Hidden oscillations in fractional-order multidimensional chaotic systems

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ABSTRACT

In this paper we propose numerical method for the study of localized hidden oscillation in multidimensional fractional chaotic dynamical systems. Implementation of the algorithm is different from the classical method of Aizerman and Kaplan. The reconstructed mapping is presented.

Keywords: chaotic system, time series, SSA – algorithm, lifting, Poincare diagram.

1. INTRODUCTION

The implementation of the tasks on the stability of multidimensional chaotic systems of fractional order can manifest hidden oscillation, that are not established after the transition process from the neighborhoods of the stationary states. Here a simple simulation can lead to erroneous results.

Therefore, numerous results dealing with mechanisms of the generation of attractors, their localization in the phase space, and the evolution of their characteristics where obtained with the use of computer modeling well – known examples of the existence of hidden attractor in multidimensional models of automated control systems are given by counterexamples to the Aizerman and Kaplan conjecture, where the unique stable-in-small equilibrium co-exists with an orbital stable cycle [1].

Effectively verified conditions for the existence hidden orbital stable cycles in some class multidimensional systems were obtained in [1].

Thus, the is a statement

2. FORMULATION OF THE PROBLEM

Consider the following n-dimensional-fractional-order chaotic system [2]

$$D^q X = F\left(X, X_0, \theta\right), \tag{1}$$

where $X = \left(x_1, x_2, \dots, x_n\right)^T \in R^n$ denotes the n-dimensional state vector of the original system; X_0 - represents the system initial state, $q = \left(q_1, q_2, \dots, q_n\right)^T \in R^n$ is a set of fractional order of the original system, and $\theta = \left(\theta_1, \theta_2, \dots, \theta_D\right)^T \in R^D$ is the value of original system parameters.

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Let the fractional-order derivative of the function $f(t)$ in the Caputo sense is defined as [2]:

$$D^q f(t) = J^{m-q} f^{(m)}(t). \tag{2}$$

Here, q is the fractional order, m is an integer that satisfies $m-1 \le q < m, f^{m}(t)$ is the ordinary m th derivative of f, and J^{μ} is the Riemann-Liouville integral operator of order $\mu > 0$, defined by

$$J^{\mu} g(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} g(\tau) d\tau, \tag{3}$$

where $\Gamma(\cdot)$ denotes the gamma function. A particularly important case in many engineering applications is $0 < q < 1$. In this situation, Eq. (2) together with Eq. (3)

$$D_*^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t (t-\tau)^{-q} f^{t}(\tau) d\tau. \tag{4}$$

The operator D_*^q is often called "q th-order Caputo differential operator" and will be used throughout the paper.

Given a fractional-order hyperchaotic systems:

Fractional-order Rabinovich-Fabrikant system following [3]:

$$\left.\begin{aligned} \dot{x}_1 &= x_2\left(x_3 - 1 + x_1^2\right) + \gamma x_1, \\ \dot{x}_2 &= x_1\left(3x_3 + 1 - x_1^2\right) + \gamma x_2, \\ \dot{x}_3 &= -2x_3\left(x_1 x_2 + \alpha\right), \\ \dot{x}_4 &= -3x_3\left(x_2 x_4 + \delta\right) + x_4^2, \end{aligned}\right\} \tag{5}$$

where $\alpha = 0.14,\ \gamma = 1.1,\ -0.01 \le \delta \le 7650$.

The fractional-order Chen system as follows [4]:

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$$
\begin{aligned}
\frac{d^{\alpha}x}{dt^{\alpha}} &= a(y-x)+w,\\
\frac{d^{\alpha}y}{dt^{\alpha}} &= bx-xz+cy,\\
\frac{d^{\alpha}z}{dt^{\alpha}} &= xy-dz,\\
\frac{d^{\alpha}w}{dt^{\alpha}} &= yz+rw,
\end{aligned}
\tag{6}
$$

where $a=35,\ b=7,\ c=12,\ d=3,\ r=0.5$ and $\alpha=0.9$.

Let $\hat{x}_{1,2,} =_{\alpha} \{x_n\}_{n=0}^{N}$ is the mapping of the (5-6) hyperchaotic fractional-order and (3) hyperchaotic systems.

2.1. SSA Algorithm

We consider a time series $Y_T=(y_1,\ldots,y_T)$. Fix $L(L\le T/2)$, the window length, and let $K=T-L+1$ [5].

Step 1. (*Computing the trajectory matrix*): this transfers a one-dimensional time series $Y_T=(y_1,\ldots,y_T)$ into the multi-dimensional series $X_1,\ldots,X_K$ with vectors $X_i=(y_i,\ldots,y_{i+L-1})' \in \mathbf{R}^L$, where $K=T-L+1$. The single parameter of the embedding is the *window length* L, an integer such that $2\le L\le T$. The result of this step is the trajectory matrix $\mathbf{X}=[X_1,\ldots,X_K]$, [5]:

$$
\mathbf{X}=(x_{ij})_{i,j=1}^{L,K}=\begin{pmatrix} y_1 & y_2 & y_3 & \cdots & y_K\\ y_2 & y_3 & y_4 & \cdots & y_{K+1}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ y_L & y_{L+1} & y_{L+2} & \cdots & y_T \end{pmatrix}. \tag{7}
$$

Note that the trajectory matrix $\mathbf{X}$ is a Hankel matrix, which means that all the elements along the diagonal $i+j=const$ are equal.

Step 2. Compute the matrix $\mathbf{XX}^T$.

Step 3. Compute the eigenvalues and eigenvectors of the matrix $\mathbf{XX}^T$ and represent it in the form $\mathbf{XX}^T=P\Lambda P^T$. Here $\Lambda=diag(\lambda_1,\ldots,\lambda_L)$ is the diagonal matrix of eigenvalues of $\mathbf{XX}^T$ ordered so that $\lambda_1\ge\lambda_2\ge\ldots\ge\lambda_L\ge 0$ and $P=(P_1,P_2,\ldots,P_L)$ is the corresponding orthogonal matrix of eigen-vectors of $\mathbf{XX}^T$.

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Step 4. *(Selection of eigen-vectors):* select a group of l $(1 \le l \le L)$ eigen-vectors $P_{i_1}, P_{i_2}, \ldots, P_{i_l}$.

The grouping step corresponds to splitting the elementary matrices $\mathbf{X}_i$ into several groups

and summing the matrices within each group. Let $I = \{i_1, \ldots, i_l\}$ be a group of indices $i_1, \ldots, i_l$. Then the matrix $\mathbf{X}_I$ corresponding to the group I is defined as $\mathbf{X}_I = \mathbf{X}_{i_1} + \cdots + \mathbf{X}_{i_l}$.

Step 5. *(Reconstruction of the one-dimensional series):* compute the matrix $\tilde{\mathbf{X}} = \left\|\tilde{x}_{i,j}\right\| = \sum_{k=1}^{l} P_{i_k} P_{i_k}^T \mathbf{X}$ as an approximation to $\mathbf{X}$. Transition to the one-dimensional series can now be achieved by averaging over the diagonals of the matrix $\tilde{\mathbf{X}}$ [5].

It is known, the singularly-spectral analysis is effective in a combination with wavelet-transformation [6]. It is connected by that the signal can have a changing frequency.

Further cleaned components related to the trend and noise. After the restoration of a number using wavelet-transform [6].

In this paper, for the purpose of localization and reconstruction of abnormal components of fractional dynamic chaotic multidimensional maps, it proposed the use singularly-spectral analysis in combination with lifting method [7].

Lifting methods of processing of the information make possible wavelet the stretching's and shifts of one function.

The advantage of the lifting scheme is:

1. the conversion process occurs quickly;
2. the set of wavelet-coefficients occupies a volume that matches the original data;
3. return transformation restores a signal very precisely.

2.2. Lifting scheme

Briefly, the mechanism is a follows [7]. Let the original signal s_j contains 2^j points. Transformation involves three steps (split-predict-update), which will yield two sets of points s_{j-1} and d_{j-1}.

Split

From in s_j shape two new not crossed sets. We note that the division of the set into two depends on the type of wavelet. For example, Lazy wavelet distinguishes $even_{j-1}$ and odd_{j-1} samples.

Formally it looks as [7]:

$$\left(even_{j-1}, odd_{j-1}\right) = S\left(s_j\right). \tag{8}$$

Predict

Here is calculated the difference between true and predicted values and defines coefficients d_{j-1} [7]:

$$d_{j-1} = odd_{j-1} - P\left(even_{j-1}\right), \tag{9}$$

where P - the predicting operator.

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Update

On this step, the help of the operator U , calculate coefficients s_{j-1} [7]:

$$s_{j-1} = even_{j-1} + U\left(d_{j-1}\right). \quad (10)$$

The described algorithm of transformation of data lifting-scheme is presented in figure 1 [7].

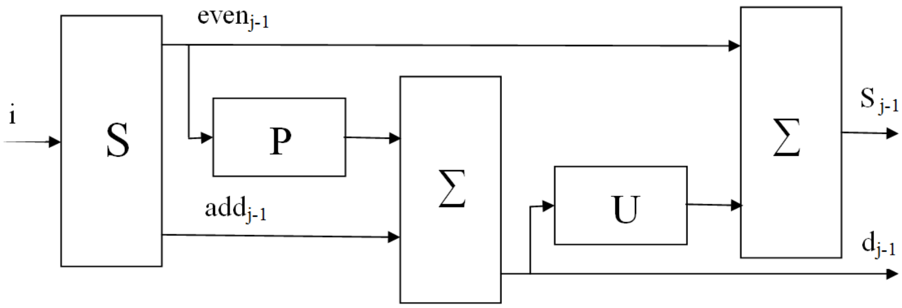

Fig. 1 Constructing the wavelet-coefficients in the lifting-scheme.

Thus, lifting schema generates two sets of coefficients s_{j-1} and d_{j-1} , each of which is less than half the length of the initial signal.

From here s_{j-1} reflects behavior of a signal in the big scale, and a coefficients d_{j-1} shows difference an initial signal from s_{j-1}.

In this paper, the realization of lifting scheme is based on the use of Haar wavelets and Doubechies [7].

The goal of the problems:

- **I** - determine the influence on the function of fractional sawtooth at hyperchaotic systems.
- **ii** - determine the stability systems.
- **iii** - approximate result with subsequent reconstruction mapping.
- **iiii** - construction Poincare recurrence diagram.

3. ALGORITHM

Step 1. Perturbation of the system (5 - 6) as [8]:

$\eta : D^q x_i \vee A\, frac\left(\frac{x}{T} + \varphi\right)$, where $frac\,(x)$ is the fractional part.

$frac\,(x) = x - [x]$, A is amplitude, T is the period of the wave, and φ is its phase.

Step 2. Determine the stability systems [8].

Step 3. Produce the singular-spectrum analysis for systems (5 - 6).

Step 4. Produce signal (Step 3) reconstruction using a lifting scheme.

Step 5. Construction Poincare recurrence diagrams.

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4. VISUALIZATION OF SIMULATION

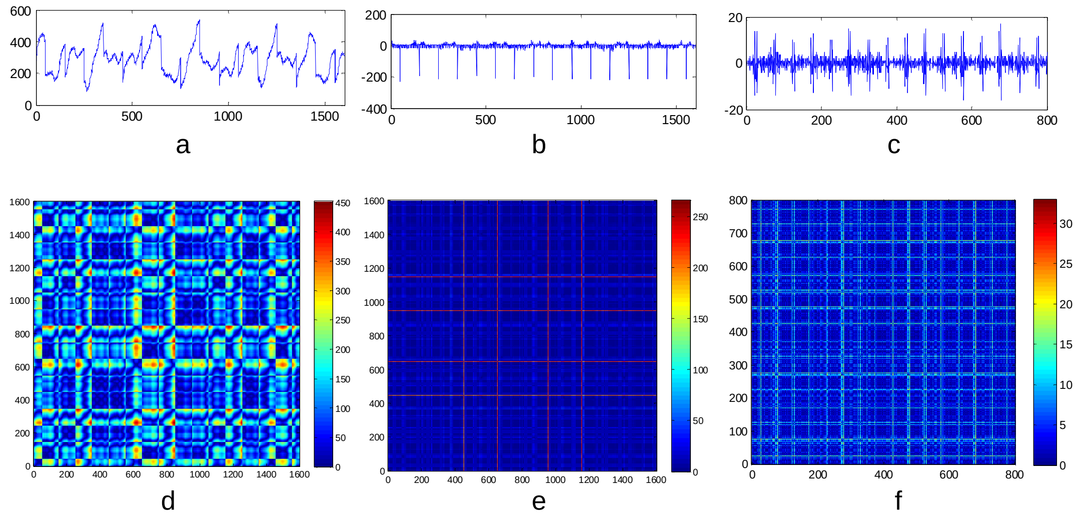

Fig. 2 Fractional-order chaotic Chen system;
a - signal with noise, b - stability, c - signal reconstruction,
d, e, f - Poincare recurrence diagrams.

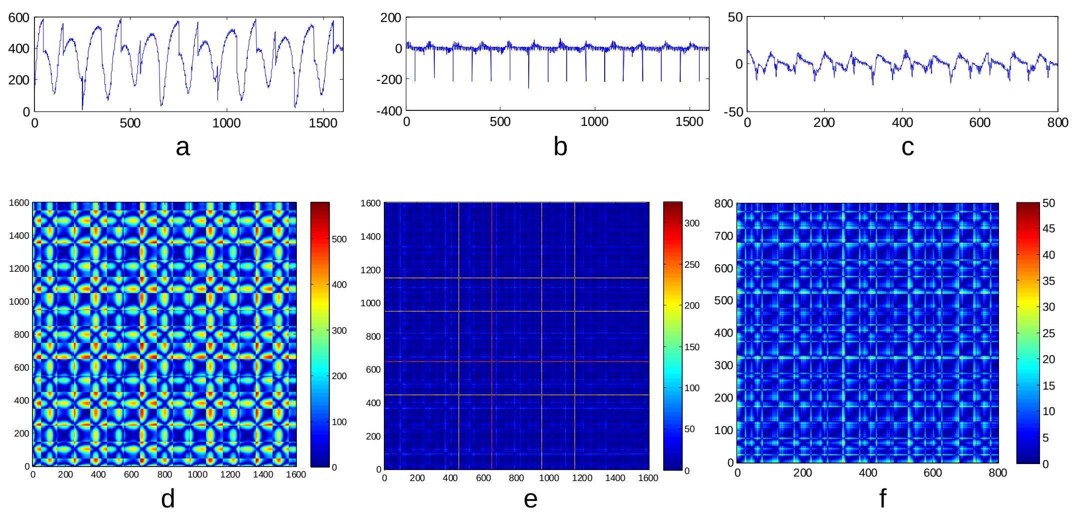

Fig. 3 Fractional-order chaotic Rabinovich-Fabricant system;
a - signal with noise, b - stability, c - signal reconstruction,
d, e, f - Poincare recurrence diagrams.

5. CONCLUSION

Proposed structure of the “SSA – lifting scheme”, produced a reconstruction signal. The proposed algorithm can be used in micro – control systems. Visualizing transient implemented in MATLAB software environment.

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