

Time Periodic Varying Deceleration Parameter In Reconstructed $f(R,T)$ Gravity

ABSTRACT

In this paper, we have investigated string cloud with perfect fluid matter distributions for Friedmann–Robertson–Walker (FRW) space-time in reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ gravity. To obtain exact solutions of the field equations we have used a time periodic varying deceleration parameter. The new reconstructed $f(R,T)$ model includes two models of Harko et al. and transforms to general relativity. However, we obtain vanishing string tension density in all models. This result agrees with Aygün and colleagues, Naidu et al. and Kiran and Reddy in different alternative gravitation theories.

Keywords: $f(R,T)$ gravity, FRW universe, deceleration parameter, string cloud, perfect fluid.

1. INTRODUCTION

Today we know that our universe is expanding and contains about 20% dark matter, 75% dark energy and 5% baryonic matter [1, 2]. Also, the dark energy and the dark matter etc., are still unresolved secrets for human beings. To solve the expansion of the universe two approaches are suggested. One of them is dark matter and dark energy models. The other is alternative gravitation theories such as $f(R,T)$ [3], Lyra [4], $f(R)$ [5], and Brans-Dicke [6] etc. After the $f(R,T)$ study of Harko et al. [3] in 2011, many scientist researched $f(R,T)$ gravitation theory [7 – 13] and some of authors represented new $f(R,T)$ models [14 – 16]. In 2011, Pradhan and Chouhan have studied a string cloud with perfect fluid matter distributions for Bianchi I space-time in the frame of general relativity [17]. Pradhan and Singh have researched massive string solutions in Bianchi I universe for Lyra geometry using constant deceleration parameter [18]. Shen and Zhao have suggested a new model which is oscillating with time periodic varying deceleration parameter for FRW universe in general relativity theory [19]. Also, Shen and Jiang have investigated plane symmetric metric with oscillating varying deceleration parameter for perfect fluid in general relativity theory [20]. Ramesh and Umadevi have studied perfect fluid matter distributions with linearly varying deceleration parameter for FRW universe in $f(R,T)$ gravity [21]. With deceleration parameter, Baffou et al. have investigated $f(R,T)$ gravitation theory in FRW universe for collisional matter [22]. Perfect fluid solutions for FRW space in $f(R,T)$ theory was researched by Myrzakulov [23]. Sahoo has investigated cosmic strings in $f(R,T)$ theory [24]. Because, the strings play an important role in the definition of the early universe [25]. Strings caused the density impairments leading to the formation of galaxies. [24, 25]. As the stress energy of the strings can be combined with the gravitational field, it may be interesting to investigate the effect of gravity on the strings [26]. Also, the homogeneous and isotropic FRW universe model best describes the present universe.

In 2011, Harko et al. proposed various $f(R,T)$ models i.e. $f(R,T) = R + 2f(T)$, $f(R,T) = f_1(R) + f_2(T)$ and $f(R,T) = f_1(R) + f_2(R)f_3(T)$ in their study for the solution of modified field equations. Also, Starobinsky [27] suggested the following $f(R)$ model such as $f(R) = R + \alpha R^2 + \beta R^n$. Where αR^2 demonstrates cosmic inflation and linear term R demonstrates cosmic acceleration. Considering the suggestions by Harko et al. [3] and Starobinsky [27], we use $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ to get solutions in our model due to the following reasons:

- i) In our suggestion, the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, the linear term R demonstrates cosmic acceleration [28]
- ii) In our suggestion, the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, $\alpha_2 f_3(T)$ represents interaction between matter and curvature.
- iii) To get viable accelerating universe it's needed that $f_R > 0$, $R > R_0 (> 0)$ [27], [29]
- iv) In our suggestion, the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model includes both geometrical terms (Ricci scalar R) and matter terms (trace of energy-momentum tensor T).

In this study, using time varying deceleration parameter, we consider string cloud with perfect fluid matter distributions for FRW space-time in reconstructed new $f(R,T)$ model as $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$. This new $f(R,T)$ model is important because it combines the first and second model of Harko et al. [3] also give general relativity solutions for $\alpha_1 = 1$ and $\alpha_2 = 0$ or $f_3(T) = 0$. From these reasons, in this study we consider string cloud with perfect fluid distributions for FRW space-time in $f(R,T)$ gravity. The paper is organized as follows. In section 2 we will give the field equation of $f(R,T)$ gravity. In section 3, we consider the reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model in $f(R,T)$ gravity. Finally, conclusions are given with various $f(R,T)$ models.

2. FIELD EQUATIONS OF $f(R,T)$ GRAVITY

The action of this theory was proposed by Harko et al. in 2011 [3] as follows

$$S = \frac{1}{16\pi} \int \left(\frac{f(R,T)}{G} + L_m \right) \sqrt{-g} d^4x \quad (1)$$

Here T is trace of T_{ik} and g is determinant of g_{ik} , R is Ricci scalar, $f(R,T)$ is an arbitrary function of R and T , also L_m is the matter of the Lagrangian [3]. T_{ik} is defined by

$$T_{ik} = L_m g_{ik} - \frac{2\partial L_m}{\partial g^{ik}} \quad (2)$$

and by varying in equation (1) we have [3]

$$\begin{aligned} f_R(R,T)R_{ik} - \frac{1}{2}f(R,T)g_{ik} + (g_{ik}\nabla_a\nabla^a - \nabla_i\nabla_k)f_R(R,T) \\ = 8\pi T_{ik} - f_T(R,T)T_{ik} - f_T(R,T)\Xi_{ik} \end{aligned} \quad (3)$$

Here $f_T(R,T)$ shows the derivative with respect to T , $f_R(R,T)$ shows the derivative with respect to R , ∇_i is the covariant derivative and Ξ_{ik} is defined as [3]

$$\Xi_{ik} = -2T_{ik} + L_m g_{ik} - 2g^{ab} \frac{\partial^2 L_m}{\partial g^{ik} \partial g^{ab}} \quad (4)$$

83 If we contract equation (3), we get
84

$$(R + 3\nabla_a \nabla^a) f_R(R, T) - 2f(R, T) = (8\pi - f_T(R, T))T - \Xi f_T(R, T) \quad (5)$$

85 If we substitute equation (5) in equation (3) we get the field equations in the f(R,T) theory [3].
86

$$\begin{aligned} f_R(R, T) \left(R_{ik} - \frac{1}{3} R g_{ik} \right) + \frac{1}{6} f(R, T) g_{ik} \\ = (8\pi - f_T(R, T)) \left(T_{ik} - \frac{1}{3} T g_{ik} \right) - f_T(R, T) \left(\Xi_{ik} - \frac{1}{3} \Xi_{ik} g \right) \\ + \nabla_i \nabla_k f_R(R, T) \end{aligned} \quad (6)$$

87 In this study, we'll discuss a new f(R,T) theory where $f(R, T)$ is linear in R and an arbitrary
88 function of T i.e. $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ for string cloud with perfect fluid matter
89 distributions. Here α_1 and α_2 are constants and $f_3(T)$ is an arbitrary function of T. The FRW
90 universe model is given by
91

$$ds^2 = dt^2 - \frac{A(t)^2}{1 - kr^2} dr^2 - A(t)^2 r^2 (d\theta^2 + \sin(\theta^2) d\phi^2) \quad (7)$$

92 here k is a curvature parameter and given as $k = 0, +1, -1$. The cosmic strings are very
93 important to solve the mystery of the universe. Then recently, various scientists have been
94 investigated string cosmology in various cosmological models and theories with different
95 matter distributions [17], [30], [31]. According to Zel'dovich [32]; strings gives leads to the
96 formation of galaxies. Also, strings have the gravitational field. Then, it is important to
97 research the gravitational effects that arise from strings [17]. However, perfect fluid matter
98 distribution could adequately characterize the majority of the thermal history of the universe
99 [33]. From these reasons, we have used string cloud with perfect fluid matter distribution in
100 f(R,T) theory. The string cloud with perfect fluid energy-momentum tensor is given by

$$T_{ik} = (\rho + p)u_i u_k - p g_{ik} - \lambda x_i x_k \quad (8)$$

101 Here u_i is the four velocity vector, x_i is the direction of string, ρ is energy density, λ is string
102 tension and p is pressure [17]. Also, u_i and x_i satisfy $u_i u^i = -1$, $x_i x^i = 1$ and $u_i x^i = 0$
103 conditions [17].
104

105 2.1 Field Equations for $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ Model

106 If we take $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ in equation (3) we reconstruct the f(R,T) gravity model as
107 follows

$$\alpha_1 R_{ik} - \frac{\alpha_1}{2} R g_{ik} = (8\pi - \alpha_2 f'_3) T_{ik} - \alpha_2 f'_3 \Xi_{ik} + \frac{\alpha_2 f_3}{2} g_{ik} \quad (9)$$

108 where $\Xi_{ik} = -2T_{ik} - p g_{ik}$ also, the prime indicates derivative w.r.t. T . Substitution of
109 equations (7) and (8) into equation (9) yields

$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi(p - \lambda) - \alpha_2 f'_3 \lambda - \frac{\alpha_2}{2} f_3 \quad (10)$$

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$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi p - \frac{\alpha_2}{2} f_3 \quad (11)$$

111

$$3\alpha_1 \frac{\dot{A}^2}{A^2} + 3\alpha_1 \frac{k}{A^2} = -8\pi\rho - \alpha_2(p + \rho)f'_3 - \frac{\alpha_2}{2} f_3 \quad (12)$$

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In this study we have three field equations and four unknowns. In order to solve equations (10)-(12) we use a time periodic deceleration parameter (DP). Because the sign of DP gives the accelerates of the universe or not [19]. To obtain the accelerated expansion of the universe many studies have been done using DP [19]. Berman [34] obtained a law of variation for Hubble parameter in general relativity for FRW universe model with a constant deceleration parameter (q) [19]. According to Berman; the DP can obtain values $-1 \leq q < 0$, corresponding to the accelerating expansion [34]. Various authors have researched with various cosmological models using constant deceleration parameter. Besides, it is important to study varying DP to explain expansion of the universe. Then in this study we have used oscillating time varying DP model. This model is given by;

$$q = -\frac{A \ddot{A}}{\dot{A}^2} = m \cos(nt) - 1 \quad (13)$$

125 Here m and n are positive constants [19]. If we solve equation (13), with the initial condition
126 $A(0) = 0$ we obtain the metric potential as

$$A = c_1 \tan\left(\frac{nt}{2}\right)^{\frac{1}{m}} \quad (14)$$

127 here c_1 is an integration constant [19]. From equations (10)-(12) and (14) we get the values
128 of pressure, energy density and string tension for the reconstructed f(R,T) theory in the FRW
129 space-time as follows
130

$$p = \frac{\alpha_1 k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{\alpha_1 n^2 (2m+3) \tan\left(\frac{nt}{2}\right)^2}{32 \pi m^2} + \frac{m^2 \alpha_2 f_3 + \alpha_1 n^2}{16 \pi m^2} - \frac{\alpha_1 n^2 (2m-3)}{32 \pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (15)$$

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$$\rho = \frac{\alpha_1 n^2 (\alpha_2 (2m-3) f_3' - 24\pi)}{32 m^2 \pi (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^2} - \frac{\alpha_1 k (\alpha_2 f_3' + 24\pi)}{8\pi c_1 (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{\alpha_1 n^2 (\alpha_2 (2m+3) f_3' + 24\pi) \tan\left(\frac{nt}{2}\right)^2}{32 m^2 \pi (\alpha_2 f_3' + 8\pi)} - \frac{\alpha_2 m^2 f_3 + 3\alpha_1 n^2}{16 \pi m^2} \quad (16)$$

132 and

$$\lambda = 0 \quad (17)$$

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134 3. RESULTS AND DISCUSSION

135 In this research, we have investigated a new f(R,T) gravitational model with FRW universe.
136 In 2011, Harko et al. proposed new f(R,T) models with $f(R,T) = R + 2\mu T$, $f(R,T) = \mu R + \mu T$
137 (here μ is a constant) and $f(R,T) = f_1(R) + f_2(R)f_3(T)$ [3]. For the solutions of modified field
138 equations in this study we proposed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$. This model contains the first
139 and second models of Harko et al. [3]. For $\alpha_1 = 1$, $\alpha_2 = 2\mu$ and $f_3(T) = T$, we obtain the first
140 model of Harko et al. i.e. $f(R,T) = R + 2\mu T$ [3]. If we take $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$, we get

141 the second model of Harko et al. i.e. $f(R, T) = \mu R + \mu T$ [3]. Also, for $\alpha_1 = 1$ and $\alpha_2 = 0$ or
 142 $f_3(T) = 0$, our modified $f(R, T)$ gravitation theory transforms to general relativity.

143

144 From equations (15)- (17) we obtain the pressure, energy density and string tension density
 145 as follows

146

$$p = \frac{n^2(8\pi + 3\mu)\cos(nt)}{2m(2\pi + \mu)(4\pi + \mu)(\cos(2nt) - 1)} - \frac{3n^2}{2m^2(2\pi + \mu)(\cos(2nt) - 1)} + \frac{k\pi}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (18)$$

147

$$\rho = \frac{n^2\mu\cos(nt)}{2m(2\pi + \mu)(4\pi + \mu)(\cos(2nt) - 1)} + \frac{3n^2}{2m^2(2\pi + \mu)(\cos(2nt) - 1)} - \frac{k(3\pi + \mu)}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (19)$$

148

$$\lambda = 0 \quad (20)$$

149 Using $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$ in the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we have the second
 150 model of Harko et al. [3]. Using equations (15)- (17) we get the pressure, energy density and
 151 string tension density as follows

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$$p = \frac{n^2\mu(16\pi + 3\mu)\cos(nt)}{m(4\pi + \mu)(8\pi + \mu)(\cos(2nt) - 1)} - \frac{3n^2\mu}{m^2(4\pi + \mu)(\cos(2nt) - 1)} + \frac{4k\mu\pi}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (21)$$

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$$\rho = \frac{n^2\mu^2\cos(nt)}{m(4\pi + \mu)(8\pi + \mu)(\cos(2nt) - 1)} + \frac{3n^2\mu}{m^2(4\pi + \mu)(\cos(2nt) - 1)} - \frac{2k\mu(6\pi + \mu)}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (22)$$

154

$$\lambda = 0 \quad (23)$$

155 If we take $\alpha_1 = 1$ and $f_3(T) = 0$, in the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we obtain general
 156 relativity solutions for pressure, energy density and string tension density as follows
 157

$$p = \frac{k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{n^2}{16\pi m^2} + \frac{n^2(2m+3)\tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} - \frac{n^2(2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (24)$$

158

$$\rho = -\frac{3k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{3n^2}{16\pi m^2} - \frac{3n^2 \tan\left(\frac{nt}{2}\right)^2}{32m^2\pi} - \frac{3n^2}{32m^2\pi \tan\left(\frac{nt}{2}\right)^2} \quad (25)$$

159 and

$$\lambda = 0 \quad (26)$$

160 4. CONCLUSION

161 In this study, for the solutions of the FRW universe model we have used a time periodic
 162 varying deceleration parameter given by Shen and Zhao [19]. Using this parameter, we have
 163 obtained the following results:

164 From equations (15) and (16) it is easily seen that cosmic pressure and energy density are
 165 functions of cosmic time. Also, as $t \rightarrow \infty$ cosmic the pressure and energy density decrease
 166 with time. From equation (17), we found $\lambda = 0$. Kiran and Reddy [35] studied the Bianchi III
 167 universe with string cosmological model in $f(R, T)$ gravity and they found zero string tension
 168 density. Naidu et al. [36] investigated Bianchi V space-time with string cosmological model.
 169 Also they found $\lambda = 0$ in $f(R, T)$ gravity. However, Çağlar and Aygün have obtained zero
 170 string tension density for FRW universe in Lyra [37], Creation field [38], Brans-Dicke [39] and
 171 self-creation [40] theories. Also, Şen and Aygün have investigated the FRW universe model
 172 in self creation cosmology with cloud of string with perfect fluid attached strange quark
 173 matter and obtained zero string tension density in their study [41]. Our solution agrees with
 174 [35 – 41]. For $\lambda = 0$, string cloud with perfect fluid matter distribution turns out to be a perfect
 175 fluid matter distribution. This universe model does not allow the formation of strings in the
 176 frame of $f(R, T)$ gravitation theory similar other models.

177 It follows from equations (18)-(26) that the cosmic pressure and energy density in the $f(R, T)$
 178 gravitation models and general relativity are functions of cosmic time t , and if we take the
 179 limit $t \rightarrow \infty$, cosmic pressure and energy density decrease with cosmic time t . Also, we found
 180 that $\lambda = 0$ in all cosmological models. The obtained solutions show that the source of the
 181 acceleration of the universe is similar to perfect fluid distributions not strings.

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