Original Research Article

A Computational Method for the Integration of Modeled Differential Equations

ABSTRACT

This paper presents the derivation and implementation of a computational method for the integration of modeled differential equations. The one-step computational hybrid block method was developed using Legendre polynomial of degree six as our basis function via interpolation and collocation techniques. The computational method developed was applied on some practical problems in electricity to generate graphical results and also interpret the natures of these results. The paper went further to analyze the basic properties of the computational method derived. From the graphical results obtained, the computed solutions converge toward the exact solutions.

Keywords: Computational Method, Electric Circuit, Hybrid, Legendre Polynomial, Model

2010 AMS Subject Classification: 65L05, 65L06, 65D30

1. INTRODUCTION

Classic application of differential equations is found in many areas of science and technology. They can be used for modeling of physical, technical or biological processes such as in the study of an electric circuit consisting of a resistor, an inductor and a capacitor driven by an electromotive force (emf), in gravitational equilibrium of a star, chemical reactions kinetic, in psychology, in models of the learning of a task involves the equation, in vibrating strings and propagation of waves, among others. The main questions of modern technology are how to increase the accuracy of calculations considering short computational time and how to decrease necessary mathematical operations.

This paper presents a one-step computational hybrid block method for the integration modeled differential equations of the form,

$$y' = f(x,y),\ y(a)=\eta,\ f:\Re\times\Re\to\Re \qquad (1)$$

The following standard theorem lays down sufficient conditions for a unique solution of (1) to exist; we shall always assume that the hypotheses of this theorem are satisfied.

Theorem 1.1 [1]

Let $f(x,y)$, where $f:\Re\times\Re\to\Re$, be defined and continuous for all (x,y) in the region D defined by $a\le x\le b,\ -\infty<y<\infty$, where a *and* b are finite and let there exist a constant L such that,

$$\|f(x,y)-f(x,y^*)\|\le L\|y-y^*\| \qquad (2)$$

holds for every $(x,y),\ (x,y^*)\in D$. Then, for $\eta\in\Re$ there exists a unique solution $y(x)$ of the problem (1), where $y(x)$ is continuous and differentiable for all $(x,y^*)\in D$. The requirement (2) is known as Lipchitz condition and the constant L as a Lipchitz constant.

It is important to note that, researchers have proposed different computational methods for the solution of problems of the form (1) ranging from predictor-corrector methods to hybrid methods. Despite the successes recorded by the predictor-corrector methods, its major setbacks are that the predictors are in reducing order of accuracy, high cost of developing separate predictor for the corrector, high cost of human and computer time involved in the execution, [2], [3]. Block methods were later proposed to carter for some of the setbacks of the predictor-corrector methods. It is important to state that Milne in 1953 first developed block method to serve as a predictor to a predictor-corrector algorithm before it was later adopted as a full method. Block method has the advantage of generating simultaneous numerical approximations at different grid points within the interval of integration, [4]. Another advantage of block method is the fact that it is less expensive in terms of the number of function evaluations compared to the linear multistep and the Runge-Kutta methods. Its major setback however is that the order of interpolation points must not exceed the order of the differential equations, thus when equations of lower order are developed, the accuracy of the developed method is reduced. This led to the development of hybrid methods which permit the incorporation of function evaluation at off-step points which affords the opportunity of circumventing the "Dahlquist Zero-Stability Barrier" and it is actually possible to obtain convergent k-step methods with order $2k+1$ up to $k=7$. The method is also useful in reducing the step number of a method and still remain zero-stable, see [5], [6], [7] and [8].

Definition 1.1 [9]

Legendre polynomial of degree n or Legendre function of the first kind is defined by,

$$y_n(x)=\sum_{r=0}^{R}(-1)^r\frac{(2n-2r)!}{2^r\,r!(n-r)!(n-2r)!}x^{n-2r} \tag{3}$$

where $R=\begin{Bmatrix} n/2, \; if \;\; n \; is \; even \\ (n-1)/2, \; if \;\; n \; is \; odd \end{Bmatrix}$. In particular, $y_0(x)=1,\ y_1(x)=x,\ y_2(x)=\left(3x^2-1\right)/2,...$

2. AN OVERVIEW OF ELECTRIC CIRCUITS

Description of circuits using differential equations is very convenient for the electrical circuits' behavior analysis. Electrical circuits are described by differential equations for time-dependent elements (capacitors, inductances) together with equations for linear and non-linear time-independent elements (resistors, diodes and transistors). Well-known Ohm's and Kirchhoff's laws are part of the electronic circuit description.

Equations of the form (1) are applicable to series circuits containing an electromotive force, resistors, inductors and capacitors. It is important to note that the emf or voltage denoted by E is measured in volt (V), current, i, is measured in ampere, charge, q, is measured in coulomb, resistance, R, is measured in ohm (Ω), inductance, L, is measured in Henry (H) and capacitance, C, is measured in Farad.

Electromotive force (for example, a battery or generator) produces a flow of current in a closed circuit and that this current produces a so called voltage drop across each resistor, inductor and capacitor [9].

We state below the three important laws concerning voltage drop across resistor, inductor and capacitor.

Law 1

The voltage drop E_R across a resistor is given by,

$$E_R=Ri \tag{4}$$

where R is a constant of proportionality called resistance and i the current.

Law 2

The voltage drop E_L across an inductor is given by,

$$E_L=L\left(\frac{di}{dt}\right) \tag{5}$$

where L is a constant of proportionality called inductance.

Law 3

The voltage drop E_C across a capacitor is given by,

$$E_C = \frac{q}{C} \tag{6}$$

where C is a constant of proportionality called capacitance and q is instantaneous charge on the capacitor.

The fundamental law in the study of electric circuits is the following.

Law 4 (The Kirchhoff's Voltage Law)

The sum of the voltage drops across resistors, inductors and capacitors is equal to the total electromotive force in a closed circuit.

Thus, the relationship between Law 4 and Laws 1,2 and 3 is,

$$L\left(\frac{di}{dt}\right) + Ri + \frac{q}{C} = E \tag{7}$$

containing two dependent variables i and q. But, we also have,

$$i = \frac{dq}{dt}, \text{ so that } \frac{di}{dt} = \frac{d^2q}{dt^2} \tag{8}$$

Using (8), (7) takes the form,

$$L\left(\frac{d^2q}{dt^2}\right) + R\left(\frac{dq}{dt}\right) + \frac{q}{C} = E \tag{9}$$

which is a second-order linear differential equation in the single independent variable q. So we can obtain q from (9). Now differentiating (7) with respect to t gives,

$$L\left(\frac{d^2i}{dt^2}\right) + R\left(\frac{di}{dt}\right) + \left(\frac{1}{C}\right)i = \frac{dE}{dt} \tag{10}$$

which is a second order linear differential equation in the single dependent variable i. So we can obtain i from (10). We now consider two very special cases in which the problem reduces to a first-order linear differential equation.

Case 1: If the circuit contains no capacitor (so that C=0), then (7) reduces to,

$$L\left(\frac{di}{dt}\right) + Ri = E \tag{11}$$

Case2: If the circuit contains no inductor (so that L=0), then (9) reduces to,

$$R\left(\frac{dq}{dt}\right) + \frac{q}{C} = E \tag{12}$$

3. DERIVATION OF THE COMPUTATIONAL METHOD

We shall derive a computational one-step hybrid block method of the form,

$$A^{(0)}\mathbf{Y}_m = E\mathbf{y}_n + hd\mathbf{f}(\mathbf{y}_n) + hb\mathbf{F}(\mathbf{Y}_m) \tag{13}$$

using the first six terms of Legendre polynomial as our basis function. This is given by,

$$y_6(x) = 9 + 22x + 69x^2 - 100x^3 - 245x^4 + 126x^5 + 231x^6 \tag{14}$$

Equation (14) is interpolated at point $x_{n+s}, s = 0$ and its first derivative is collocated at points $x_{n+r}, r = 0\left(\frac{1}{5}\right)1$, where s and r are the numbers of interpolation and collocation points respectively. This leads to the system of equations of the form,

$$XA = U \tag{15}$$

where

$$A = [a_0 \;\; a_1 \; a_2 \;\; a_3 \;\; a_4 \;\; a_5 \;\; a_6]^T \qquad U = \left[y_n \;\; f_n \;\; f_{n+\frac{1}{5}} \;\; f_{n+\frac{2}{5}} \;\; f_{n+\frac{3}{5}} \;\; f_{n+\frac{4}{5}} \;\; f_{n+1} \right]^T$$

and

$$X = \begin{bmatrix} 9 & 22x_n & 69x_n^2 & -100x_n^3 & -245x_n^4 & 126x_n^5 & 231x^6 \\ 0 & 22 & 138x_n & -300x_n^2 & -980x_n^3 & 630x_n^4 & 1386x_n^5 \\ 0 & 22 & 138x_{n+\frac{1}{5}} & -300x^2_{n+\frac{1}{5}} & -980x^3_{n+\frac{1}{5}} & 630x^4_{n+\frac{1}{5}} & 1386x^5_{n+\frac{1}{5}} \\ 0 & 22 & 138x_{n+\frac{2}{5}} & -300x^2_{n+\frac{2}{5}} & -980x^3_{n+\frac{2}{5}} & 630x^4_{n+\frac{2}{5}} & 1386x^5_{n+\frac{2}{5}} \\ 0 & 22 & 138x_{n+\frac{3}{5}} & -300x^2_{n+\frac{3}{5}} & -980x^3_{n+\frac{3}{5}} & 630x^4_{n+\frac{3}{5}} & 1386x^5_{n+\frac{3}{5}} \\ 0 & 22 & 138x_{n+\frac{4}{5}} & -300x^2_{n+\frac{4}{5}} & -980x^3_{n+\frac{4}{5}} & 630x^4_{n+\frac{4}{5}} & 1386x^5_{n+\frac{4}{5}} \\ 0 & 22 & 138x_{n+1} & -300x^2_{n+1} & -980x^3_{n+1} & 630x^4_{n+1} & 1386x^5_{n+1} \end{bmatrix}$$

Solving (15) for $a_j\text{'s}, j = 0(1)6$ and substituting into the basis function gives a continuous linear multistep method of the form,

$$y(x) = \alpha_0(x)y_n + h\sum_{j=0}^{1}\beta_j(x)f_{n+j}, \;\; j = 0\left(\frac{1}{5}\right)1 \qquad (16)$$

where

$$\left.\begin{aligned} \alpha_0 &= 1 \\ \beta_0 &= -\frac{1}{288}(1250t^6 - 4500t^5 + 6375t^4 - 4500t^3 + 1644t^2 - 288t) \\ \beta_{\frac{1}{5}} &= \frac{25}{288}(250t^6 - 840t^5 + 1065t^4 - 616t^3 + 144t^2) \\ \beta_{\frac{2}{5}} &= -\frac{25}{288}(250t^6 - 780t^5 + 885t^4 - 428t^3 + 72t^2) \\ \beta_{\frac{3}{5}} &= \frac{25}{288}(250t^6 - 720t^5 + 735t^4 - 312t^3 + 48t^2) \\ \beta_{\frac{4}{5}} &= -\frac{25}{288}(250t^6 - 660t^5 + 615t^4 - 244t^3 + 36t^2) \\ \beta_1 &= \frac{1}{288}(1250t^6 - 3000t^5 + 2625t^4 - 1000t^3 + 144t^2) \end{aligned}\right\} \qquad (17)$$

$t=\dfrac{x-x_n}{h}$, $\alpha(t)$ *and* $\beta(t)$ are continuous functions. Evaluating (16) at $t=\frac{1}{5}\left(\frac{1}{5}\right)1$, gives a discrete computational block method of the form (13), where

$$\mathbf{Y}_m=\begin{bmatrix} y_{n+\frac{1}{5}} & y_{n+\frac{2}{5}} & y_{n+\frac{3}{5}} & y_{n+\frac{4}{5}} & y_{n+1}\end{bmatrix}^T, \quad \mathbf{y}_n=\begin{bmatrix} y_{n-\frac{4}{5}} & y_{n-\frac{3}{5}} & y_{n-\frac{2}{5}} & y_{n-\frac{1}{5}} & y_n\end{bmatrix}^T$$

$$\mathbf{F}(\mathbf{Y}_m)=\begin{bmatrix} f_{n+\frac{1}{5}} & f_{n+\frac{2}{5}} & f_{n+\frac{3}{5}} & f_{n+\frac{4}{5}} & f_{n+1}\end{bmatrix}^T, \quad \mathbf{f}(\mathbf{y}_n)=\begin{bmatrix} f_{n-\frac{4}{5}} & f_{n-\frac{3}{5}} & f_{n-\frac{2}{5}} & f_{n-\frac{1}{5}} & f_n\end{bmatrix}^T$$

$$A^{(0)}=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix} \qquad E=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix} \qquad d=\begin{bmatrix}0&0&0&0&\frac{19}{288}\\0&0&0&0&\frac{14}{225}\\0&0&0&0&\frac{51}{800}\\0&0&0&0&\frac{14}{225}\\0&0&0&0&\frac{19}{288}\end{bmatrix}$$

$$b=\begin{bmatrix}\frac{1427}{7200} & \frac{-133}{1200} & \frac{241}{3600} & \frac{-173}{7200} & \frac{3}{800} \\ \frac{43}{150} & \frac{7}{225} & \frac{7}{225} & \frac{-1}{75} & \frac{1}{4500} \\ \frac{219}{800} & \frac{57}{400} & \frac{57}{400} & \frac{-21}{800} & \frac{3}{800} \\ \frac{64}{225} & \frac{8}{75} & \frac{64}{225} & \frac{14}{225} & 0 \\ \frac{25}{96} & \frac{25}{144} & \frac{25}{144} & \frac{25}{96} & \frac{19}{288}\end{bmatrix}$$

It is important to note here that the computational method developed above is implicit in nature, meaning that it requires some starting values before it can be implemented. Starting values for $y_{n+j},\ j=\frac{1}{5}\left(\frac{1}{5}\right)1$ are predicted using the Taylor series up to the order of each individual scheme.

4. ANALYSIS OF BASIC PROPERTIES OF THE COMPUTATIONAL METHOD

In this section, we shall analyze the basic properties of the computational method derived.

4.1. Order of Accuracy and Error Constant

The block method (13) is said to be of uniform accurate order p, if p is the largest positive integer for which $\bar{c}_0=\bar{c}_1=\bar{c}_2=...=\bar{c}_p=0$ but $\bar{c}_{p+1}\neq 0$, [1]. It is the largest positive integer p that quantifies the rate of convergence of a numerical approximation of a differential equation to that of the exact solution. On the other hand, the error constant is the accumulated error when the order of a method has been computed. Thus, $\bar{c}_0=\bar{c}_1=\bar{c}_2=\bar{c}_3=\bar{c}_4=\bar{c}_5=\bar{c}_6=0$

$\bar{c}_7=\left[-1.83\times10^{-7}\ \ -1.25\times10^{-7}\ \ -1.66\times10^{-7}\ \ -1.08\times10^{-7}\ \ -2.91\times10^{-7}\right]^T$

Therefore, our computational method is of uniform sixth order.

4.2. Root Condition and Zero Stability

Definition 4.1 [1]: The block method (13) is said to satisfy root condition, if the roots $z_{s,} s=1,2,...,k$ of the first characteristic polynomial $\rho(z)$ defined by $\rho(z)=\det(zA^{(0)}-E)$ satisfies $|z_s|\le 1$ and every root satisfying $|z_s|=1$ have multiplicity not exceeding the order of the differential equation. The method (13) is said to be zero-stable if it satisfies the root condition. Moreover, as $h\to 0, \rho(z)=z^{r-\mu}(z-1)^{\mu}$, where μ is the order of the matrices $A^{(0)}$ *and* E, see [10] for details.

We shall now verify whether or not the computational method derived satisfies root condition.

$$\rho(z)=\left|z\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\right| \tag{18}$$

$$\rho(z)=z^4(z-1)=0 \Rightarrow z_1=z_2=z_3=z_4=0,\ z_5=1.$$

Thus, the computational method (13) is said to satisfy root condition.

Theorem 4.1 [1]

The necessary and sufficient condition for the method given by (13) to be zero-stable is that it satisfies the root condition.

It is important to note that the main consequence of zero-stability is to control the propagation of the error as the integration proceeds.

4.3. Consistency

The computational method (13) is consistent since it has order $p=6\geq 1$. According to [11], consistency controls the magnitude of the local truncation error committed at each stage of the computation.

4.4. Convergence

The computational method (13) is convergent by consequence of Dahlquist theorem below.

Theorem 4.2 [12]

The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable.

4.5. Symmetry

According to [13], a linear multistep method (16) is symmetric if,

$$\left\{\begin{matrix}\alpha_j=\alpha_{k-j}\\ \beta_j=\beta_{k-j}\end{matrix}\right\},\ j=0(1)\left(\frac{k}{2}\right)\ \ for\ even\ k \tag{19}$$

and

$$\left\{\begin{matrix}\alpha_j=-\alpha_{k-j}\\ \beta_j=-\beta_{k-j}\end{matrix}\right\},\ j=0(1)k\ \ for\ odd\ k \tag{20}$$

The newly derived computational method is symmetric since from equation (17), it is clear that the condition in equation (20) holds because $\beta_0=-\beta_1,\ \beta_{\frac{1}{5}}=-\beta_{\frac{4}{5}}\ and\ \beta_{\frac{2}{5}}=-\beta_{\frac{3}{5}}$.

4.6. Region of Absolute Stability

Definition 4.3 [1]

The linear multistep method (16) is said to have region of absolute stability R_A, where R_A is a region of the complex $\bar{h}$-plane, if it is absolutely stable for all $\bar{h}\in R_A$. The intersection of R_A with the real axis is called the interval of absolute stability.

In plotting the stability region, we shall adopt the boundary locus method. The stability polynomial of the newly derived computational method is given by,

$$\bar{h}(w) = -h^5\left(\frac{1}{18750}w^5 + \frac{1}{18750}w^4\right) - h^4\left(\frac{137}{112500}w^4 - \frac{137}{112500}w^5\right) - h^3\left(\frac{3}{200}w^5 + \frac{3}{200}w^4\right) - h^2\left(\frac{17}{150}w^4 - \frac{17}{150}w^5\right) - h\left(\frac{1}{2}w^5 + \frac{1}{2}w^4\right) + w^5 - w^4 \quad (21)$$

The stability region is shown in Figure 1.

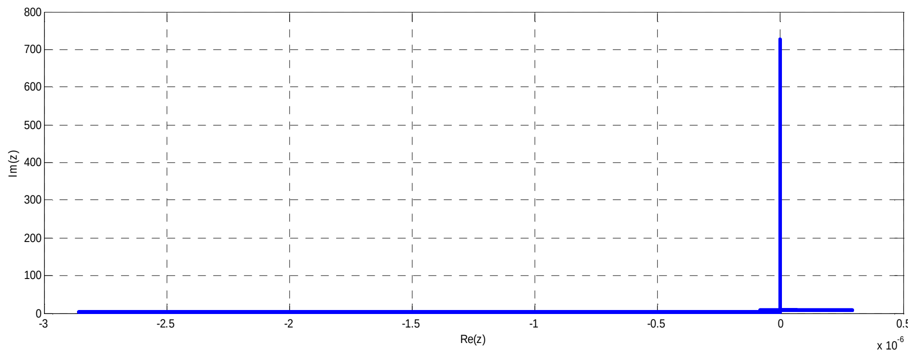

Figure 1: Region of Absolute Stability for the Computational Method

According to [11], stiff and oscillatory algorithms have unbounded RAS. Also, the author in [1] showed that the stability region for L-stable schemes must encroach into the positive half of the complex plane. Thus, the stability region in the Figure 1 is L-stable

5. IMPLEMENTATION, NUMERICAL EXPERIMENTS AND DISCUSSION OF RESULTS

5.1. Implementation

The computational method (13) was derived using the Scientific Work Place 5.5 and the graphical results were generated with the aid of MATLAB 2010a programming language.

5.2. Numerical Experiments

Two important problems in electricity shall be considered. These problems have been successfully modeled into first-order initial value problem of the form (1) and the computational method developed shall be applied on them. Graphical results shall be generated in order to interprete the nature of these problems.

Problem 5.1:

A circuit has in series an emf given by E=100sin40t V, a resistor of 10Ω and an inductor of 0.5H. If the initial current is 0, find the current at time $t > 0$.

Source: [9]

Applying Laws 1,2 and 4 earlier stated, the differential equation modeling this problem is given by the first-order linear equation,

$$\frac{di}{dt} + 20i = 200\sin 40t \quad (22)$$

Since the initial current is 0, the initial condition is,

$$i(0) = 0 \quad (23)$$

It is important to note that, the Integrating Factor (I.F) of (22) is,

$$I.F = e^{\int 20dt} = e^{20t} \quad (24)$$

Hence, its solution is,

$$ie^{20t} = \int\left\{(200\sin 40t)\times e^{20t}\right\}dt + C = 200\int e^{20t}\sin 40t dt + C \quad (25)$$

But from integral calculus,

$$\int e^{ax}\sin bx dx = \left\{e^{ax}\left(a\sin bx - b\cos bx\right)\right\}/(a^2+b^2) \tag{26}$$

Hence,

$$e^{20t}\sin 40t dt = \frac{e^{20t}\left(200\sin 40t - 40\cos 40t\right)}{20^2+40^2} = \frac{e^{20t}\left(\sin 40t - 2\cos 40t\right)}{100} \tag{27}$$

Equation (25) reduces to,

$$ie^{20t} = 2e^{20t}\left(\sin 40t - 2\cos 40t\right) + C$$

Or

$$i = 2\left(\sin 40t - 2\cos 40t\right) + Ce^{-20t} \tag{28}$$

Applying the initial condition $i = 0$ when $t = 0$, equation (28) gives C=4. Hence, (28) becomes,

$$i = 2\left(\sin 40t - 2\cos 40t\right) + 4e^{-20t} \tag{29}$$

We then transform (29) into a "phase-angle" form as follows,

$$\sin 40t - 2\cos 40t = \sqrt{5}\left\{\left(\frac{1}{\sqrt{5}}\right)\sin 40t - \left(\frac{2}{\sqrt{5}}\right)\cos 40t\right\} = \sqrt{5}\sin\left(40+\phi\right) \tag{30}$$

where

$$\cos\phi = \frac{1}{\sqrt{5}} \; and \; \sin\phi = \frac{2}{\sqrt{5}} \tag{31}$$

From (31), $\phi = -1.11$ radians. Hence,

$$\sin 40t - 2\cos 40t = \sqrt{5}\sin\left(40t - 1.11\right)$$

Therefore, (29) transforms to

$$i(t) = \sqrt[2]{5}\sin\left(40t - 1.11\right) + 4e^{-20t} \tag{32}$$

The graph of the current, i, as a function of time, t, is shown in Figure 2 for Problem 5.1.

Problem 5.2:

A 12V battery is connected to a series circuit in which the inductance is (1/2) H and the resistance is 10Ω. Determine the current i if $i(0) = 0$ at time $t > 0$.

Source:[9]

If a circuit has in series an emf E volt, a resistor R ohm and an inductor L henries, then the current i in amperes at time t is given by,

$$L\left(\frac{di}{dt}\right) + Ri = E \tag{33}$$

Thus, the initial value problem modeling the problem is given by,

$$\frac{di}{dt} + 20i = 24,\; i(0) = 0 \tag{34}$$

The I.F of (34) is,

$$e^{\int 20dt} = e^{20t} \tag{35}$$

and the solution is given by,

$$ie^{20t} = \int\left(24e^{20t}\right)dt + C = \left(\frac{6}{5}\right)e^{20t} + C \tag{36}$$

Thus,

$$i = \left(\frac{6}{5}\right) + e^{-20t} \tag{37}$$

Since $i(0)=0$, putting $i=0$ and $t=0$ in (37), we get $C=-(6/5)$, thus (37) reduces to,

$$i(t) = \left(\frac{6}{5}\right)\left(1-e^{-20t}\right) \tag{38}$$

The graph of the current, i, as a function of time, t, is shown in Figure 3 for Problem 5.2.

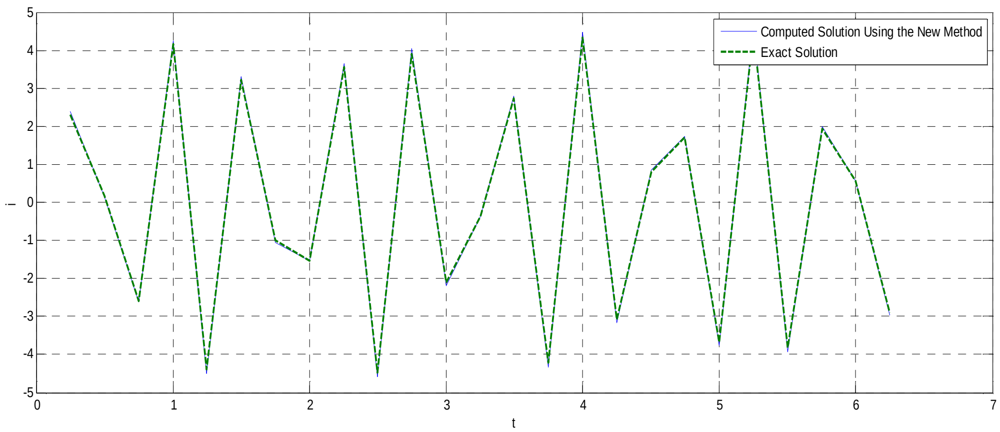

Figure 2: Graphical Results for Problem 5.1

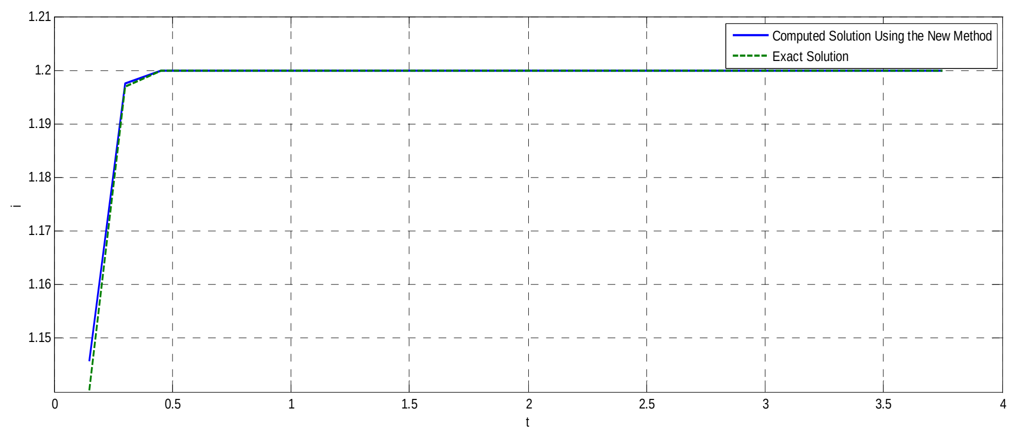

Figure 3: Graphical Results for Problem 5.2

5.3. Discussion of Results

We considered two numerical examples in this paper. For problem 5.1 (see Figure 2), the current is presented as the sum of sinusoidal term and an exponential term. The exponential term becomes very small in a short time that its effect is soon practically negligible; it is the transient term. Thus, after a short time, essentially all that remains is the sinusoidal term; it is the steady current. Observe that its period (π/2) is the same as that of the emf. However, the phase angle $\phi = -1.11$ radians indicates that the emf leads to a steady-state current by approximately (1/46)×1.11. From the graphical result, one can easily conclude that the computational method is convergent. For problem 5.2 (see Figure 3), the current is presented as an exponential term. The exponential term becomes very small in a short time that its effect is soon practically negligible; this is the transient term (in order words the current is not steady). However, the graphical results in general show that the computational method is convergent, since the computed solutions converge toward the exact solutions.

6. CONCLUSION

We developed a computational method for the solution of electric circuit problems of the form (1) using Legendre polynomial of degree six as our basis function. The method developed was found to be L-stable and that explains why it performed well on this class of problems. The computational method was also found to be zero-stable, symmetric, consistent and convergent. The graphical results obtained in Figures 2 and 3 show that the computational method developed is computationally reliable and it is also important to note that, at any value of time, t, one would be able to know the current, i, that flows through the circuit.

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