

Original Research Article

Time Periodic Varying Deceleration Parameter In Reconstructed $f(R,T)$ Gravity

ABSTRACT

In this paper, we have investigated string cloud with perfect fluid matter distributions for Friedmann–Robertson–Walker (FRW) space-time in reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ gravity. To obtain exact solutions of the field equations we have used a time periodic varying deceleration parameter. The new reconstructed $f(R,T)$ model includes two models of Harko et al. and transforms to general relativity. However, we obtain vanishing string tension density in all models. This result agrees with Aygün and colleagues, Naidu et al. and Kiran and Reddy in different alternative gravitation theories.

Keywords: $f(R,T)$ gravity, FRW universe, deceleration parameter, string cloud, perfect fluid.

1. INTRODUCTION

Today we know that our universe is expanding and contains about 20% dark matter, 75% dark energy and 5% baryonic matter [1, 2]. Also, the dark energy and the dark matter etc., are still unresolved secrets for human beings. To solve the expansion of the universe two approaches are suggested. One of them is dark matter and dark energy models. The other is alternative gravitation theories such as $f(R,T)$ [3], Lyra [4], $f(R)$ [5], and Brans-Dicke [6] etc. After the $f(R,T)$ study of Harko et al. [3] in 2011, many scientist researched $f(R,T)$ gravitation theory [7 – 13] and some of authors represented new $f(R,T)$ models [14 – 16].

In 2011, Pradhan and Chouhan have studied a string cloud with perfect fluid matter distributions for Bianchi I space-time in the frame of general relativity [17]. Pradhan and Singh have researched massive string solutions in Bianchi I universe for Lyra geometry using constant deceleration parameter [18]. Shen and Zhao have suggested a new model which is oscillating with time periodic varying deceleration parameter for FRW universe in general relativity theory [19]. Also, Shen and Jiang have investigated plane symmetric metric with oscillating varying deceleration parameter for perfect fluid in general relativity theory [20]. Ramesh and Umadevi have studied perfect fluid matter distributions with linearly varying deceleration parameter for FRW universe in $f(R,T)$ gravity [21]. With deceleration parameter, Baffou et al. have investigated $f(R,T)$ gravitation theory in FRW universe for collisional matter [22]. Perfect fluid solutions for FRW space in $f(R,T)$ theory was researched by Myrzakulov [23]. Sahoo has investigated cosmic strings in $f(R,T)$ theory [24]. Because, the strings play an important role in the definition of the early universe [25]. Strings caused the density impairments leading to the formation of galaxies [24, 25]. As the stress energy of the strings can be combined with the gravitational field, it may be interesting to investigate the effect of gravity on the strings [26]. Also, the homogeneous and isotropic FRW universe model best describes the present universe.

fluctuations

investigated

* This is unclear. Usually 'inflation' means accelerated expansion

In 2011, Harko et al. proposed various $f(R, T)$ models i.e. $f(R, T) = R + 2f(T)$, $f(R, T) = f_1(R) + f_2(T)$ and $f(R, T) = f_1(R) + f_2(R)f_3(T)$ in their study for the solution of modified field equations. Also, Starobinsky [27] suggested the following $f(R)$ model, such as $f(R) = R + \alpha R^2 + \beta R^n$, where αR^2 implies cosmic inflation and linear term R demonstrates cosmic acceleration. Considering the suggestions by Harko et al. [3] and Starobinsky [27], we use $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ to get solutions in our model due to the following reasons:

- In our suggestion, the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, the linear term R demonstrates cosmic acceleration [28].
- In our suggestion, the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, $\alpha_2 f_3(T)$ represents interaction between matter and curvature.
- To get viable accelerating universe it's needed that $f_R > 0$, $R > R_0 (> 0)$ [27], [29].
- In our suggestion, the $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model includes both geometrical terms (Ricci scalar R) and matter terms (trace of energy-momentum tensor T).

In this study, using time varying deceleration parameter, we consider string cloud with perfect fluid matter distributions for FRW space-time in reconstructed new $f(R, T)$ model as $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$. This new $f(R, T)$ model is important because it combines the first and second model of Harko et al. [3] and also give general relativity solutions for $\alpha_1 = 1$ and $\alpha_2 = 0$ or $f_3(T) = 0$. From these reasons, in this study we consider string cloud with perfect fluid distributions for FRW space-time in $f(R, T)$ gravity. The paper is organized as follows. In section 2 we will give the field equation of $f(R, T)$ gravity. In section 3, we consider the reconstructed $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model in $f(R, T)$ gravity. Finally, conclusions are given with various $f(R, T)$ models.

2. FIELD EQUATIONS OF $f(R, T)$ GRAVITY

The action of this theory was proposed by Harko et al. in 2011 [3] as follows

$$S = \frac{1}{16\pi} \int \left(\frac{f(R, T)}{G} + L_m \right) \sqrt{-g} d^4x \quad (1)$$

Here T is trace of T_{ik} and g is determinant of g_{ik} , R is Ricci scalar, $f(R, T)$ is an arbitrary function of R and T , also L_m is the matter of the Lagrangian [3]. T_{ik} is defined by

$$T_{ik} = L_m g_{ik} - \frac{2\partial L_m}{\partial g^{ik}} \quad (2)$$

and by varying in equation (1) we have [3]

$$f_R(R, T)R_{ik} - \frac{1}{2}f(R, T)g_{ik} + (g_{ik}\nabla_a\nabla^a - \nabla_i\nabla_k)f_R(R, T) = 8\pi T_{ik} - f_T(R, T)T_{ik} - f_T(R, T)\Xi_{ik} \quad (3)$$

Here $f_T(R, T)$ shows the derivative with respect to T , $f_R(R, T)$ shows the derivative with respect to R , ∇_i is the covariant derivative and Ξ_{ik} is defined as [3]

$$\Xi_{ik} = -2T_{ik} + L_m g_{ik} - 2g^{ab} \frac{\partial^2 L_m}{\partial g^{ik} \partial g^{ab}} \quad (4)$$

If we contract equation (3), we get

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$$(R + 3\nabla_a \nabla^a) f_R(R, T) - 2f(R, T) = (8\pi - f_T(R, T))T - \Xi f_T(R, T) \quad (5)$$

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If we substitute equation (5) in equation (3) we get the field equations in the $f(R, T)$ theory [3].

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$$\begin{aligned} f_R(R, T) \left(R_{ik} - \frac{1}{3} R g_{ik} \right) + \frac{1}{6} f(R, T) g_{ik} \\ = (8\pi - f_T(R, T)) \left(T_{ik} - \frac{1}{3} T g_{ik} \right) - f_T(R, T) \left(\Xi_{ik} - \frac{1}{3} \Xi_{ik} g \right) \\ + \nabla_i \nabla_k f_R(R, T) \end{aligned} \quad (6)$$

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In this study, we'll discuss a new $f(R, T)$ theory where $f(R, T)$ is linear in R and an arbitrary function of T i.e. $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ for string cloud with perfect fluid matter distributions. Here α_1 and α_2 are constants and $f_3(T)$ is an arbitrary function of T . The FRW universe model is given by

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$$ds^2 = dt^2 - \frac{A(t)^2}{1 - kr^2} dr^2 - A(t)^2 r^2 (d\theta^2 + \sin(\theta^2) d\phi^2) \quad (7)$$

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here k is a curvature parameter and given as $k = 0, +1, -1$. The cosmic strings are very important to solve the mystery of the universe. Then recently, various scientists have been investigated string cosmology in various cosmological models and theories with different matter distributions [17], [30], [31]. According to Zel'dovich [32], strings gives leads to the formation of galaxies. Also, strings have the gravitational fields. Then, it is important to research the gravitational effects that arise from strings [17]. However, perfect fluid matter distribution could adequately characterize the majority of the thermal history of the universe [33]. From these reasons, we have used string cloud with perfect fluid matter distribution in $f(R, T)$ theory.

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The string cloud with perfect fluid energy-momentum tensor is given by

$$T_{ik} = (\rho + p)u_i u_k - p g_{ik} - \lambda x_i x_k \quad (8)$$

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Here u_i is the four velocity vector, x_i is the direction of string, ρ is energy density, λ is string tension and p is pressure [17]. Also, u_i and x_i satisfy $u_i u^i = -1$, $x_i x^i = 1$ and $u_i x^i = 0$ conditions [17].

2.1 Field Equations for $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ Model

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If we take $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ in equation (3) we reconstruct the $f(R, T)$ gravity model as follows

$$\alpha_1 R_{ik} - \frac{\alpha_1}{2} R g_{ik} = (8\pi - \alpha_2 f'_3) T_{ik} - \alpha_2 f'_3 \Xi_{ik} + \frac{\alpha_2 f_3}{2} g_{ik} \quad (9)$$

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where $\Xi_{ik} = -2T_{ik} - p g_{ik}$ also, the prime indicates derivative w.r.t. T . Substitution of equations (7) and (8) into equation (9) yields

$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi(p - \lambda) - \alpha_2 f'_3 \lambda - \frac{\alpha_2}{2} f_3 \quad (10)$$

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$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi p - \frac{\alpha_2}{2} f_3 \quad (11)$$

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$$3\alpha_1 \frac{\dot{A}^2}{A^2} + 3\alpha_1 \frac{k}{A^2} = -8\pi\rho - \alpha_2(p + \rho)f'_3 - \frac{\alpha_2}{2} f_3 \quad (12)$$

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whether the universe is accelerated

In this study we have three field equations and four unknowns. In order to solve equations (10)-(12) we use a time periodic deceleration parameter (DP). Because the sign of DP gives determines the accelerates of the universe or not [19]. To obtain the accelerated expansion of the universe many studies have been done using DP [19]. Berman [34] obtained a law of variation for Hubble parameter in general relativity for FRW universe model with a constant deceleration parameter (q) [19]. According to Berman; the DP can obtain values $-1 \leq q < 0$ corresponding to the accelerating expansion [34]. Various authors have researched with various cosmological models using constant deceleration parameter. Besides, it is important to study varying DP to explain expansion of the universe. Then in this study we have used oscillating time varying DP model. This model is given by; considered an

$$q = -\frac{A \ddot{A}}{\dot{A}^2} = m \cos(nt) - 1 \quad (13)$$

Here m and n are positive constants [19]. If we solve equation (13), with the initial condition $A(0) = 0$ we obtain the metric potential as

$$A = c_1 \tan\left(\frac{nt}{2}\right)^{\frac{1}{m}} \quad (14)$$

here c_1 is an integration constant [19]. From equations (10)-(12) and (14) we get the values of pressure, energy density and string tension for the reconstructed f(R,T) theory in the FRW space-time as follows

$$p = \frac{\alpha_1 k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{\alpha_1 n^2 (2m+3) \tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} + \frac{m^2 \alpha_2 f_3 + \alpha_1 n^2}{16\pi m^2} - \frac{\alpha_1 n^2 (2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (15)$$

$$\rho = \frac{\alpha_1 n^2 (\alpha_2 (2m-3) f_3' - 24\pi)}{32m^2 \pi (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^2} - \frac{\alpha_1 k (\alpha_2 f_3' + 24\pi)}{8\pi c_1 (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{\alpha_1 n^2 (\alpha_2 (2m+3) f_3' + 24\pi) \tan\left(\frac{nt}{2}\right)^2}{32m^2 \pi (\alpha_2 f_3' + 8\pi)} - \frac{\alpha_2 m^2 f_3 + 3\alpha_1 n^2}{16\pi m^2} \quad (16)$$

$$\text{and } \lambda = 0 \quad (17)$$

3. RESULTS AND DISCUSSION

In this research, we have investigated a new f(R,T) gravitational model with FRW universe. In 2011, Harko et al. proposed new f(R,T) models with $f(R,T) = R + 2\mu T$, $f(R,T) = \mu R + \mu T$ (here μ is a constant) and $f(R,T) = f_1(R) + f_2(R)f_3(T)$ [3]. For the solutions of modified field equations in this study we proposed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$. This model contains the first and second models of Harko et al. [3]. For $\alpha_1 = 1$, $\alpha_2 = 2\mu$ and $f_3(T) = T$, we obtain the first model of Harko et al. i.e. $f(R,T) = R + 2\mu T$ [3]. If we take $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$, we get the second model of Harko et al. i.e. $f(R,T) = \mu R + \mu T$ [3]. Also, for $\alpha_1 = 1$ and $\alpha_2 = 0$ or $f_3(T) = 0$, our modified f(R,T) gravitation theory transforms to general relativity.

$$p = \frac{k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{n^2}{16\pi m^2} + \frac{n^2(2m+3)\tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} - \frac{n^2(2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (24)$$

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$$\rho = -\frac{3k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{3n^2}{16\pi m^2} - \frac{3n^2 \tan\left(\frac{nt}{2}\right)^2}{32m^2\pi} - \frac{3n^2}{32m^2\pi \tan\left(\frac{nt}{2}\right)^2} \quad (25)$$

158 and

$$\lambda = 0 \quad (26)$$

159 4. CONCLUSION

160 In this study, for the solutions of the FRW universe model we have used a time periodic varying
161 deceleration parameter given by Shen and Zhao [19]. Using this parameter, we have obtained
162 the following results:

163 From equations (15) and (16) it is easily seen that cosmic pressure and energy density are
164 functions of cosmic time. Also, as $t \rightarrow \infty$ cosmic the pressure and energy density decrease
165 with time. From equation (17), we found $\lambda = 0$. Kiran and Reddy [35] studied the Bianchi III
166 universe with string cosmological model in $f(R,T)$ gravity and they found zero string tension
167 density. Naidu et al. [36] investigated Bianchi V space-time with string cosmological model.
168 Also they found $\lambda = 0$ in $f(R,T)$ gravity. However, Çağlar and Aygün have obtained zero string
169 tension density for FRW universe in Lyra [37], Creation field [38], Brans-Dicke [39] and self-
170 creation [40] theories. Also, Sen and Aygün have investigated the FRW universe model in self
171 creation cosmology with cloud of string with perfect fluid attached strange quark matter and
172 obtained zero string tension density in their study [41]. Our solution agrees with [35 – 41]. For

173 A $\lambda = 0$, string cloud with perfect fluid matter distribution turns out to be a perfect fluid matter
174 distribution. This universe model does not allow the formation of strings in the frame of $f(R,T)$
175 gravitation theory similar other models.

176 It follows from equations (18)-(26) that the cosmic pressure and energy density in the $f(R,T)$
177 gravitation models and general relativity are functions of cosmic time t , and if we take the limit for large t the
178 $t \rightarrow \infty$, cosmic pressure and energy density decrease with cosmic time t . Also, we found that
179 $\lambda = 0$ in all cosmological models. The obtained solutions show that the source of the
180 acceleration of the universe is similar to perfect fluid distributions, not strings.

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