Variance Estimation in absence and presence of non-response

Abstract: In this present study, we have discussed the issue of presence and absence of non-response that we often face in survey estimation. We have suggested the estimators to estimate the population Variance in the presence and absence of non-response, using the linear combination of coefficient of skewness and quartiles as auxiliary information. The mean square errors of suggested estimators have been compared with existing estimators to carry out the precision and efficiency of modified estimators.

Key words: Simple random sampling, bias, mean square error, skewness and quartiles efficiency.

Introduction: First of all let us address the issue of absence of non-response. Here, we consider the finite population $U = \{U_1, U_2, ..., U_N\}$, consists of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on Ui, i=1,2,3,…,N giving a vector $Y = \{y_1, y_2, ..., y_N\}$. The goal is to estimate the population means $\bar{Y} = \frac{1}{N}\sum_{I=1}^{N} y_I$ or its variance $S_Y^2 = \frac{1}{N-1}\sum_{I=1}^{N}\left(y_i - \bar{y}\right)^2$ on the basis of random sample selected from a population “U’’ in the absence of non-response. The effort of modifying the efficient estimators for the population variance has been widely discussed by various authors such as Isaki [1], who proposed ratio and regression estimators. Latter on authors such as Upadhyaya and Singh [2], have also put their efforts to improve the efficiency of modified estimators over existing estimators. On the same lines, Kadilar and Cingi [3] , Singh et al (2015) [4] , have discussed the variance estimators to accelerate the efficiency of estimators by using auxiliary information Recreantly, M. A. Bhat et al (2018) [9] , Usman SHAHZAD (2017) [5] have contributed their great efforts to enhance the precision and efficiency of variance estimators in theory of variance estimation These efforts are now being regularly carried out by the authors to improve and develop the efficiency of estimators. Non-response, which is one of the serious issue has been discussed by various authors such as Isaki [1], Upadhyaya and Singh [2] , Kadilar and Cingi, [3], by using auxiliary information to estimate the population variance in presence of non response. Recently, Riaz et al (2014) [6], Singh et al (2015) [4] and Usman SHAHZAD (2017) [5], Sarandal et al. [7] and M. H. Hansen and W. N Hurtiz [8] have also addressed the issue of non-response by utilizing the auxiliary information to enhance the efficiency of estimators. In this paper our aim is to have the precise and efficient estimators for the estimation of population variance in the presence and absence of non-response.

MATERIALS AND METHODS

Notations: $N =$ Population size. $n =$ Sample size. $\gamma = \frac{1}{n}$, Y= study variable. X= Auxiliary variable. $\bar{X}$, $\bar{Y}$ = Population means. $\bar{x}$, $\bar{y} =$ Sample means. S_Y^2, $S_x^2 =$ population variances. s_y^2, s_x^2 = sample variances. C_x, $C_y =$ Coefficient of variation $\rho =$ coefficient $\beta_{1(x)} =$ Skewness of auxiliary variable $\beta_{2(x)} =$ Kurtosis of the auxiliary variable. $\beta_{2(y)} =$ Kurtosis of the study variable. M_d= Median of the auxiliary variable. $Q_1 =$ first quartile, $Q_2 =$ second quartile $Q_3 =$ third quartile $\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$ Where $\mu_{rs} = \frac{1}{N}\sum\left(Y_i - \bar{Y}\right)^r \left(X_i - \bar{X}\right)^s$.

B (.)=Bias of the estimator and MSE (.)= Mean square error.

$\hat{S}_{Is}^2 = s_y^2\left[\frac{S_x^2}{s_x^2}\right]$ Existing estimator proposed by Isaki

$\hat{S}_{US1}^2 =$ Existing estimator proposed by Upadhyaya &Singh.

$\hat{S}_{Kc1}^2$,= Existing modified ratio estimator (proposed by Kadilar & Cingi).

2. Section-1 existing estimators under absence of non-response:

2.1 Ratio type variance estimator proposed by Isaki [1]

$$\hat{S}_{Is}^2 = s_y^2\left[\frac{S_x^2}{s_x^2}\right] \quad (1)$$

The bias and mean square error of the estimator up to first order of approximation is given by the following expressions

$$\text{Bias } (\hat{S}_{IS}^2) = \gamma S_y^2 \left[\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right] \quad (2)$$

$$\text{MSE}((\hat{S}_{IS}^2) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right] \quad (3)$$

2.2 Ratio type variance estimator proposed by Upadhyaya and Singh [2]

$$\hat{S}^2_{US} = s_y^2\left[\frac{S_x^2+\beta_2(x)}{s_x^2+\beta_2(x)}\right] \tag{4}$$

The bias and mean square error of the estimator up to first order of approximation is give the following expressions

$$\text{Bias}((\hat{S}^2_{US}) = \gamma\, S_y^2 A_{US}\ \left[A_{US}\left(\beta_{2(x)}-1\right)-(\lambda_{22}-1)\ \right] \tag{5}$$

$$\text{MSE}((\hat{S}^2_{US}) = \gamma\, S_y^4\ \left[\left(\beta_{2(y)}-1\right)+A_{US}^2\ (\beta_{2(x)}-1)-\ 2A_{US}\,(\lambda_{22}-1)\ \right] \tag{6}$$

2.3 Ratio type variance estimator proposed by Kadilar and Cingi [3]

$$\hat{S}^2_{kc1} = s_y^2\left[\frac{S_x^2+C_x}{s_x^2+C_x}\right] \tag{7}$$

The bias and mean square error of the estimator up to first order of approximation is given by the following expressions

$$\text{Bias}((\hat{S}^2_{kc1}) = \gamma\, S_y^2 A_{kc}\ \left[A_1\left(\beta_{2(x)}-1\right)-(\lambda_{22}-1)\ \right] \tag{8}$$

$$\text{MSE}((\hat{S}^2_{kc1}) = \gamma\, S_y^4\ \left[\left(\beta_{2(y)}-1\right)+A_{kc}^2\ (\beta_{2(x)}-1)-\ 2A_{kc}\,(\lambda_{22}-1)\ \right] \tag{9}$$

2.4 Ratio type Variance estimator proposed by Singh et al (2015) [4]:

$$\hat{S}_s = S_y'^2\left[\frac{S_x^2+Q_1^2}{s_x^2+Q_1^2}\right] \tag{10}$$

The bias and mean square error of the estimator up to first order of approximation is given by the following expressions

$$\text{Bias}((\hat{S}^2_s) = \gamma\, S_y^2 A_s\ \left[\ A_s\left(\beta_{2(x)}-1\right)'\ -(\lambda_{22}-1)\ \right] \tag{11}$$

$$\text{MSE}((\hat{S}^2_s) = \gamma\, S_y^4\ \left[\left(\beta_{2(y)}-1\right)+A_s^2\ (\beta_{2(x)}-1)-\ 2A_s\,(\lambda_{22}-1)\ \right] \tag{12}$$

Table-1 Mean square error of existing estimators in absence of non response

Existing Estimators	Mean square error
by Isaki [1]	29216846.227
Upadhyaya and Singh [2]	29187686.03
Kadilar and Cingi [3]	29216846.224
Singh et al (2015) [4]:	28839478.44

3. Suggested estimators under the absence of non-response:

We have suggested, new modified ratio type variance estimators to estimate the population variance in presence and absence of non-response which are given below:-

$$\hat{S}^2_{p_1} = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_1^2)}{s_x^2+(\beta_{1x})(Q_1^2)}\right] \quad \hat{S}^2_{p_2} = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_2^2)}{s_x^2+(\beta_{1x})(Q_2^2)}\right] \quad \hat{S}^2_{p_3} = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_3^2)}{s_x^2+(\beta_{1x})(Q_3^2)}\right]$$

Where K is a Characterizing scalar to be determined such that the MSE of the proposed estimators is minimized. The bias and mean square error of the proposed estimators has been carried out by the following mathematical expression.

The bias and mean square error of proposed estimators up to first order approximation has been carried out by the following expression

Let $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$. Further we can write $s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_0)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0, \quad E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)}-1), \quad E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)}-1), \quad E[e_0e_1] = \frac{1-f}{n}(\lambda_{22}-1)$$

The proposed estimator can be written as

$$\hat{R}_{MS} = Ks_y^2(1+e_0)(1+R_ie_1)^{-1} \qquad (1)$$

Expanding the right hand side of above equation up to the first order approximation we ge

$$\hat{R}_{MS} = Ks_y^2(1+e_0 - R_ie_1 - R_ie_0e_1 + R_i^2e_1^2) \qquad (2)$$

After subtracting the population variance S_y^2 of study variable on both sides of above equation we get

$$\hat{R}_{MS} - S_y^2 = Ks_y^2(1+e_0 - R_i e_1 - R_i e_0 e_1 + R_i^2 e_1^2) - S_y^2 \qquad (3)$$

By taking expectations on both sides of above equation, we get the bias of the proposed estimators

$$\text{Bias}= \gamma K S_y^2\left[R_i^2\left(\beta_{2y}-1\right) - R_i\left(\lambda_{22-1}\right)\right] + S_y^2\left(K-1\right) \qquad (4)$$

The mean square error is obtained by squaring both sides of equation (1) and taking expectations on both sides up to first order of approximation as:-

$$\text{MSE}= S_y^4\left[K^2\gamma\left(\beta_{2y}-1\right) + \left(3K^2-2K\right)R_i^2\left(\beta_{2x}-1\right) - 2\left(2K^2-K\right)R_i\gamma\left(\lambda_{22}-1\right) + \left(K-1\right)^2\right] \qquad (5)$$

Where $\gamma = \frac{1-f}{n}$, MSE is minimum for $K= \frac{1+R_i^2\gamma\left(\beta_{2x}-1\right) - R_i\gamma\left(\lambda_{22}-1\right)}{1+\gamma\left(\beta_{2y}-1\right) + 3R_i^2\gamma\left(\beta_{2x}-1\right) - 4R_i\gamma\left(\lambda_{22}-1\right)}$

The Minimum MSE for the estimator, optimum value of K is :

$$\text{MSE } MSE_{\min.}\hat{R}_{MS} = S_y^4\left[1 - \frac{\left\{\left(1+R_i^2\gamma\left(\beta_{2x}-1\right) - R_i\gamma\left(\lambda_{22}-1\right)\right)^2\right\}}{1+\gamma\left(\beta_{2y}-1\right) + 3R_i^2\gamma\left(\beta_{2x}-1\right) - 4R_i\gamma\left(\lambda_{22}-1\right)}\right]$$

The based on the condition if efficiency conditions of proposed estimators is

$$\textbf{MS } MSE_{\min.}(\ ) - MSE(\ ) = s_y^2\left[1 - \frac{\left\{1-R_1^2\left(\beta_{2x}-1\right) - R_1\left(\lambda_{22}-1\right)\right\}}{\left\{1+\left(\beta_{2y}-1\right) + 3R_1^2\left(\beta_{2x}-1\right) - 4R_1\left(\lambda_{22}-1\right)\right\}}\right] < \mathbf{0}$$

Numerical Illustration:

We use the data set presented in Sarandal et al (1992) concerning (P85) 1985 population considered as Y and RMT85 revenue from 1985 municipal taxation in millions of kronor considered as X. Descriptive statistics is given below.

Sarandal population [7]

$$N = 234, n = 35, \bar{Y} = 29.3626, \bar{X} = 245.088, S_y = 51.556, S_x = 596.332, \rho = 0.96, \beta_{2y} = 89.231, \beta_{2x} = 89.189$$
$$\lambda_{22} = 4.041, \beta_{1x} = 8.83, \beta_{1y} = 8.27, TM = 167.4, Q_1 = 67.75, Q_2 = 113.5, Q_3 = 230.25, C_x = 2.43, D_1 = 49.0,$$
$$D_2 = 63.0, D_3 = 75.0, D_4 = 90.0, D_5 = 113.5, D_6 = 145.9, D_7 = 197.9, D_8 = 271.1, D_9 = 467.5, D_{10} = 6720.0$$

We consider 20% weight for non-response (missing values) and have considered last 47 values as non-respondents results are as follows:-

$l=2, S_{y2}^2=2.9167, \beta_2(y_2)=11.775, N_2=47$. We apply the proposed and existing estimators to this data set and the data statistics is given below:

Table-2 Mean square error of existing estimators in absence of non response

Existing Estimators	**Mean square error**
Isaki [1]	**29216846.227**
Upadhyaya and Singh [2]	**29187686.03**
Kadilar and Cingi [3]	**29216846.224**
Singh et al (2015) [4]:	**28839478.44**

Table-3 Mean square error of proposed estimators in absence of non response

Proposed Estimators	**Mean square error**
$\hat{S}_{p_1}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_1^2)}{s_x^2+(\beta_{1x})(Q_1^2)}\right]$	**2690922.57**
$\hat{S}_{p_2}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_2^2)}{s_x^2+(\beta_{1x})(Q_2^2)}\right]$	**2817688.72**
$\hat{S}_{p_3}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_3^2)}{s_x^2+(\beta_{1x})(Q_3^2)}\right]$	**3930301.6**

Table-4 Percent relative efficiency of existing and proposed estimators

	Existing Estimators			
Proposed Estimators	**Isaki [1]**	**Upadhyaya and Singh [2]**	**Kadilar and Cingi [3]**	**Singh et al (2015) [4]:**
$\hat{S}_{p_1}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_1^2)}{s_x^2+(\beta_{1x})(Q_1^2)}\right]$	1085.75	1084.67	1085.75	1071.73
$\hat{S}_{p_2}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_2^2)}{s_x^2+(\beta_{1x})(Q_2^2)}\right]$	1036.90	1035.87	1036.90	1023.51
$\hat{S}_{p_3}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_3^2)}{s_x^2+(\beta_{1x})(Q_3^2)}\right]$	743.37	742.63	743.37	733.77

Section-2.

4. Existing estimators addressed in presence of non response

Hansen and Hurwitz (1946) sub Sampling scheme is the most popular scheme, used for the Non-response problems Let us consider a finite population consisting of *N* units. Let *y* be the character under study and a simple random sample of size *n* is drawn without replacement, of which n_1 units respond and n_2 units do not respond. From the n_2 non-respondents we select a sample of size $r = \frac{n_2}{k}, (k \geq 1)$ where *k* is the inverse sampling rate at the second phase sample of size *n* (fixed in advance) and from whom we collect the required information. It is assumed here that all the *r* units respond fully this time.

Let N_1 and $N_2 = N - N_1$ be the sizes of the responding and non-responding units from the finite population $N; W_1 = \frac{N_1}{N}, W_2 = \frac{N_2}{N}$ are the corresponding weights

Hansen and Hurwitz (1946) unbiased estimator under non-response is given by

$Var(\hat{T}') = S_y^4(\beta_{2y} - 1) + WS_{y2}^4\beta_2(y_2)^*$ where W= $\frac{N_2(l-1)}{nN}$, Where, l= sampling inverse rate

4.1 Ratio type variance estimator proposed by Isaki [1]

$$\hat{S}_{Is}^2 = s_y'^2\left[\frac{S_x^2}{s_x^2}\right] \qquad (1)$$

The bias and mean square error of the estimator up to first order of approximation is given by the following expressions

$$\text{Bias } (\hat{S}_{IS}^2) = \gamma S_y^2\left[\left(\beta_{2(x)} - 1\right)' - (\lambda_{22} - 1)\right] \qquad (2)$$

$$\text{MSE}((\hat{S}_{IS}^2) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right)' + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right] \qquad (3)$$

Where, $(\beta_{2y} - 1)' = \left(\beta_{2y} - 1\right) + \frac{WS_{y2}^4(\beta_{2y})^*}{S_y^4} = \frac{Var(\hat{T}')}{S_y^4}$

4.2 Ratio type variance estimator proposed by Upadhyaya and Singh [2]

$$\hat{S}_{US}^2 = s_y'^2\left[\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right] \qquad (4)$$

The bias and mean square error of the estimator up to first order of approximation is give the following expressions

$$\text{Bias}((\hat{S}^2_{US}) = \gamma S^2_y A_{US} \left[A_{US}\left(\beta_{2(x)} - 1\right)' - (\lambda_{22} - 1) \right] \quad (5)$$

$$\text{MSE}((\hat{S}^2_{US}) = \gamma S^4_y \left[\left(\beta_{2(y)} - 1\right)' + A^2_{US}\ (\beta_{2(x)} - 1) - \ 2A_{US}\ (\lambda_{22} - 1) \right] \quad (6)$$

Where $(\beta_{2y} - 1)' = \left(\beta_{2y} - 1\right) + \dfrac{WS^4_{y2}(\beta_{2y})^*}{S^4_y} = \dfrac{Var(\hat{T}')}{S^4_y}$

4.4 Ratio type variance estimator proposed by Singh (2015) [4]

$$\hat{S}^2_s = s'^2_y \left[\frac{S^2_x + Q^2_1}{s^2_x + Q^2_1} \right] \quad (7)$$

The bias and mean square error of the estimator up to first order of approximation is given by the following expressions

$$\text{Bias}((\hat{S}^2_s) = \gamma S^2_y A_1 \left[A_s\left(\beta_{2(x)} - 1\right)' - (\lambda_{22} - 1) \right] \quad (8)$$

$$\text{MSE}((\hat{S}^2_s) = \gamma S^4_y \left[\left(\beta_{2(y)} - 1\right)' + A^2_s\ (\beta_{2(x)} - 1) - \ 2A_s\ (\lambda_{22} - 1) \right] \quad (9)$$

Where $(\beta_{2y} - 1)' = \left(\beta_{2y} - 1\right) + \dfrac{WS^4_{y2}(\beta_{2y})^*}{S^4_y} = \dfrac{Var(\hat{T}')}{S^4_y}$

Proposed Estimators under presence of non-response

$$\hat{S}^2_{p_1} = Ks'^2_y \left[\frac{S^2_x + (\beta_{1x})(Q^2_1)}{s^2_x + (\beta_{1x})(Q^2_1)} \right] \quad \hat{S}^2_{p_2} = Ks'^2_y \left[\frac{S^2_x + (\beta_{1x})(Q^2_2)}{s^2_x + (\beta_{1x})(Q^2_2)} \right] \quad \hat{S}^2_{p_3} = Ks'^2_y \left[\frac{S^2_x + (\beta_{1x})(Q^2_3)}{s^2_x + (\beta_{1x})(Q^2_3)} \right]$$

Where K is a Characterizing scalar to be determined such that the MSE of the proposed estimators is minimized. The bias and mean square error of the proposed estimators has been carried out by the following mathematical expression.

The bias and mean square error of proposed estimators up to first order approximation has been carried out by the following expression

Let $e_0 = \dfrac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \dfrac{s_x^2 - S_x^2}{S_x^2}$. Further we can write $s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_0)$ and from the definition of e_0 and e_1 we obtain:

$$E[e_0] = E[e_1] = 0, \quad E[e_0^2] = \frac{1-f}{n}(\beta_{2(y)} - 1), \quad E[e_1^2] = \frac{1-f}{n}(\beta_{2(x)} - 1), \quad E[e_0 e_1] = \frac{1-f}{n}(\lambda_{22} - 1)$$

The proposed estimator can be written as

$$\hat{R}_{MS} = K s_y^2 (1+e_0)(1+R_i e_1)^{-1} \qquad (1)$$

Expanding the right hand side of above equation up to the first order approximation we ge

$$\hat{R}_{MS} = K s_y^2 (1 + e_0 - R_i e_1 - R_i e_0 e_1 + R_i^2 e_1^2) \qquad (2)$$

After subtracting the population variance S_y^2 of study variable on both sides of above equation we get

$$\hat{R}_{MS} - S_y^2 = K s_y^2 (1 + e_0 - R_i e_1 - R_i e_0 e_1 + R_i^2 e_1^2) - S_y^2 \qquad (3)$$

By taking expectations on both sides of above equation, we get the bias of the proposed estimators

Bias= $\gamma K S_y^2 \left[R_i^2(\beta_{2y} - 1) - R_i(\lambda_{22-1})\right] + S_y^2(K-1)$ (4)

The mean square error is obtained by squaring both sides of equation (1) and taking expectations on both sides up to first order of approximation as:-

MSE= $S_y^4 \left[K^2 \gamma(\beta_{2y} - 1) + (3K^2 - 2K) R_i^2 (\beta_{2x} - 1) - 2(2K^2 - K) R_i \gamma (\lambda_{22} - 1) + (K-1)^2\right]$ (5)

Where $\gamma = \dfrac{1-f}{n}$, MSE is minimum for K= $\dfrac{1 + R_i^2 \gamma(\beta_{2x} - 1) - R_i \gamma(\lambda_{22} - 1)}{1 + \gamma(\beta_{2y} - 1) + 3R_i^2 \gamma(\beta_{2x} - 1) - 4R_i \gamma(\lambda_{22} - 1)}$

The Minimum MSE for the estimator, optimum value of K is :

MSE $MSE_{\min}.\hat{R}_{MS} = S_y^4 \left[1 - \dfrac{\left\{\left(1 + R_i^2 \gamma(\beta_{2x} - 1) - R_i \gamma(\lambda_{22} - 1)\right)^2\right\}}{1 + \gamma(\beta_{2y} - 1) + 3R_i^2 \gamma(\beta_{2x} - 1) - 4R_i \gamma(\lambda_{22} - 1)}\right]$

The efficiency conditions of proposed estimators is based on the condition if

MS $MSE_{\min.}(\ )-MSE(\ ) = s_y^2\left[1-\frac{\left\{1-R_1^2\left(\beta_{2x}-1\right)-R_1\left(\lambda_{22}-1\right)\right\}}{\left\{1+\left(\beta_{2y}-1\right)+3R_1^2\left(\beta_{2x}-1\right)-4R_1\left(\lambda_{22}-1\right)\right\}}\right]$ **< 0**

Table-5 Mean square error of existing estimators in absence of non response

Existing Estimators	Mean square error
Isaki [1]	**29215130.96**
Upadhyaya and Singh [2]	**29185970.72**
Singh et al (2015) [4]:	**28837763.13**

Table-6 Mean square error of proposed estimators in absence of non response

Proposed Estimators	Mean square error
$\hat{S}_{p_1}^2 = Ks_y'^2\left[\frac{S_x^2+(\beta_{1x})(Q_1^2)}{s_x^2+(\beta_{1x})(Q_1^2)}\right]$	**22343807.45**
$\hat{S}_{p_2}^2 = Ks_y'^2\left[\frac{S_x^2+(\beta_{1x})(Q_2^2)}{s_x^2+(\beta_{1x})(Q_2^2)}\right]$	**2012750.82**
$\hat{S}_{p_3}^2 = Ks_y'^2\left[\frac{S_x^2+(\beta_{1x})(Q_3^2)}{s_x^2+(\beta_{1x})(Q_3^2)}\right]$	**3855736.26**

Table-7 Percent relative efficiency of existing and proposed estimators

Proposed Estimators	**Isaki [1]**	**Upadhyaya and Singh [2]**	**Singh et al (2015) [4]:**
$\hat{S}_{p_1}^2 = Ks_y'^2\left[\frac{S_x^2+(\beta_{1x})(Q_1^2)}{s_x^2+(\beta_{1x})(Q_1^2)}\right]$	1246.48	1245.23	1230.38
$\hat{S}_{p_2}^2 = Ks_y'^2\left[\frac{S_x^2+(\beta_{1x})(Q_2^2)}{s_x^2+(\beta_{1x})(Q_2^2)}\right]$	1451.50	1450.05	1432.75
$\hat{S}_{p_3}^2 = Ks_y^2\left[\frac{S_x^2+(\beta_{1x})(Q_3^2)}{s_x^2+(\beta_{1x})(Q_3^2)}\right]$	757.70	756.94	731.68

Discussion and Conclusion

We suggest the estimators in presence and absence of non-response whose mean square errors have been obtained and compared with usual unbiased estimator and these suggested estimators performed better than the existing estimators. From table-1 and table-2 percent relative efficiency clearly reveals that suggested estimators proven better than the existing estimators, when comparison is made between existing and proposed estimators.

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