Hidden oscillations in fractional-order multidimensional chaotic systems

ABSTRACT

In this paper proposes a combined approach of analysis and annulations hidden oscillation with are taking place after transients processes. Shows a visualization of the results.

SAMPLE ABSTRACT:

The research paper is a part of the research area "Physics of open systems," the problem of the Poincaré recurrence in the study of transients multidimensional fractional chaotic systems. The use of fractional dynamics opens up new possibilities for the solution of problems of forecasting and decision-making in open systems. The mathematical basis for modeling of such phenomena is a fractional calculus. In the context of the use of fractional dynamics in problems of studying physical systems, important place is occupies an problem Poincare returns. This is one of the fundamentally important features of dynamic processes (systems) with a limited type of steady motions. Poincaré return time is the main indicators and characteristics, show a certain state of a dynamic system are repeated over time. The paper include the the results of simulation of fractional order Rabinovich-Fabrikant and Chen systems using the proposed algorithm. This algorithm includes lifting and wavelet processing and non-linear recurrence analysis of noisy added fractional-order time series. The results of the simulations are presented in the form of time series, the recurrence plots and graphs of functions.

Keywords: chaotic system, time series, Poincare diagram, Lyapunov exponent

1. INTRODUCTION

At the decision of problems of stability of fractional dynamical chaotic systems there can be hidden oscillations which are not established after transient from the neigh bowheads of the stationary states. Such phenomena can result undesirable consequences, such as in aircraft control systems [1]. In order to research and localization of hidden oscillation used singularly-spectrum analyses (SSA) [2].

2. GENERAL PROVISIONS

2.1. Singular-spectral analyses to identify hidden oscillations in chaotic time series

Let $\{x_i\}$ be a time series of values of function $f(t): x_i = f[i] = f(i\Delta t)$, $i = 1,\ldots,N$. Let number of $M < N$ - length of a window. To introduce k, $k = N - M + 1$ and construct k M - dimensional vectors $X_i = (x_i,\ldots,x_{i+M-1})^T$, $1 \le i \le k$, $X_i \in R^M$. Let's make a matrix [3]:

$$X = \begin{pmatrix} x_1 & x_2 & x_3 \ldots & x_k \\ x_2 & x_3 & x_4 \ldots & x_{k+1} \\ \ldots & \ldots & \ldots & \ldots \\ x_M & x_{M+1} & x_{M+2} \ldots & x_k \end{pmatrix} \quad (1)$$

Remark. The condition of $M < N/2$ - limit on the integer parameter M.

Let $S = XX^T \in R^{M \times M}$ - non-negative and symmetric matrix. Eigen values S are non-negative: $\lambda_1, \ldots, \lambda_M \geq 0$. Arrange the Eigen values in not increase order:

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M \geq 0.$$

$u_1, \ldots, u_M$ - ortonormality system eigenvectors of a matrix S.

$d = \max\{i : \lambda_i > 0\}$ - order singular value decomposition.

We introduce the matrix [3]:

$$V_i = \frac{1}{\sqrt{\lambda_i}} x^T u_i, \ i = 1, \ldots, d. \quad (2)$$

Than the singular value decomposition of the matrix has the form:

$$x = x_1 + x_2 + \ldots + x_d, \quad x_i = \sqrt{\lambda_i} u_i V_i^T. \quad (3)$$

It is known, that the singularly-spectral analyses is effective in a combination with wavelet-transformation [3]. It is connected by that the signal can have a changing frequency.

Further cleaned components related to trend and noise. After the restoration of a number of used wavelet-transform [4].

In this paper, for the purpose of localization and reconstruction of abnormal components of fractional dynamic chaotic multidimensional maps, it proposed the use singularly-spectral analyses in combination with lifting method [5].

Lifting methods of processing of the information make possible wavelet the stretching's and shifts of one function.

Advantage of the lifting scheme is:

1. the conversion process occurs quickly;
2. the set of wavelet-coefficients occupies a volume that matches the original data;
3. return transformation restores a signal very precisely.

2.2. Lifting scheme

Briefly, the mechanism is a follows [6]. Let the original signal s_j contains 2^j points. Transformation involves three steps (split-predict-update), which will yield two sets of points s_{j-1} and d_{j-1}.

1. Split

From in s_j shape two new not crossed sets. We note that the division of the set into two depends on the type of wavelet. For example, Lazy wavelet distinguishes $even_{j-1}$ and odd_{j-1} samples.

Formally it looks as [6]:

$$(even_{j-1}, odd_{j-1}) = S(s_j). \quad (4)$$

Predict

Here is calculated the difference between true and predicted values and defines coefficients d_{j-1} [6]:

$$d_{j-1} = odd_{j-1} - P\left(even_{j-1}\right), \tag{5}$$

where P - the predicting operator.

Update

On this step, the help of the operator U, calculate coefficients s_{j-1} [6]:

$$s_{j-1} = even_{j-1} + U\left(d_{j-1}\right). \tag{6}$$

The described algorithm of transformation of data lifting-scheme is presented in figure [6].

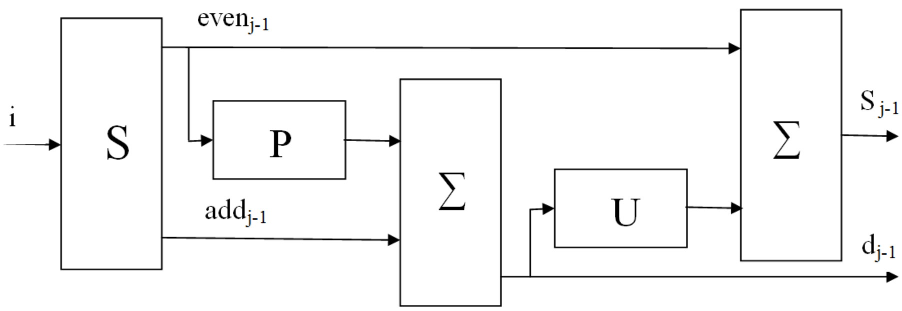

Fig. 1 Constructing the wavelet-coefficients in the lifting-scheme.

Thus, lifting schema generates two sets of coefficients s_{j-1} and d_{j-1}, each of which is less than half the length of the initial signal.

From here s_{j-1} reflects behavior of a signal in the big scale, and a coefficients d_{j-1} shows difference an initial signal from s_{j-1}.

In this paper, the realization of lifting scheme is based on the use of Haar wavelets and Doubechies [7].

3. ALGORITM

Step 1. Let given the fractional-order Rabinovich-Fabrikant system following [8]:

$$\left.\begin{aligned} \dot{x}_1 &= x_2\left(x_3 - 1 + x_1^2\right) + \gamma x_1, \\ \dot{x}_2 &= x_1\left(3x_3 + 1 - x_1^2\right) + \gamma x_2, \\ \dot{x}_3 &= -2x_3\left(x_1 x_2 + \alpha\right), \\ \dot{x}_4 &= -3x_3\left(x_2 x_4 + \delta\right) + x_4^2, \end{aligned}\right\} \tag{7}$$

where $\alpha = 0.14,\ \gamma = 1.1,\ -0.01 \le \delta \le 7650.$

The fractional-order Chen system as follows [9-11]:

$$\frac{d^{\alpha}x}{dt^{\alpha}} = a(y-x)+w,$$
$$\frac{d^{\alpha}y}{dt^{\alpha}} = bx - xz + cy, \tag{8}$$
$$\frac{d^{\alpha}z}{dt^{\alpha}} = xy - dz,$$
$$\frac{d^{\alpha}w}{dt^{\alpha}} = yz + rw,$$

where $a = 35,\ b = 7,\ c = 12,\ d = 3,\ r = 0.5$ and $\alpha = 0.9$.

Step 2. Simulation of the system (1 – 2) according to the algorithm [8].

Step 3. Let $\hat{x} =_{\alpha} \{x_n\}_{n=0}^{N}$ is related observable two fractional-order hyperchaotic (1 – 2) systems.

Step 4. The related observable $\hat{x}$ perturb of sawtooth wave $\eta : D^q x_i \vee A\, frac\left(\frac{x}{T} + \varphi\right)$, where $frac\,(x)$ is the fractional part.

$frac\,(x) = x - [x]$, A is amplitude, T is the period of the wave, and φ is its phase.

Step 5. We determine the stability of noisy systems (1 – 2).

Step 6. Lifting step 5 produce the singular-spectrum analysis for systems (1 – 2).

Step 7. Produce signals (*Step 6*) conversion on lifting scheme.

Step 8. Determine the overage return time of Poincare.

Step 9. We make visualization of the results.

4. VISUALIZATION OF SIMULATION

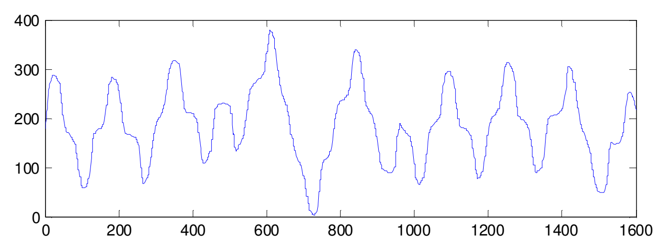

Fig. 2 Chen system time series.

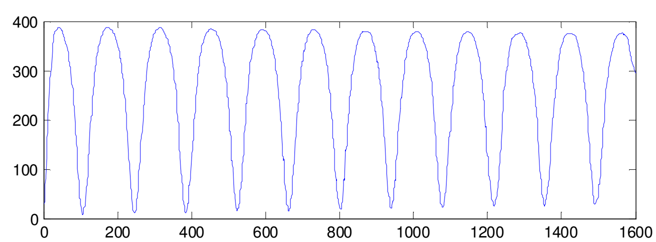

Fig. 3 Rabinovich-Fabrikant system time series.

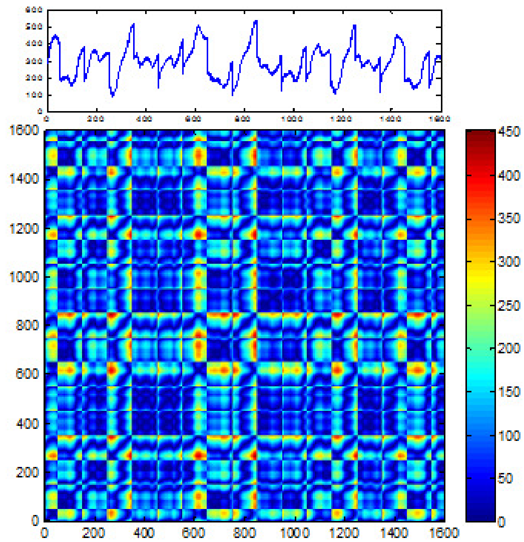

Fig. 4 Poincare recurrence diagram of Chen system time series with noise.

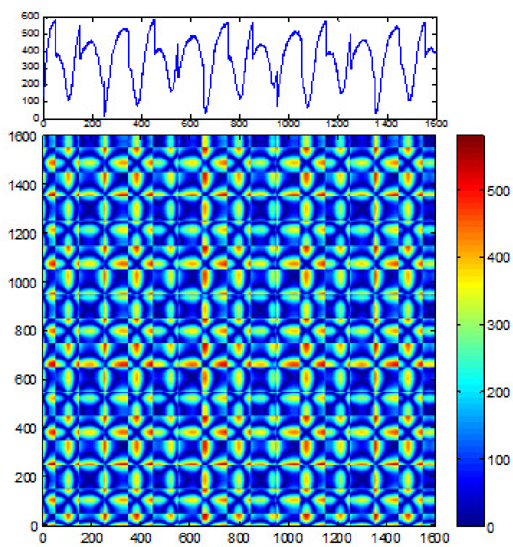

Fig. 5 Poincare recurrence diagram of Rabinovich-Fabrikant system with noise.

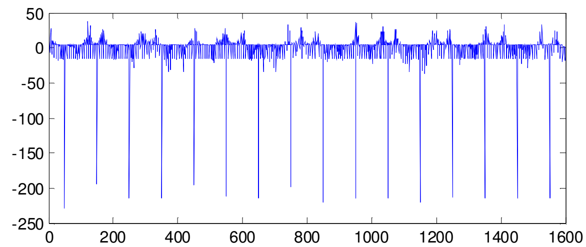

Fig. 6 Chen system stability.

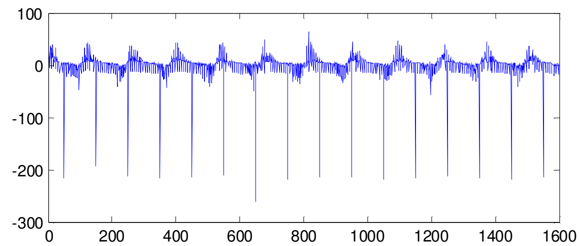

Fig. 7 Rabinovich-Fabrikant system stability.

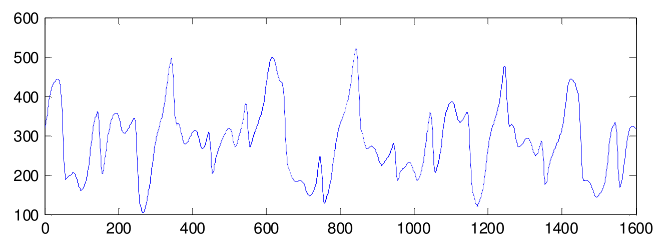

Fig. 8 Lifting Chen system time series.

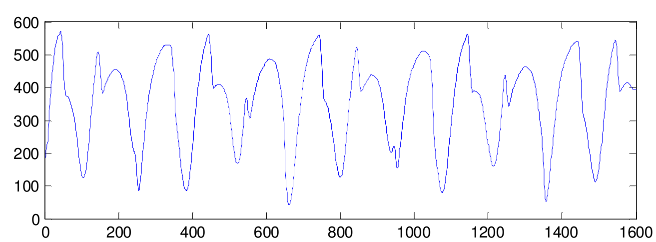

Fig. 9 Lifting Rabinovich-Fabrikant system time series.

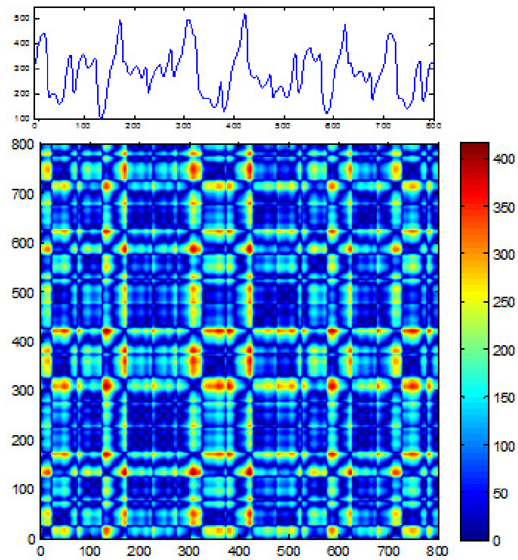

Fig. 10 Poincare recurrence diagram of Haar wavelet of Chen system time series.

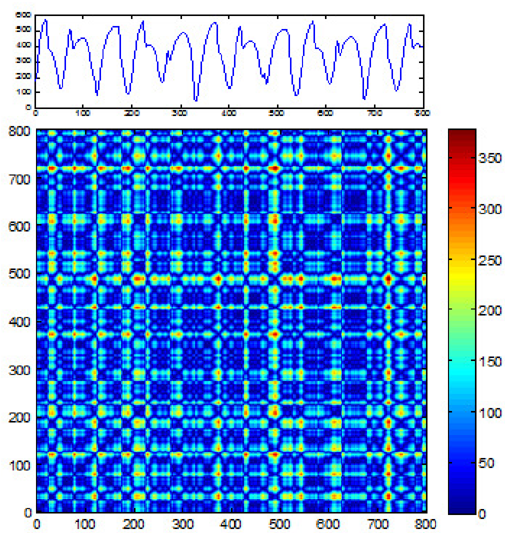

Fig. 11 Poincare recurrence diagram of Haar wavelet of Rabinovich-Fabrikant system time series.

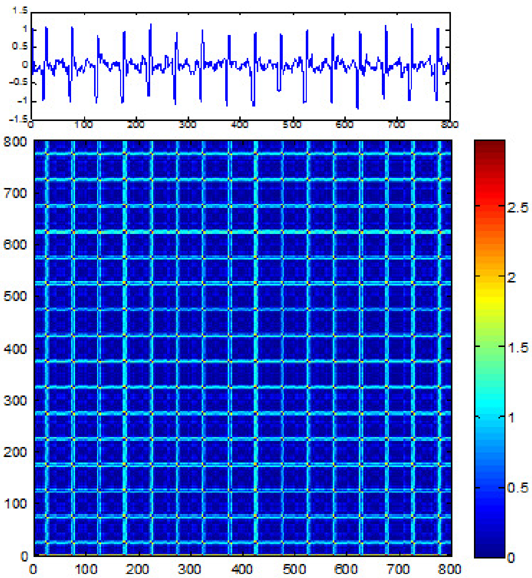

Fig. 12 Poincare diagram of Daubechies wavelet of Chen system time series.

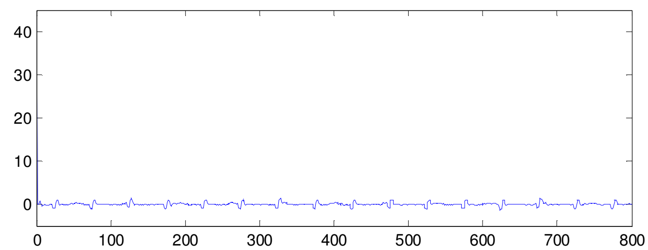

Fig. 13 Daubechies wavelet of Rabinovich-Fabrikant system time series.

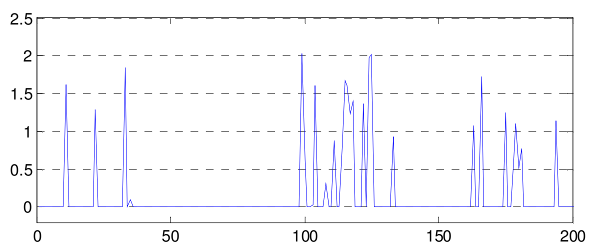

Fig. 14 Lyapunov exponent of Chen system

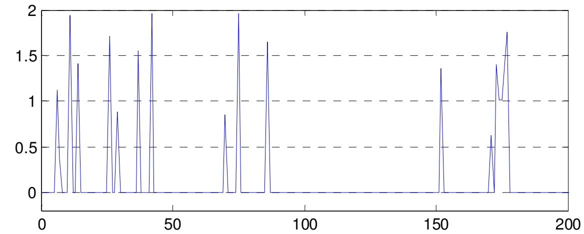

Fig. 15 Lyapunov exponent of Rabinovich-Fabrikant system

Article implementation of Pitting scheme is based on the use of Haar and Doubechies wavelets.

4. CONCLUSION

The results showed that when using the Haar wavelet hold the transient type “hyperchaos – chaos” and wavelet Doubechies “hyperchaos – quasiperiodic” transient. Visualization of transients implemented software MATLAB. It shows the numerical values of the overage Poincare return time.

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