (i.e. $y=0$) with velocity $u_w(x) = c x^n$; origin is taken as fixed and the fluid
89 flow is confined to $y>0$. Here c is constant and $n(n \geq 0)$ is the nonlinear stretching sheet
90 parameter; $n=1$ represents the linear sheet case and $n \neq 1$ is for nonlinear case. The magnetic
91 Reynolds number of the flow is taken to be small enough so that induced magnetic field is assumed to
92 be negligible in comparison with applied magnetic field so that $B = (0, B(x), 0)$, where $B(x)$ is the
93 applied magnetic field acting normal to the plate and varies in strength as a function of x . The flow is
94 assumed to be in the x-direction which is taken along the plate and y-axis is normal to it. There is a
95 constant suction/injection velocity v_w normal to the plate.

96 Under these assumptions the rheological equation for incompressible flow of Casson fluid is given by
97 (Sharada et al. [12], Mukhopadhyay, S., et al. [13])

$$98 \tau_{ij} = \begin{cases} 2(\mu_B + p_y / \sqrt{2\pi}) e_{ij} & \pi > \pi_c, \\ 2(\mu_B + p_y / \sqrt{2\pi_c}) e_{ij} & \pi < \pi_c. \end{cases}$$

99 where $\pi = e_{ij} e_{ij}$ and e_{ij} is the (i, j) – th component of the deformation rate, π is the product of
100 the components of deformation, π_c is critical value of the product based on the non-Newtonian
101 model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, and p_y is the yield stress of
102 the fluid. The viscosity and thermal conductivity of the fluid are assumed to be constant. There is
103 thermo-diffusion effect as well as diffusion-thermo effect. The pressure gradient, body forces and

104 Joule heating are neglected compared with the effect of viscous dissipation. The temperature and
 105 concentration of the stretching surface are always greater than their free stream values. The flow
 106 configuration and the coordinate system are shown in Figure 1.
 107



108

109 **Figure 1 Physical model and coordinate system.**

110

111

112 Under the above assumptions and using Bossinesqu approximation, boundary layer equations for
 113 flow with heat and mass transfer in Casson fluid (Ullah, I., Sharidan et al. [17]) are given by the
 114 following.

115 The continuity equation:

$$116 \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

117 The equation of momentum:

$$118 \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u \quad (2)$$

119 Energy Equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \left(\frac{1}{\rho C_p} \right) \frac{\partial q_r}{\partial y} \quad (3)$$

Mass equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K(x)(C - C_\infty) \quad (4)$$

where u and v are velocity components along x and y axes, respectively, ρ is fluid density, ν is kinematic viscosity, μ is dynamic viscosity, $\beta = \mu_B \sqrt{2\pi} / p_y$ is the Casson fluid parameter, σ is the electrical conductivity of the fluid and is assumed to be constant, T_w is the temperature of the fluid at the stretching sheet, T is the temperature of the fluid within the boundary layer, T_∞ is the temperature of the fluid outside the boundary layer (free stream temperature), k is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure p , q_r is radiative heat flux, C_w is the concentration of the fluid at the stretching sheet, C is the concentration of the fluid within the boundary layer, C_∞ is the concentration of the fluid outside the boundary layer (free stream concentration), D is the chemical molecular diffusivity. The applied magnetic field is $B = B_0 x^{\frac{n-1}{2}}$ where B_0 is assumed to be constant.

Boundary conditions:

$$\begin{aligned} \text{At } y=0: u &= cx^n = U, \quad v = V(x) = V_0 x^{\frac{n-1}{2}}, \\ T &= T_w = T_\infty + T_0 x^m, \quad C = C_w = C_\infty + C_0 x^t. \\ \text{As } y &\rightarrow \infty: u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty. \end{aligned} \quad (5)$$

c is a parameter related to the surface stretching speed; for surface stretching $c > 0$ and for surface shrinking $c < 0$. n is the velocity power index related to the surface stretching speed. $V(x)$ is suction velocity; V_0 is constant. T_0 is constant; m is temperature power index parameter. C_0 is constant; t is concentration power index parameter.

The radiative heat flux q_r is simplified by using Rosseland approximation (Bataller, R.C. [30]) as

$$q_r = \frac{4\sigma^*}{3k'} \frac{\partial T^4}{\partial y}. \quad (6)$$

142 Where σ^* is the Stefan-Boltzmann constant and k' is the mean absorption coefficient.

143 This approximation is valid at points optically far away from the boundary surface and it is
144 good for intensive absorption, which is for optically thick boundary layer. It is assumed that the
145 temperature difference within the flow is such that the term T^4 may be expressed as a linear function
146 of temperature. Hence expanding T^4 by Taylor's series about T_∞

147
$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \dots$$

148 and then neglecting higher order terms beyond the first degree in $(T - T_\infty)$ we get

149
$$T^4 = 4T_\infty^3T - 3T_\infty^4.$$

150 And so

151
$$q_r = -(4\sigma^*/3k^*)(\partial(4T_\infty^3T - 3T_\infty^4)/\partial y) = -(16\sigma^*/3k^*)T_\infty^3(\partial T/\partial y)$$

152 And
$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (7)$$

153 Using equations (6) and (7), equation (3) reduces to:

154
$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \left(k + \frac{16\sigma^*T_\infty^3}{3k'} \right) \frac{\partial^2 T}{\partial y^2}. \quad (8)$$

155

156 Dimensional Analysis: we define (Ullah, I., Sharidan et al. [17])

157
$$\psi = \left(\frac{2\nu c}{n+1} \right)^{\frac{1}{2}} x^{\frac{n+1}{2}} f(\eta), \quad \eta = \left(\frac{c(n+1)}{2\nu} \right)^{\frac{1}{2}} x^{\frac{n-1}{2}} y, \quad \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \quad \phi(\eta) = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \quad (9)$$

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \quad u = cx^n f'(\eta), \quad v = -\left(\frac{c\nu(n+1)}{2} \right)^{\frac{1}{2}} x^{\frac{n+1}{2}} \left(f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right)$$

158 Here η is the similarity variable. ψ is stream function.

159

160 Using (9), introducing these variables in the Equations (2), (4) and (8), we get the following
161 dimensionless forms of the equations:

$$(1 + \frac{1}{\beta})f''' + f f'' - \frac{2n}{n+1}f'^2 - \frac{2}{n+1}Mf' = 0 \quad (10)$$

$$\left(1 + \frac{1}{R}\right)\frac{1}{\text{Pr}}\theta'' - \frac{2m}{n+1}f'\theta + f\theta' = 0 \quad (11)$$

$$\frac{1}{\text{Sc}}\phi'' - \frac{2t}{n+1}\phi f' + f\phi' - \frac{k_1}{n+1}\phi = 0 \quad (12)$$

With parameters:

$$\left. \begin{aligned} M &= \frac{\sigma B_0^2}{\rho c}, \text{Pr} = \frac{v}{k} = \frac{\rho v C_p}{k} = \frac{\mu C_p}{k}, \\ u &= cx^n, \quad v = \frac{\mu}{\rho}, \quad R = \frac{3k k'}{16 \sigma^* T_\infty^3} \\ \text{Sc} &= \frac{v}{D}, \quad K(x) = \frac{cx^{n-1}L}{V_0}, \quad k_1 = \frac{2L}{V_0}. \end{aligned} \right\} \quad (13)$$

M is Magnetic parameter (Hartmann number), α is thermal diffusion coefficient, Pr is Prandtl number, D is mass diffusion coefficient, Sc is Schmidt number, $K(x)$ is variable reaction rate, k_1 is chemical reaction parameter. L is reference length. And corresponding boundary conditions are as follows:

$$\begin{aligned} f(0) &= S, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \\ f'(\infty) &= 0, \theta(\infty) = 0, \phi(\infty) = 0 \end{aligned} \quad (14)$$

$$\text{Where } f(0) = -V_0 \left(\frac{2}{c v (n+1)} \right)^{\frac{1}{2}} = S \quad (15)$$

S is called suction parameter.

The physical quantities of Engineering interest are the Skin friction coefficient (rate of shear stress), the couple stress coefficients at the sheet the Nusselt number (rate of heat transfer), and the Sherwood number (rate of mass transfer).

The local Skin-friction C_f , local Nusselt Number Nu_x and local Sherwood Number Sh_x which are defined as

$$\left. \begin{aligned}
 181 \quad C_f &= \frac{\tau_w}{\frac{\rho U_w^2}{2}} = \frac{\mu_B \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}} \Rightarrow C_f = \left(\frac{2\nu(n+1)}{c}\right)^{\frac{1}{2}} x^{-\frac{(n+1)}{2}} f''(0), \\
 C_f &= \left(\frac{2(n+1)}{\text{Re}}\right)^{\frac{1}{2}} \left(1 + \frac{1}{\beta}\right) f''(0), \quad \text{Re} = \frac{cx^{(n+1)}}{\nu}.
 \end{aligned} \right\} \quad (16)$$

$$\left. \begin{aligned}
 182 \quad Nu &= -\frac{x \left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -\left(\frac{c(n+1)}{2\nu}\right)^{\frac{1}{2}} x^{\frac{(n+1)}{2}} \theta'(0), \\
 Nu &= -\left(\frac{(n+1)cx^{(n+1)}}{2\nu}\right)^{\frac{1}{2}} \theta'(0) = -\left(\frac{(n+1)}{2} \text{Re}\right)^{\frac{1}{2}} \theta'(0).
 \end{aligned} \right\} \quad (17)$$

183 Here $-\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is a heat flux from the surface of the sheet and Re is the local Reynold Number.

$$\left. \begin{aligned}
 184 \quad Sh &= -\frac{x \left(\frac{\partial C}{\partial y}\right)_{y=0}}{C_w - C_\infty} = -\left(\frac{c(n+1)}{2\nu}\right)^{\frac{1}{2}} x^{\frac{(n+1)}{2}} \phi'(0), \\
 Sh &= -\left(\frac{(n+1)cx^{(n+1)}}{2\nu}\right)^{\frac{1}{2}} \phi'(0) = -\left(\frac{(n+1)}{2} \text{Re}\right)^{\frac{1}{2}} \phi'(0).
 \end{aligned} \right\} \quad (18)$$

185 Here $-\left(\frac{\partial C}{\partial y}\right)_{y=0}$ is a mass flow rate from the surface of the sheet and Re is the local Reynold
 186 Number.

187 3. Method of Numerical Solution

188 The numerical solutions are obtained using the equations (10)-(12) and boundary conditions (14) for
 189 some values of the governing parameters, namely, Magnetic parameter (M), Casson parameter (b),
 190 Stretching Index parameter (n1), Suction parameter (S), Prandtl number (Pr), Radiation
 191 parameter(R), Schmith number (Sc), Thermal index parameter (m), Concentration index
 192 parameter(t), and Chemical reaction parameter(k₁) on the steady boundary layer flow are discussed in
 193 detail. The numerical computation is done using the MATLAB in-built Numerical Solver bvp4c. In
 194 the computation we have taken $\eta_\infty = 12$ and axis according to the clear figure-visibility.

195 4. Result Analysis and discussion

In order to analyze the behaviour of non-dimensional linear velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ profiles of the physical problem, numerical calculations are carried out for various values of Magnetic parameter (M), Casson parameter (b), Stretching Index parameter (n1), Suction parameter (S), Prandtl number (Pr), Radiation parameter (R), Schmidt number (Sc), Thermal index parameter (m), Concentration index parameter (t), and Chemical reaction parameter (c). Also, the Skin-friction, Nusselt and Sherwood numbers are discussed. For illustrations of the results, the numerical data are tabulated in tables 1-13 and plotted in Figures. 2–19.

The results for skin friction and local Nusselt number and reduced Nusselt numbers are compared with the previous published results, and are shown in Tables 1–3. It is observed that the obtained results are in good agreement with the published results.

Tables 1, 2 and 3 present the values of skin friction coefficient and local Nusselt number, and reduced Nusselt number for different values of nonlinear stretching parameter n and Prandtl number Pr, respectively. The present results are compared with the results of Cortell (2007), Wang (1989), Gorla and Sidawi (1994), Khan and Pop (2010), Ullah et al. (2016).

It is also observed from Table 1 that magnitude of skin friction coefficient $\left(1 + \frac{1}{b}\right) f''(0)$ increases with the increase in n1 and Table 2 shows local Nusselt number decreases with the increase in n1 and increases with increase in Pr.

Table 3 shows the comparison of the present results for reduced Nusselt number with various values of Pr with the results of Wang (1989), Gorla and Sidawi (1994), and Khan and Pop (2010). It is found that the rate of heat transfer coefficient increases as Pr increases.

For the convenience in calculation in Matlab following symbols are used for respective symbols used in modelling equations and boundary conditions:

$M = \text{Magnetic parameter}$, $b = \beta$ Casson Parameter, $n1 = n$ Stretching index parameter,
 $S = \text{Suction parameter}$, $Pr = \text{Prandtl number}$, $R = \text{Radiation parameter}$
 $Sc = \text{Schmidt number}$, $c = \text{Chemical reaction parameter } k_1$,
 $m = \text{Thermal index parameter}$, $t = \text{Concentration index parameter}$,

Table 1 Comparison of $-f''(0)$ for different values of n1 with

M=0; n1=1; b=10.^{^8}; Pr=1; R=inf; t=0; m=0; Sc=0.22; S=0.0; c=0;

M=0; n1=1; b=10. ^{^8} ; Pr=1; R=inf; t=0; m=0; Sc=0.22; S=0.0; c=0;
--

$-f''(0)$			
n1	Cortell(2007)	Ullah et al.(2016)	Present
0.0	0.627547	0.6276	0.627554111495105
0.2	0.766758	0.7668	0.766835311872827
0.5	0.889477	0.8896	0.889537395487564
1	1.0	1.0	1.000001131721189
3	1.148588	1.1486	1.148604214083713
10	1.234875	1.2349	1.234882849405469
100	1.276768	1.2768	1.276784084458797

225

226 **Table 2** Comparison of local Nusselt number $-\theta'(0)$, for different values of n1 with

227 $M=0; n1=0.2; b=10.^8; Pr=1$ and $5; R=\inf; t=0; m=0; Sc=0.22; S=0.0; k_1=0;$

228

$M=0; n1=0.2; b=10.^8; R=\inf; t=0; m=0; Sc=0.22; S=0.0; c=0;$				$M=0; n1=0.2; b=10.^8; R=\inf; t=0; m=0; Sc=0.22; S=0.0; c=0;$		
$-\theta'(0), Pr=1,$				$-\theta'(0), Pr=5,$		
n1	Cortell (2007)	Ullah et al.(2016)	Present	Cortell(2007)	Ullah et al. (2016)	Present
0.2	0.610262	0.6102	0.610203981361	1.607175	1.6076	1.607799764754
0.5	0.595277	0.5949	0.595204129502	1.586744	1.5868	1.586794023622
1.5	0.574537	0.5747	0.574736444244	1.557463	1.5576	1.557705129027
3.0	0.564472	0.5647	0.564671395375	1.542337	1.5430	1.543190746978

229

230 Table 3 Comparison of results for reduced Nusselt number $-\theta'(0)$, for different values of Pr with

231 $M=0; n1=1; b=10.^8; Pr=0.7; R=\inf; t=0; m=0; Sc=0.22; S=0.0; c=0;$

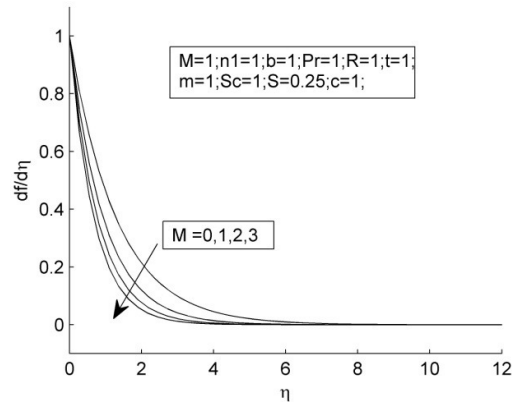
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$M=0; n1=1; b=10.^8; Pr=0.7; R=\inf; t=0; m=0; Sc=0.22; S=0.0; c=0;$					
$-\theta'(0), Pr=1,$					
Pr	Wang(1989)	Gorla and Sidawi(1994)	Khan and Pop(2010)	Ullah et al.(2016)	Present
0.7	0.4539	0.5349	0.4539	0.4544	0.454049789569805

2	0.9114	0.9114	0.9113	0.9113	0.911361122074545
7	1.8954	1.8905	1.8954	1.8953	1.895415847905391
20	3.3539	3.3539	3.3539	3.3538	3.353940662862386
70	6.4622	6.4622	6.4621	6.4638	6.462331549437675

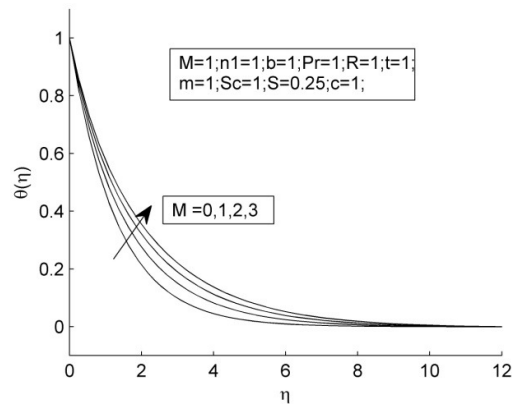
233 .

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235

236 Figure 2 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values
237 of Magnetic parameter M.



238



239 Figure 3 Temperature profile $\theta(\eta)$ with respect to η for some values of Magnetic parameter M.

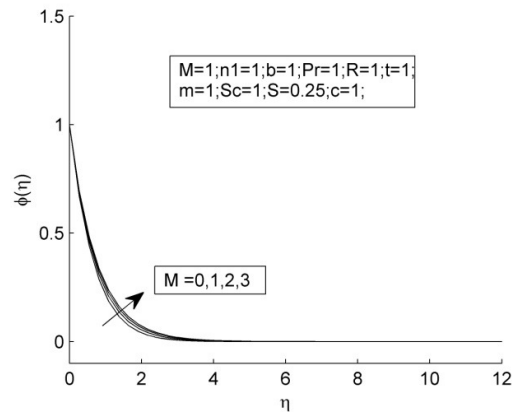


Figure 4 Concentration profile $\phi(\eta)$ with respect to η for some values of Magnetic parameter M .

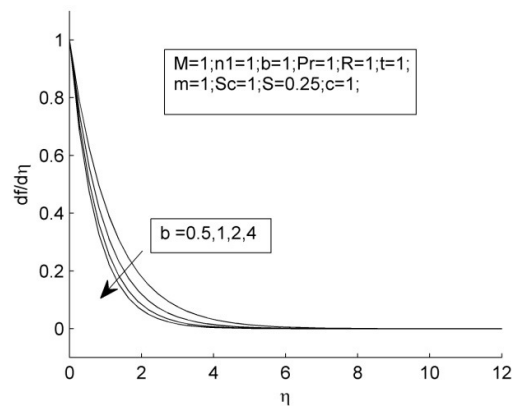


Figure 5 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Casson parameter b .

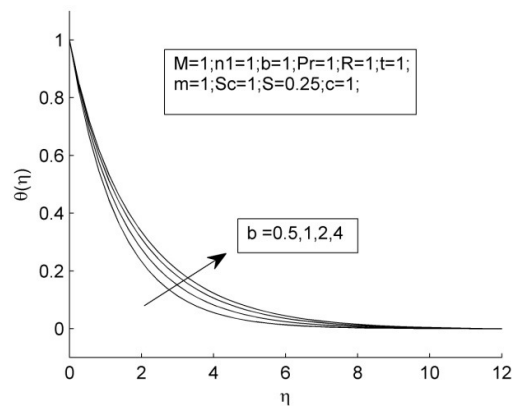


Figure 6 Temperature profile $\theta(\eta)$ with respect to η for some values of Casson parameter b .

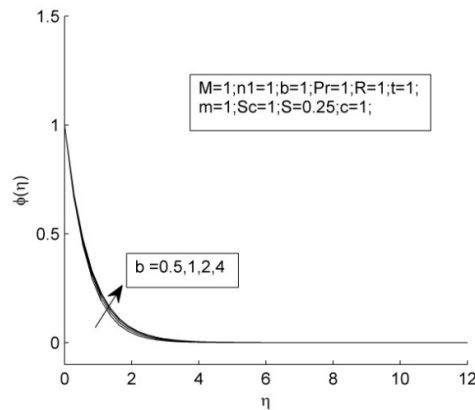


Figure 7 Concentration profile $\phi(\eta)$ with respect to η for some values of Casson parameter b .

The effect of M is illustrated in Fig. 2. All the trajectories are of same family with the magnetic number in between 0 and 3. It is seen that fluid velocity reduces as M increases. It is due to the fact that drag force, also known as Lorentz force is produced when magnetic field is applied to the fluid. This force has the tendency to slow down the velocity of fluid in the boundary layer. The decreasing pattern of velocity field with the increase in similarity variable shows that transverse magnetic field opposes the transport phenomenon. Thus the momentum boundary layer thickness decreases as M increases.

It is seen in Fig.3 that the temperature profile increases with increasing magnetic parameter and causes thickening of the corresponding boundary layer.

It is seen in Fig. 4 that the concentration profile increases with increasing Magnetic parameter and causes thickening of the corresponding boundary layer.

Fig. 5 shows that velocity profile decreases with the decrease in the values of Casson parameter and this causes decrease in corresponding boundary layer thickness.

Fig. 6 shows the temperature profile increases with the increase in Casson parameter and this causes increase in corresponding boundary layer thickness.

Fig. 7 shows the concentration profile increases with the increases with the increase in Casson parameter and thus causes increase is corresponding boundary layer thickness.

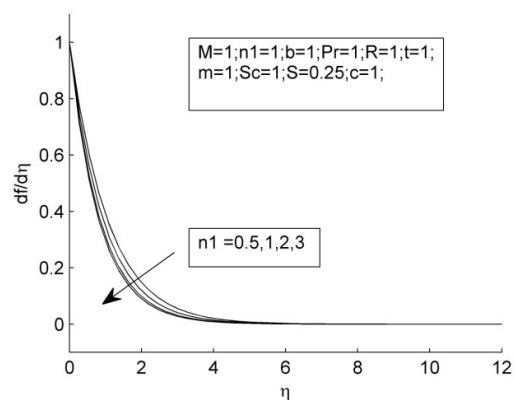


Figure 8 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Stretching index parameter n_1 .

Fig. 8 shows the velocity profile decreases with the increase in stretching index parameter and thus causes decrease in corresponding boundary layer thickness.

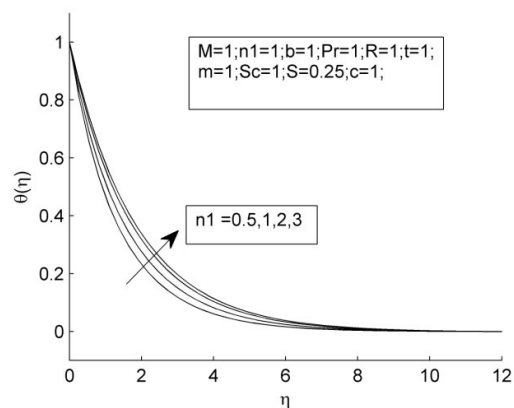


Figure 9 Temperature profile $\theta(\eta)$ with respect to η for some values of Stretching index parameter.

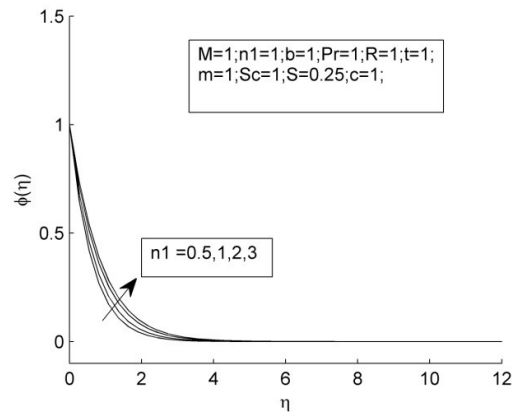


Figure 10 Concentration profile $\phi(\eta)$ with respect to η for some values of Stretching index parameter n_1 .

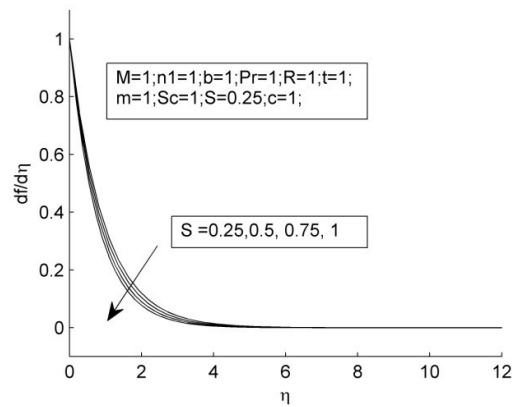


Figure 11 Velocity profile $f'(\eta)$ with respect to similarity transformation η for some values of Suction parameter S .

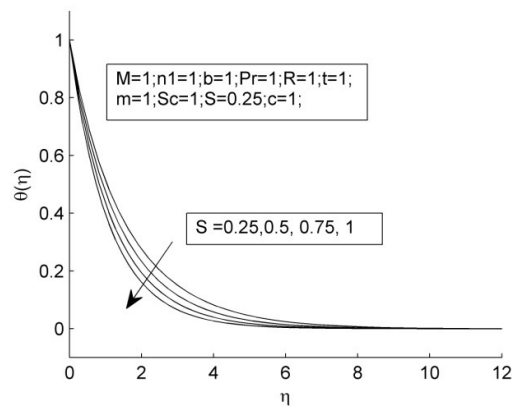
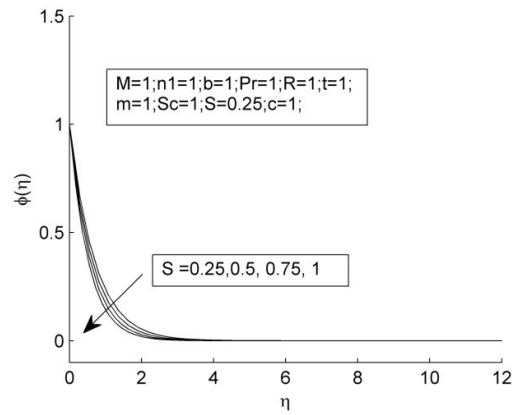


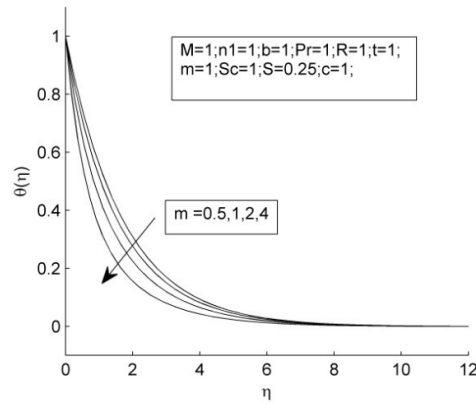
Figure 12 Temperature profile $\theta(\eta)$ with respect to η for some values of Suction parameter S .

282



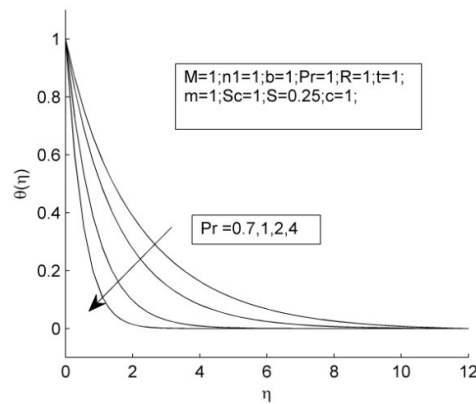
283

284 Figure 13 Concentration profile $\phi(\eta)$ with respect to η for some values of Suction parameter S .



285

286 Figure 14 Temperature profile $\theta(\eta)$ with respect to η for some values of Thermal index parameter
287 m .



288

289 Figure 15 Temperature profile $\theta(\eta)$ with respect to η for some values of Prandtl number Pr .

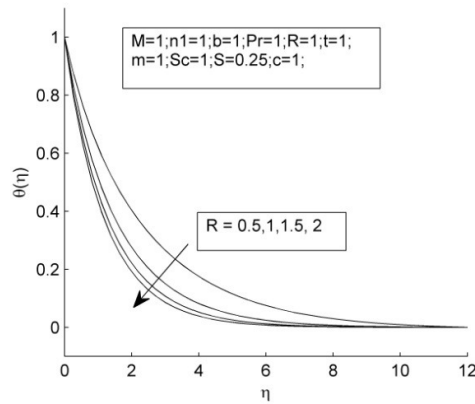


Figure 16 Temperature profile $\theta(\eta)$ with respect to η for some values of Radiation parameter R .

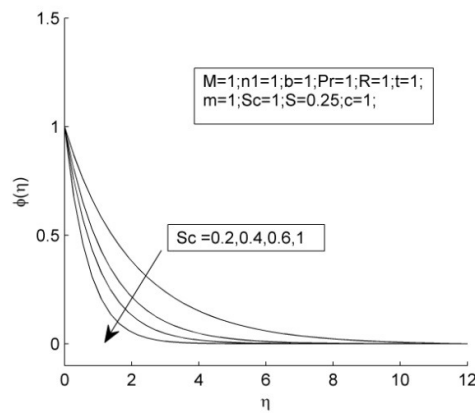


Figure 17 Concentration profile $\phi(\eta)$ with respect to η for some values of Schmidt parameter Sc .

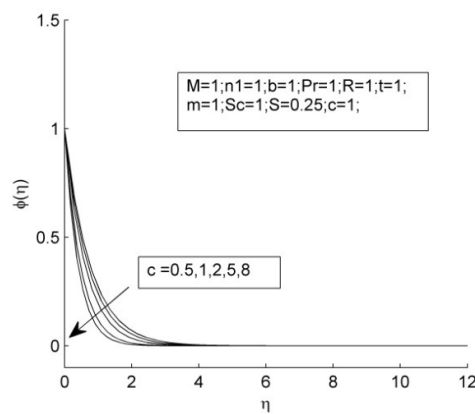
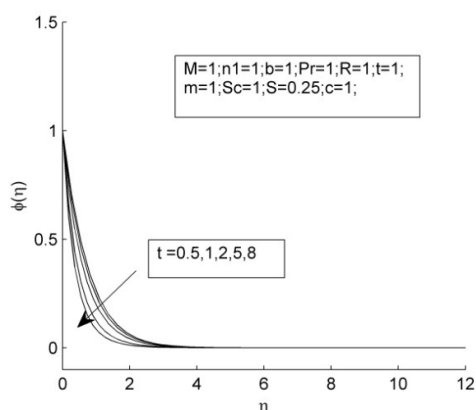


Figure 18 Concentration profile $\phi(\eta)$ with respect to η for some values of Chemical reaction parameter c .

298



299

300 Figure 19 Concentration profile $\phi(\eta)$ with respect to η for some values of Concentration Index
301 parameter t .

302 Fig 9 shows the temperature profile increases with the increase in stretching index parameter and thus
303 causes increase in corresponding boundary layer thickness.

304 Fig. 10 shows the concentration profile increases with the increase in stretching index parameter
305 and thus causes increase in corresponding boundary layer thickness.

306 Fig.11 shows the velocity profile decreases with the increase in suction parameter and thus causes
307 decrease in corresponding boundary layer thickness.

308 Fig. 12 shows the temperature profile decreases with the increase in suction parameter and thus causes
309 decrease in corresponding boundary layer thickness.

310 Fig. 13 shows the concentration profile decreases with the increases in suction parameter S , and thus
311 causes decrease in corresponding boundary layer thickness.

312 Fig. 14 shows the temperature profile decreases with the increases in Thermal index parameter m ,
313 and thus causes corresponding decreases in boundary layer thickness.

314 Fig. 15 shows the temperature profile decreases with the increases in Prandtl number and thus causes
315 decreases in corresponding boundary layer thickness.

316 Fig. 16 shows the temperature profile decreases with the increases in Radiation parameter R , and thus
317 causes decreases in corresponding boundary layer thickness.

318 Fig.17 shows the concentration profile decreases with the increases in Schmidt number Sc , and
319 thus causes decreases in corresponding boundary layer thickness.

Fig. 18 shows the concentration profile decreases with the increase in the value of Chemical reaction parameter c , and thus causes decrease in corresponding boundary layer thickness.

Fig. 19 shows the concentration profile decreases with the increases in the value of Concentration Index parameter t , and thus causes decreases in corresponding boundary layer thickness.

Table 4 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Magnetic parameter M .

$M=1;n1=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;$			
M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	-0.772380765882430	0.772380765882400	1.443691334729526
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.288838554005237	0.654907768407752	1.365372722305526
3	-1.478093956613316	0.617912526785317	1.339875593768916

Table 5 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Casson parameter b .

b	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-0.859225894421490	0.751334137204969	1.429688788817794
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.241036996916409	0.664825218572031	1.372092721488271
4	-1.368857754042093	0.638820030080323	1.354376225410678

Table 6 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Stretching Index parameter $n1$.

$M=1;n1=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;$			
$n1$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-0.918105532143194	0.810336626979573	1.578794600600632
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.192839815888933	0.600512982322554	1.201288066348144
3	-1.252024024566633	0.550357874663092	1.094265501542168

Table 4 shows with the increase in Magnetic parameter (M), Skin -friction, Nusselt number and Sherwood number decrease.

Table 5 shows with the increase in Casson parameter(b), Skin -friction, Nusselt number and Sherwood number decrease.

Table6 shows with the increase in Stretching Index parameter($n1$), Skin -friction, Nusselt number and Sherwood number decrease.

Table 7 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Suction parameter S.

M=1;n=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
S	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.25	-1.064451227933708	0.703353442961857	1.397884038796368
0.5	-1.132782222290359	0.781250513161171	1.548889097761843
0.75	-1.204926289151033	0.864402649566321	1.710870598067398
1.0	-1.280776407766019	0.952182976893372	1.882814891094115

Table 8 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Temperature power index parameter m.

M=1;n=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
m	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-1.064451228004112	0.579972075082871	1.397884039133279
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.064451229001629	0.922520745033117	1.397884051444187
4	-1.064451229007984	1.285239044709142	1.397888512222279

Table 7 shows with the increase in Suction parameter(S), Skin -friction decreases, Nusselt number and Sherwood number increase.

Table 8 shows with the increase in Thermal power Index parameter (m), Skin -friction first increases and then decreases, Nusselt number increases and Sherwood number first decreases and then increases.

Table 9 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of parameter Pr.

M=1;n=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
Pr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.7	-1.064451227935287	0.539046293903134	1.397884038793309
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.064451229001544	1.148963683105466	1.397884051475590
4	-1.064451229007306	1.828562782440114	1.397884279456529

Table 10 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Radiation parameter R.

M=1;n=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
R	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-1.064451227935287	0.519510958603281	1.397884038793309
1.0	-1.064451227933708	0.703353442961857	1.397884038796368
1.5	-1.064451229000330	0.803015948852864	1.397884051476741
2.0	-1.064451229000330	0.865904761663387	1.397884051476745

Table 9 shows with the increase in Prandtl number (Pr), Skin -friction first increases and then decreases, Nusselt number and Sherwood number increase.

Table 10 shows with the increase in Radiation parameter(R), Skin -friction first increases and then decreases, Nusselt number and Sherwood number increase.

Table 11 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Schmidt parameter Sc.

M=1;n l=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
Sc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.2	-1.064451227307101	0.703353431798897	0.517087265237630
0.4	-1.064451227305958	0.703353429832318	0.795180570166533
0.6	-1.064451227306725	0.703353431000844	1.021457499285938
1.0	-1.064451227933708	0.703353442961857	1.397884038796368

Table 12 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Chemical reaction parameter c.

M=1;n l=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
c	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-1.064451227935144	0.703353444895451	1.282872003805204
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.064451228001790	0.703353444160419	1.595139951113721
5	-1.064451228799512	0.703353036214121	2.055987515833043

Table 13 Comparison of local Skin-friction $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ for various values of Concentration power index parameter t.

M=1;n l=1;b=1;Pr=1;R=1;t=1;m=1;Sc=1;S=0.25;c=1;			
t	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	-1.064451227932965	0.703353443754825	1.246986619289261
1	-1.064451227933708	0.703353442961857	1.397884038796368
2	-1.064451228585952	0.703353100846418	1.671140561168520
5	-1.064451229008724	0.703353060524090	2.340352375400558

Table 11 shows with the increase in Schmidt number (Sc), Skin -friction first increases and then decreases, and Nusselt number first decreases and then increases, and Sherwood number increases.

Table 12 shows with the increase in Chemical reaction parameter(c), Skin -friction first increases and then decreases, and Nusselt number first decreases and then increases and Sherwood number increase.

Table 13 shows with the increase in Concentration power index (t), Skin -friction, Nusselt number decrease and Sherwood number increases.

5. Conclusion

In the present work we studied the effect of chemical reaction, and radiation on magnetohydrodynamic Casson fluid flow over non-linearly stretching surface with fluid suction. The effects of Magnetic, Casson, and, Prandtl number, Radiation parameters and Schmid number, Chemical reaction parameter, Stretching index, Suction, Thermal index, and Concentration index parameters on velocity, heat transfer, and concentration profiles, Skin-friction, Nusselt Number and Sherwood Number are computed and discussed numerically and presented through tables and graphs. From the work following conclusion are obtained.

1. Figs 2, 5, 8, 11 shows with the increase in the values of Magnetic parameter, Casson parameter Suction parameter, (M , b , n_1 , S) the velocity profile decreases, and this causes decrease in corresponding boundary layer thickness.
2. Figs 3, 6, 9 with the increase in Magnetic parameter, Casson parameter, Stretching index parameter (M , b , n_1), the temperature profile increases and thus causes increase in corresponding boundary layer thickness.
3. Figs 4, 7, 10 with the increase Magnetic parameter, Casson parameter, Stretching index parameter (M , b , n_1), the concentration profile increases with the increase in stretching index parameter and thus causes increase in corresponding boundary layer thickness.
4. Figs 12, 14, 15, 16 show with the increases in Prandtl number, Radiation parameters, Suction parameter, Thermal index parameter, (Pr , R , S , m ,) the temperature profile decreases, and thus causes decreases in corresponding boundary layer thickness.
5. Figs 13, 17, 18, 19 show with the increases in the value of Schmid number, Chemical reaction parameter, Suction parameter, concentration index parameter, (Sc , c , S , t) the concentration profile decreases, and thus causes decreases in corresponding boundary layer thickness.
6. Table 8, 9, 10, 11, 12 show Skin -friction first increases and then decreases, with the increase in the value of Prandtl number, Radiation parameters, Schmid number, Chemical reaction parameter, Thermal index parameter, (m , Pr , R , Sc , c) each.
7. Table 4, 5, 6, 7, 13 shows Skin -friction decreases, with the increase in the value of Magnetic parameter, Casson fluid parameter, stretching index parameter, Suction parameter, concentration index parameter (M , b , n_1 , S , t) each.

8. Table 7, 8, 9, 10 shows Nassult number increases, with the increase in the value of Prandtl number, Radiation parameters , Suction parameter, Thermal index parameter (Pr, R, S, m).
9. Table 11, 12, show, Nassult number first decreases and then increases, with the increase in the value of Schmith number, Chemical reaction parameter(Sc, c) each.
10. Table 4, 5, 6, 13 shows Nassult number decreases, with the increase in the value of Magnetic parameter, Casson fluid parameter, stretching index parameter, concentration index parameter(M, b, n1, t) each.
11. Table 7, 9, 10, 11, 12, 13 shows Sherwood number increases, with the increase in the value of Suction parameter, Prandtl parameter, Radiation parameter, Schmith parameter, Chemical reaction parameter, concentration index parameter(S, Pr, R, Sc, c, t) each.
12. Table 8 shows Sherwood number first decreases and then increases with the increase in Thermal power Index parameter (m).
13. Table 4, 5, 6 shows Sherwood number decreases, with the increase in the value of Magnetic parameter, Casson fluid parameter, stretching index parameter (M, b, n1) each.

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