

A New Modified approach for the improvement of new estimator using known value of Downton's method for estimating the population variance

Abstract: In the present study we have suggested a new modified ratio estimator for estimating the population variance by utilizing the auxiliary information of Downton's method. Properties associated with proposed estimators are assessed by mean square error and bias. We have also provided theoretical efficiency comparison and numerical illustration.

Keywords: Ratio estimator, Downton's Method; SRSWOR, MSE, Bias and Efficiency.

1. Introduction

Here we consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i=1,2,3,\dots,N$ given a vector $[Y_1, Y_2, Y_3, \dots, Y_N]$. Sometimes in sample surveys information on auxiliary variable X correlated with study variable Y , is available can be utilized to obtain the efficient estimator for the estimation of Population variance. "Here in this modification our aim is to estimate the population variance $S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2$ on the bases of random sample selected from population U " to obtain the improved and precise estimates. Various authors have proposed different ratio estimators for finite population variance by utilizing the ancillary information of different parameters of population such as Isaki [2], Kadilar and Cingi [3], Subramani and Kumarapandiyan [5] and Bhat *et al.* [1].

2. Material and Methods:

2.1 Notations: N = Population size. n = sample size. $\gamma = \frac{1}{n}$, Y = study variable. X = Auxiliary variable. $\bar{X}, \bar{Y}$ = Population means. $\bar{x}, \bar{y}$ = Sample means. S_y^2, S_x^2 = Population variances. s_y^2, s_x^2 = sample variances. C_x, C_y = Coefficient of variation. ρ = Correlation coefficient. $\beta_{1(x)}$ = Skewness of the auxiliary variable. $\beta_{2(x)}$ = Kurtosis of the auxiliary variable. $\beta_{2(y)}$ = Kurtosis of the study variable. M_d = Median of the auxiliary variable. $B(.)$ = Bias of the estimator. $MSE(.)$ = Mean square error. $\hat{S}_R^2$ = Ratio type variance estimator. $\hat{S}_{Kcl}^2, \hat{S}_{JG}^2$ = Existing modified ratio estimators. D = Downton's method. We have derived here the bias and mean square error of the proposed estimator to first order of approximation as given below:

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$. Further we can write $s_y^2 = S_y^2(1 + e_0)$ and

$s_x^2 = S_x^2(1 + e_1)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0, \quad E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1), \quad E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1),$$

$$E[e_0 e_1] = \frac{1-f}{n}(\lambda_{22} - 1)$$

The proposed estimator is given below:

$$\begin{aligned}
 42 \quad \hat{S}_p^2 &= S_y^2 \left[\frac{S_x^2 + \alpha D}{S_x^2 + \alpha D} \right] \\
 43 \quad \Rightarrow \quad \hat{S}_p^2 &= S_y^2 (1 + e_0) \left[\frac{S_x^2 + \alpha D}{S_x^2 + e_1 S_x^2 + \alpha D} \right] \\
 44 \quad \Rightarrow \quad \hat{S}_p^2 &= \frac{S_y^2 (1 + e_0)}{(1 + A_p e_1)} \text{ where } A_p = \frac{S_x^2}{(S_x^2 + \alpha D)} \\
 45 \quad \Rightarrow \quad \hat{S}_p^2 &= S_y^2 (1 + e_0) (1 + A_p e_1)^{-1} \\
 46 \quad \Rightarrow \quad \hat{S}_p^2 &= S_y^2 (1 + e_0) (1 - A_p e_1 + A_p^2 e_1^2 - A_p^3 e_1^3 + \dots) \\
 47 \quad &\text{Expanding and neglecting the terms more than 3rd order, we get} \\
 48 \quad \hat{S}_p^2 &= S_y^2 + S_y^2 e_0 - S_y^2 A_p e_1 - S_y^2 A_p e_0 e_1 + S_y^2 A_p^2 e_1^2 \\
 49 \quad \Rightarrow \quad \hat{S}_p^2 - S_y^2 &= S_y^2 e_0 - S_y^2 A_p e_1 - S_y^2 A_p e_0 e_1 + S_y^2 A_p^2 e_1^2 \\
 50 \quad (1) \\
 51 \quad &\text{By taking expectation on both sides of (1), we get} \\
 52 \quad E(\hat{S}_p^2 - S_y^2) &= S_y^2 E(e_0) - S_y^2 A_p E(e_1) - S_y^2 A_p E(e_0 e_1) + S_y^2 A_p^2 E(e_1^2) \\
 53 \quad Bias E(\hat{S}_p^2) &= S_y^2 A_p^2 E(e_1^2) - S_y^2 A_p E(e_0 e_1) \\
 54 \quad Bias E(\hat{S}_p^2) &= \gamma S_y^2 A_p [A_p (\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \\
 55 \quad (2) \\
 56 \quad &\text{Squaring both sides of (1) and (2), neglecting the terms more than 2nd order and taking} \\
 57 \quad &\text{expectation, we get} \\
 58 \quad E(\hat{S}_p^2 - S_y^2)^2 &= S_y^4 E(e_0^2) - S_y^4 A_p^2 E(e_1^2) - 2 S_y^4 A_p E(e_0 e_1) \\
 59 \quad MSE(\hat{S}_p^2) &= \gamma S_y^4 [(\beta_{2(y)} - 1) + A_p^2 (\beta_{2(x)} - 1) - 2 A_p (\lambda_{22} - 1)] \\
 60 \quad \mathbf{2.2 Efficiency conditions} &\text{ In this section, we have derived the efficiency conditions of} \\
 61 \quad &\text{proposed estimators with other existing estimators under which proposed estimator are} \\
 62 \quad &\text{performing better than the existing estimators} \\
 63 \quad &\text{The bias and Mean square error of existing ratio type estimators up to the first order of} \\
 64 \quad &\text{approximation is given by} \\
 65 \quad Bias E(\hat{S}_K^2) &= \gamma S_y^2 R_K [R_K (\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \\
 66 \quad (3) \\
 67 \quad MSE(\hat{S}_K^2) &= \gamma S_y^4 [(\beta_{2(y)} - 1) + R_K^2 (\beta_{2(x)} - 1) - 2 R_K (\lambda_{22} - 1)] \\
 68 \quad (4) \\
 69 \quad &\text{Where } R_K = \text{Constant, } K = 1, 2, 3 \\
 70 \quad &\text{Bias, MSE and constant of proposed estimators is given by} \\
 71 \quad Bias(\hat{S}_p^2) &= \gamma S_y^2 R_p [R_p (\beta_{2x} - 1) - (\lambda_{22} - 1)] \\
 72 \quad (5) \\
 73 \quad MSE(\hat{S}_p^2) &= \gamma S_y^4 [(\beta_{2y} - 1) + R_p^2 (\beta_{2x} - 1) - 2 R_p (\lambda_{22} - 1)] \\
 74 \quad (6) \\
 75 \quad R_p &= \text{Constant, } p = 1 \\
 76 \quad &\text{From Equation (4) and (6), we have} \\
 77 \quad MSE(\hat{S}_p^2) &\leq MSE(\hat{S}_K^2) \text{ if } \lambda_{22} \geq 1 + \frac{(R_p + R_K)(\beta_{2x} - 1)}{2} \\
 78 \quad MSE(\hat{S}_p^2) &\leq MSE(\hat{S}_K^2)
 \end{aligned}$$

$$\gamma S_y^4 [(\beta_{2y} - 1) + R_p^2(\beta_{2x} - 1) - 2R_p(\lambda_{22} - 1)] \leq \gamma S_y^4 [(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{22} - 1)]$$

(7)

$$\Rightarrow [(\beta_{2y} - 1) + R_p^2(\beta_{2x} - 1) - 2R_p(\lambda_{22} - 1)] \leq [(\beta_{2y} - 1) + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{22} - 1)]$$

(8)

$$\Rightarrow [1 + R_p^2(\beta_{2x} - 1) - 2R_p(\lambda_{22} - 1)] \leq [1 + R_K^2(\beta_{2x} - 1) - 2R_K(\lambda_{22} - 1)]$$

(9)

$$\Rightarrow (\beta_{2x} - 1)(R_p^2 - R_K^2) [-2R_p(\lambda_{22} - 1)] \leq [-2R_K(\lambda_{22} - 1)]$$

(10)

$$\Rightarrow (\beta_{2x} - 1)(R_p^2 - R_K^2) [-2(\lambda_{22} - 1)(R_p - R_K)] \leq 0$$

(11)

$$\Rightarrow (\beta_{2x} - 1)(R_p^2 - R_K^2) \leq [2(\lambda_{22} - 1)(R_p - R_K)]$$

(12)

$$\Rightarrow (\beta_{2x} - 1) \leq \frac{2(\lambda_{22} - 1)(R_p - R_K)}{(R_p^2 - R_K^2)}$$

(13)

$$\Rightarrow (\beta_{2x} - 1) \leq \frac{2(\lambda_{22} - 1)(R_p - R_K)}{(R_p - R_K)(R_p + R_K)}$$

(14)

$$\Rightarrow (\beta_{2x} - 1)(R_p + R_K) \leq 2(\lambda_{22} - 1)$$

(15)

By solving equation (15), we get

$$MSE(\hat{S}_p^2) \leq MSE(\hat{S}_K^2) \text{ if } \lambda_{22} \geq 1 + \frac{(R_p + R_K)(\beta_{2x} - 1)}{2}$$

2.3 Numerical Illustration:

We use the data of Murthy [4] page 228 in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y (study variable). We apply the proposed and existing estimators to this data set and the data statistics are given below:

N=80, $S_x=8.4542$, $A_2=0.9615$, $n=20$, $C_x=0.7507$, $A_3=0.9964$, $\bar{X} = 11.2624$, $\beta_{2(x)} = 2.8664$, $\bar{Y} = 51.8264$, $\beta_{2(y)} = 2.8664$, $\rho = 0.9413$, $\beta_{1(x)} = 1.05$, $\lambda_{22} = 2.2209$, $S_y=18.3569$, $M_d=7.5750$, $Q_1=9.318$, $C_y=0.3542$, $G = 9.0408$, $D = 8.0138$, $S_{pw} = 7.9136$

Table 1: Bias and Mean Square Error of the existing and the proposed estimators

Estimators	Bias	Mean Square Error
Isaki	10.8762	3925.1622
Kadilar&Cingi	10.4399	3850.1552
Subramani&Kumarapandiyan	6.1235	3180.7740
Proposed	5.1964	2469.7695

Table 2: Percent relative efficiency of proposed estimators with existing estimators

Estimators	Isaki	Kadilar and Cingi	Subramani and Kumarapandiyan
Proposed	158.9282	155.8912	128.7882

Decision

From the above study we revealed that our suggested estimator performs better than the existing estimators as the mean square error as well as bias is too much lower than existing estimators.

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