

Time Periodic Varying Deceleration Parameter In Reconstructed $f(R,T)$ Gravity

ABSTRACT

In this paper, we have investigated a string cloud with perfect fluid matter distributions for Friedmann–Robertson–Walker (FRW) space-time in reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ gravity. To obtain exact solutions of the field equations we have used a time periodic varying deceleration parameter. The new reconstructed $f(R,T)$ model includes two models of Harko et al. and transforms to general relativity. However, we obtain vanishing string tension density in all models. This result agrees with Aygün and colleagues, Naidu et al. and Kiran and Reddy in different alternative gravitation theories.

Keywords: $f(R,T)$ gravity, FRW universe, deceleration parameter, string cloud, perfect fluid.

1. INTRODUCTION

Today we know that our universe is expanding and contains about 20% dark matter, 75% dark energy and 5% baryonic matter [1, 2]. Also, the dark energy and the dark matter etc., are still unresolved secrets for human beings. To solve the expansion of the universe two approaches are suggested. One of them is dark matter and dark energy models. The other is alternative gravitation theories such as $f(R,T)$ [3], Lyra [4], $f(R)$ [5], and Brans-Dicke [6] etc. After the $f(R,T)$ study of Harko et al. [3] in 2011, many scientist researched $f(R,T)$ gravitation theory [7 – 13] and some of authors represented new $f(R,T)$ models [14 – 16]. In 2011, Pradhan and Chouhan have studied a string cloud with perfect fluid matter distributions for Bianchi I space-time in the frame work of general relativity [17]. Pradhan and Singh have researched massive string solutions in Bianchi I universe for Lyra geometry using constant deceleration parameter [18]. Shen and Zhao have suggested a new model which is oscillating with time periodic varying deceleration parameter for FRW universe in general theory of relativity [19]. Also, She and Jiang have investigated plane symmetric metric with oscillating varying deceleration parameter for perfect fluid in general relativity theory [20]. Ramesh and Umadevi have studied perfect fluid matter distributions with linearly varying deceleration parameter for FRW universe in $f(R,T)$ gravity [21]. Baffou et al. have investigated $f(R,T)$ gravitation theory in a FRW universe for collisional matter [22]. Perfect fluid solutions for FRW space in $f(R,T)$ theory were investigated by Myrzakulov [23]. Sahoo has investigated cosmic strings in $f(R,T)$ theory. Strings caused density fluctuations leading to the formation of galaxies [24, 25]. As the stress energy of the strings can be combined with the gravitational field, it may be interesting to investigate the effect of gravity on the strings [26]. Also, the homogeneous and isotropic FRW universe model best describes the present universe.

In 2011, Harko et al. proposed various $f(R,T)$ models i.e. $f(R,T) = R + 2f(T)$, $f(R,T) = f_1(R) + f_2(T)$ and $f(R,T) = f_1(R) + f_2(R)f_3(T)$ in their study for the solution of modified field equations. Also, Starobinsky [27] suggested the following $f(R)$ model, $f(R) = R + \alpha R^2 + \beta R^n$, where αR^2 implies cosmic inflation and linear term R implies cosmic acceleration. Considering the suggestions by Harko et al. [3] and Starobinsky [27], we use $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ to get solutions in our model for the following reasons:

- i) In the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, the linear term R demonstrates cosmic acceleration [28]
- ii) In the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, $\alpha_2 f_3(T)$ represents interaction between matter and curvature.
- iii) To get viable accelerating universe it's needed that $f_R > 0$, $R > R_0 (> 0)$ [27], [29]
- iv) The $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model includes both geometrical terms (Ricci scalar R) and matter terms (trace of energy-momentum tensor T).

In this study, using a time varying deceleration parameter, we consider a string cloud with perfect fluid matter distributions for FRW space-time in a reconstructed new $f(R,T)$ model as $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$. This new $f(R,T)$ model is important because it combines the first and second model of Harko et al. [3] and also give general relativity solutions for $\alpha_1 = 1$ and $\alpha_2 = 0$ or $f_3(T) = 0$. For these reasons, in this study we consider a string cloud with perfect fluid distributions for FRW space-time in $f(R,T)$ gravity.

The paper is organized as follows. In section 2 we presented the field equations of $f(R,T)$ gravity. In section 3, we consider the reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model in $f(R,T)$ gravity. Finally, conclusions are given with various $f(R,T)$ models.

2. FIELD EQUATIONS OF $f(R,T)$ GRAVITY

The action of this theory was proposed by Harko et al. in 2011 [3] as follows

$$S = \frac{1}{16\pi} \int \left(\frac{f(R,T)}{G} + L_m \right) \sqrt{-g} d^4x \quad (1)$$

Here T is the trace of T_{ik} and g is the determinant of g_{ik} , R is the Ricci scalar, $f(R,T)$ is an arbitrary function of R and T , also L_m is the matter of the Lagrangian [3]. T_{ik} is defined by

$$T_{ik} = L_m g_{ik} - \frac{2\partial L_m}{\partial g^{ik}} \quad (2)$$

and by varying equation (1) we have

$$\begin{aligned} f_R(R,T)R_{ik} - \frac{1}{2}f(R,T)g_{ik} + (g_{ik}\nabla_a\nabla^a - \nabla_i\nabla_k)f_R(R,T) \\ = 8\pi T_{ik} - f_T(R,T)T_{ik} - f_T(R,T)\Xi_{ik} \end{aligned} \quad (3)$$

Here $f_T(R,T)$ shows the derivative with respect to T , $f_R(R,T)$ shows the derivative with respect to R , ∇_i is the covariant derivative and Ξ_{ik} is defined as [3]

$$\Xi_{ik} = -2T_{ik} + L_m g_{ik} - 2g^{ab} \frac{\partial^2 L_m}{\partial g^{ik} \partial g^{ab}} \quad (4)$$

82 If we contract equation (3), we get
83

$$(R + 3\nabla_a \nabla^a) f_R(R, T) - 2f(R, T) = (8\pi - f_T(R, T))T - \Xi f_T(R, T) \quad (5)$$

84 If we substitute equation (5) in equation (3) we get the field equations in the $f(R, T)$ theory [3].
85

$$\begin{aligned} f_R(R, T) \left(R_{ik} - \frac{1}{3} R g_{ik} \right) + \frac{1}{6} f(R, T) g_{ik} \\ = (8\pi - f_T(R, T)) \left(T_{ik} - \frac{1}{3} T g_{ik} \right) - f_T(R, T) \left(\Xi_{ik} - \frac{1}{3} \Xi_{ik} g \right) \\ + \nabla_i \nabla_k f_R(R, T) \end{aligned} \quad (6)$$

86 In this study, we'll discuss a new $f(R, T)$ theory where $f(R, T)$ is linear in R and an arbitrary
87 function of T i.e. $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ for string cloud with perfect fluid matter
88 distributions. here α_1 and α_2 are constants and $f_3(T)$ is an arbitrary function of T . The FRW
89 universe model is given by
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$$ds^2 = dt^2 - \frac{A(t)^2}{1 - kr^2} dr^2 - A(t)^2 r^2 (d\theta^2 + \sin(\theta^2) d\phi^2) \quad (7)$$

91 here k is a curvature parameter and given as $k = 0, +1, -1$. The cosmic strings are very
92 important to solve the mystery of the universe. Recently, many scientists have been
93 investigated string cosmology in various cosmological models and theories with different
94 matter distributions [17], [30], [31]. According to Zel'dovich [32]; strings lead to the formation
95 of galaxies. Also, strings have gravitational fields. It is important to investigate the
96 gravitational effects that arise from strings [17]. However, perfect fluid matter distribution
97 could adequately characterize the majority of the thermal histories of the universe [33]. For
98 these reasons, we have used a string cloud with perfect fluid matter distribution in $f(R, T)$
99 theory. The string cloud with perfect fluid energy-momentum tensor is given by

$$T_{ik} = (\rho + p)u_i u_k - p g_{ik} - \lambda x_i x_k \quad (8)$$

100 Here u_i is the four velocity vector, x_i is the direction of string, ρ is energy density, λ is string
101 tension and p is pressure [17]. Also, u_i and x_i satisfy $u_i u^i = -1$, $x_i x^i = 1$ and $u_i x^i = 0$
102 conditions [17].
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104 2.1 Field Equations for $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ Model

105 If we take $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ in equation (3) we reconstruct the $f(R, T)$ gravity model as
106 follows

$$\alpha_1 R_{ik} - \frac{\alpha_1}{2} R g_{ik} = (8\pi - \alpha_2 f'_3) T_{ik} - \alpha_2 f'_3 \Xi_{ik} + \frac{\alpha_2 f_3}{2} g_{ik} \quad (9)$$

107 where $\Xi_{ik} = -2T_{ik} - p g_{ik}$ also, the prime indicates derivative w.r.t. T . Substitution of
108 equations (7) and (8) into equation (9) yields

$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi(p - \lambda) - \alpha_2 f'_3 \lambda - \frac{\alpha_2}{2} f_3 \quad (10)$$

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$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi p - \frac{\alpha_2}{2} f_3 \quad (11)$$

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$$3\alpha_1 \frac{\dot{A}^2}{A^2} + 3\alpha_1 \frac{k}{A^2} = -8\pi\rho - \alpha_2(p + \rho)f'_3 - \frac{\alpha_2}{2} f_3 \quad (12)$$

111 In this study we have three field equations and four unknowns. In order to solve equations
 112 (10)-(12) we use a time periodic deceleration parameter (DP). The sign of DP can indicates
 113 whether the universe is accelerated or not [19]. To obtain the accelerated expansion of the
 114 universe many studies have been done using DP [19]. Berman [34] obtained a law of
 115 variation for Hubble parameter in general relativity for a FRW universe model with a constant
 116 deceleration parameter (q) [19]. According to Berman; the DP can obtain values $-1 \leq q < 0$,
 117 corresponding to the accelerating expansion [34]. Various authors have investigated various
 118 cosmological models using constant deceleration parameter. Besides, it is important to study
 119 varying DP to explain the expansion of the universe. So in this study we have considered an
 120 oscillating DP model. This model is given by;

$$q = -\frac{A\ddot{A}}{\dot{A}^2} = m\cos(nt) - 1 \quad (13)$$

124 Here m and n are positive constants [19]. If we solve equation (13), we obtain the metric
 125 potential as

$$A = c_1 \tan\left(\frac{nt}{2}\right)^{\frac{1}{m}} \quad (14)$$

126 here c_1 is an integration constant as suggested by [19]. From equations (10)-(12) and (14)
 127 we get the values of pressure, energy density and string tension for the reconstructed f(R,T)
 128 theory in the FRW space-time as follows
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$$p = \frac{\alpha_1 k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{\alpha_1 n^2 (2m+3) \tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} + \frac{m^2 \alpha_2 f_3 + \alpha_1 n^2}{16\pi m^2} - \frac{\alpha_1 n^2 (2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (15)$$

$$\rho = \frac{\alpha_1 n^2 (\alpha_2 (2m-3) f_3' - 24\pi)}{32m^2 \pi (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^2} - \frac{\alpha_1 k (\alpha_2 f_3' + 24\pi)}{8\pi c_1 (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{\alpha_1 n^2 (\alpha_2 (2m+3) f_3' + 24\pi) \tan\left(\frac{nt}{2}\right)^2}{32m^2 \pi (\alpha_2 f_3' + 8\pi)} - \frac{\alpha_2 m^2 f_3 + 3\alpha_1 n^2}{16\pi m^2} \quad (16)$$

131 and

$$\lambda = 0 \quad (17)$$

132 3. RESULTS AND DISCUSSION

133 In this research, we have used a new f(R,T) gravitational model to investigate a FRW
 134 universe. In 2011, Harko et al. proposed new f(R,T) models with $f(R,T) = R + 2\mu T$,
 135 $f(R,T) = \mu R + \mu T$ (here μ is a constant) and $f(R,T) = f_1(R) + f_2(R)f_3(T)$ [3]. For the
 136 solutions of the modified field equations in this study we proposed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$.
 137 This model contains the first and second models of Harko et al. [3]. For $\alpha_1 = 1$, $\alpha_2 = 2\mu$ and
 138 $f_3(T) = T$, we obtain the first model of Harko et al. i.e. $f(R,T) = R + 2\mu T$ [3]. If we take
 139 $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$, we get the second model of Harko et al. i.e. $f(R,T) = \mu R + \mu T$
 140 [3]. Also, for $\alpha_1 = 1$ and $\alpha_2 = 0$ or $f_3(T) = 0$, our modified f(R,T) gravitation theory transforms
 141 to general relativity.
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From equations (15)- (17) we obtain the pressure, energy density and string tension density as follows

$$p = \frac{n^2(8\pi + 3\mu)\cos(nt)}{2m(2\pi + \mu)(4\pi + \mu)(\cos(2nt) - 1)} - \frac{3n^2}{2m^2(2\pi + \mu)(\cos(2nt) - 1)} + \frac{k\pi}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (18)$$

147

$$\rho = \frac{n^2\mu\cos(nt)}{2m(2\pi + \mu)(4\pi + \mu)(\cos(2nt) - 1)} + \frac{3n^2}{2m^2(2\pi + \mu)(\cos(2nt) - 1)} - \frac{k(3\pi + \mu)}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (19)$$

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$$\lambda = 0 \quad (20)$$

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Using $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$ in the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we have the second model of Harko et al. [3]. Using equations (15)- (17) we get the pressure, energy density and string tension density as follows

$$p = \frac{n^2\mu(16\pi + 3\mu)\cos(nt)}{m(4\pi + \mu)(8\pi + \mu)(\cos(2nt) - 1)} - \frac{3n^2\mu}{m^2(4\pi + \mu)(\cos(2nt) - 1)} + \frac{4k\mu\pi}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (21)$$

153

$$\rho = \frac{n^2\mu^2\cos(nt)}{m(4\pi + \mu)(8\pi + \mu)(\cos(2nt) - 1)} + \frac{3n^2\mu}{m^2(4\pi + \mu)(\cos(2nt) - 1)} - \frac{2k\mu(6\pi + \mu)}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (22)$$

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$$\lambda = 0 \quad (23)$$

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If we take $\alpha_1 = 1$ and $f_3(T) = 0$, in the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we obtain general relativity solutions for pressure, energy density and string tension density as follows

$$p = \frac{k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{n^2}{16\pi m^2} + \frac{n^2(2m+3)\tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} - \frac{n^2(2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (24)$$

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$$\rho = -\frac{3k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{3n^2}{16\pi m^2} - \frac{3n^2 \tan\left(\frac{nt}{2}\right)^2}{32m^2\pi} - \frac{3n^2}{32m^2\pi \tan\left(\frac{nt}{2}\right)^2} \quad (25)$$

159 and

$$\lambda = 0 \quad (26)$$

160 4. CONCLUSION

161 In this study, for the solutions of the FRW universe model we have used a time periodic
162 deceleration parameter given by Shen and Zhao [19]. Using this parameter, we have
163 obtained the following results:

164 From equations (15) and (16) it is easily seen that cosmic pressure and energy density are
165 functions of cosmic time. Also, as $t \rightarrow \infty$ the pressure and energy density decrease with
166 time. From equation (17), we found $\lambda = 0$. Kiran and Reddy [35] studied the Bianchi III
167 universe with a string cosmological model in $f(R,T)$ gravity and they found zero string tension
168 density. Naidu et al. [36] investigated Bianchi V space-time with a string cosmological model.
169 Also they found $\lambda = 0$ in $f(R,T)$ gravity. However, Çağlar and Aygün have obtained zero
170 string tension density for the FRW universe in Lyra [37], Creation field [38], Brans-Dicke [39]
171 and self-creation [40] theories. Also, Şen and Aygün have investigated the FRW universe
172 model in self creation cosmology with clouds of string with a perfect fluid of strange quark
173 matter and obtained zero string tension density in their study [41]. Our solution agrees with
174 [35 – 41]. A $\lambda = 0$, string cloud with perfect fluid matter distribution turns out to be a perfect
175 fluid matter distribution. This model universe does not allow the formation of strings in the
176 frame work of $f(R,T)$ gravitation theory similar with other models.

177 It follows from equations (18)-(26) that the cosmic pressure and energy density in the $f(R,T)$
178 gravitation models and general relativity are functions of cosmic time t , and for large t the
179 cosmic pressure and energy density decrease with cosmic time t . Also, we found that $\lambda = 0$
180 in all cosmological models. The obtained solutions show that the source of the acceleration
181 of the universe is similar to perfect fluid distributions.

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