

Original Research Article

**Time Periodic Varying Deceleration Parameter
In Reconstructed $f(R,T)$ Gravity**

ABSTRACT

In this paper, we have investigated string cloud with perfect fluid matter distributions for Friedmann–Robertson–Walker (FRW) space-time in reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ gravity. To obtain exact solutions of field equations we have used time periodic varying deceleration parameter. The new represented $f(R,T)$ model includes two models of Harko et al. and transforms to general relativity. However, we obtain vanish string tension density in all models. This result agrees with Aygün and colleagues, Naidu et al. and Kiran and Reddy in different alternative gravitation theories.

Keywords: $f(R,T)$ gravity, FRW universe, deceleration parameter, string cloud, perfect fluid.

1. INTRODUCTION

Today we know that our universe is expanding and contains about 20% dark matter, 75% dark energy and 5% baryonic matter [1, 2]. Also, the dark energy and the dark matter etc., are still unresolved secrets for human beings. To solve the expansion of the universe two approaches are suggested. One of them is dark matter and dark energy models. The other is alternative gravitation theories such as $f(R,T)$ [3], Lyra [4], $f(R)$ [5], and Brans-Dicke [6] etc. After the $f(R,T)$ study of Harko et al. [3] in 2011, many scientist researched $f(R,T)$ gravitation theory [7 – 13] and some of authors represented new $f(R,T)$ models [14 – 16]. In 2011, Pradhan and Chouhan have studied string cloud with perfect fluid matter distributions for Bianchi I space-time in presence of general relativity [17]. Pradhan and Singh have researched massive string solutions in Bianchi I universe for Lyra geometry using constant deceleration parameter [18]. Shen and Zhao have suggested new model which is oscillating with time periodic varying deceleration parameter for FRW universe in general relativity theory [19]. Also, Shen and Jiang have investigated plane symmetric metric with oscillating varying deceleration parameter for perfect fluid in general relativity theory [20]. Ramesh and Umadevi have studied perfect fluid matter distributions with linearly varying deceleration parameter for FRW universe in $f(R,T)$ gravity [21]. With deceleration parameter, Baffou et al. have investigated $f(R,T)$ gravitation theory in FRW universe for collisional matter [22]. Perfect fluid solutions for FRW space in $f(R,T)$ theory was researched by Myrzakulov [23]. Sahoo has investigated cosmic strings in $f(R,T)$ theory [24]. Because, the strings play an important role in the definition of the early universe [25]. Strings caused the density impairments leading to the formation of galaxies. [24, 25]. As the stress energy of the strings can be combined with the gravitational field, it may be interesting to research the effect of gravity on the strings [26]. Also, homogeneous and isotropic FRW universe model best describes the present universe.

investigate

In 2011, Harko et al. proposed various $f(R, T)$ models i.e. $f(R, T) = R + 2f(T)$, $f(R, T) = f_1(R) + f_2(T)$ and $f(R, T) = f_1(R) + f_2(R)f_3(T)$ in their study for the solution of modified field equations. Also, Starobinsky [27] suggested the following $f(R)$ model such as $f(R) = R + \alpha R^2$. Where αR^2 demonstrates cosmic inflation and linear term R demonstrates cosmic acceleration. Considering suggestions Harko et al. [3] and Starobinsky [27], we use $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ to get solutions in our model due to the following reasons:

- i) In our suggestion, $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, linear term R demonstrates cosmic acceleration [28]
- ii) In our suggestion, $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, $\alpha_2 f_3(T)$ represents interaction between matter and curvature.
- iii) To get viable accelerating universe it's needed $f_R > 0$, $R > R_0 (> 0)$ [27], [29]
- iv) In our suggestion, $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model includes both geometrical terms (Ricci scalar R) and matter terms (trace of energy-momentum tensor T).
- v) Our proposed $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model is important because combines first and second model of Harko et al. [3] includes linear R and arbitrary function of T .

From these reasons, in this study we consider string cloud with perfect fluid distributions for FRW space-time in $f(R, T)$ gravity. The paper is organized as follows. In section 2 we will give field equation of $f(R, T)$ gravity. In section 3, we consider reconstructed $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model in $f(R, T)$ gravity. Finally, conclusions are given with various $f(R, T)$ model.

2. FIELD EQUATIONS OF $f(R, T)$ GRAVITY

The action of this theory was proposed by Harko et al. in 2011 [3] as follows

$$S = \frac{1}{16\pi} \int \left(\frac{f(R, T)}{G} + L_m \right) \sqrt{-g} d^4x \quad (1)$$

Here T is trace of T_{ik} and g is determinant of g_{ik} , R is Ricci scalar, $f(R, T)$ is arbitrary function of R and T , also L_m is Lagrangian [3]. T_{ik} is defined by

$$T_{ik} = L_m g_{ik} - \frac{2\partial L_m}{\partial g^{ik}} \quad (2)$$

by varying equation (1) we have [3]

$$\begin{aligned} f_R(R, T) R_{ik} - \frac{1}{2} f(R, T) g_{ik} + (g_{ik} \nabla_a \nabla^a - \nabla_i \nabla_k) f_R(R, T) \\ = 8\pi T_{ik} - f_T(R, T) T_{ik} - f_R(R, T) \Xi_{ik} \end{aligned} \quad (3)$$

Here $f_T(R, T)$ shows derivative with respect to T , $f_R(R, T)$ shows derivative with respect to R , ∇_i is covariant derivative and Ξ_{ik} is defined as [3]

$$\Xi_{ik} = -2T_{ik} + L_m g_{ik} - 2g^{ab} \frac{\partial^2 L_m}{\partial g^{ik} \partial g^{ab}} \quad (4)$$

If we contract equation (3), we get

$$(R + 3\nabla_a \nabla^a) f_R(R, T) - 2f(R, T) = (8\pi - f_T(R, T))T - \Xi f_T(R, T) \quad (5)$$

82 If we substitute equation (5) in equation (3) we get ^{the} field equations in ^{the} $f(R,T)$ theory [3].
83

$$f_R(R,T) \left(R_{ik} - \frac{1}{3} R g_{ik} \right) + \frac{1}{6} f(R,T) g_{ik} \\ = (8\pi - f_T(R,T)) \left(T_{ik} - \frac{1}{3} T g_{ik} \right) - f_T(R,T) \left(\Xi_{ik} - \frac{1}{3} \Xi_{ik} g \right) \\ + \nabla_i \nabla_k f_R(R,T) \quad (6)$$

84 In this study, we'll discuss ^a new $f(R,T)$ theory model ^{where $f(R,T)$ is linear in R and an} which linear R and arbitrary function of T .
85 i.e. $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ for string cloud with perfect fluid matter distributions. ^{and} α_1
86 and α_2 are constants ^{and} also $f_3(T)$ is arbitrary function of T . ^{The} FRW universe model is given by
87

$$ds^2 = dt^2 - \frac{A(t)^2}{1 - kr^2} dr^2 - A(t)^2 r^2 (d\theta^2 + \sin(\theta^2) d\phi^2) \quad (7)$$

88 here $k = 0, \pm 1$ is constant. For the solutions of new $f(R,T)$ theory the T_{ik} of string cloud with
89 perfect fluid is given by

$$T_{ik} = (\rho + p) u_i u_k - p g_{ik} - \lambda x_i x_k \quad (8)$$

90 Here u_i is ^{the} four velocity vector, x_i is the direction of string, ρ is energy density, λ is string
91 tension, and p is pressure [17]. Also, u_i and x_i ^{four vectors} satisfy $u_i u^i = -1$, $x_i x^i = 1$ and
92 $u_i x^i = 0$ conditions [17].
93

94 2.1 Field Equations for $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ Model

95 If we take $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ in equation (3) we reconstruct ^{the} $f(R,T)$ gravity model as
96 follows

$$\alpha_1 R_{ik} - \frac{\alpha_1}{2} R g_{ik} = (8\pi - \alpha_2 f'_3) T_{ik} - \alpha_1 \Xi_{ik} + \frac{\alpha_2 f_3}{2} g_{ik} \quad (9)$$

97 ^{into} Where prime indicates derivative w.r.t. T . Substitution of equations (7) and (8) ^{into} equation (9)
98 yields

$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi(p - \lambda) - \alpha_2 f'_3 \lambda - \frac{\alpha_2}{2} f_3 \quad (10)$$

99

$$2\alpha_1 \frac{\ddot{A}}{A} + \alpha_1 \frac{\dot{A}^2}{A^2} + \alpha_1 \frac{k}{A^2} = 8\pi p - \frac{\alpha_2}{2} f_3 \quad (11)$$

100

$$3\alpha_1 \frac{\dot{A}^2}{A^2} + 3\alpha_1 \frac{k}{A^2} = -8\pi\rho - \alpha_2(p + \rho) f'_3 - \frac{\alpha_2}{2} f_3 \quad (12)$$

101

102 In this study we have three field equations and four unknowns. In order to solve equations
103 (10)-(12) we use time periodic deceleration parameter

$$q = -\frac{A \ddot{A}}{\dot{A}^2} = m \cos(nt) - 1 \quad (13)$$

104 Here m and n are positive constants [19]. If we solve equation (13), we obtain ^{the} metric
105 potential as

$$A = c_1 \tan\left(\frac{nt}{2}\right)^{\frac{1}{m}} \quad (14)$$

here c_1 is an integration constant [19]. From equations (10)-(12) and (14) we get the values of pressure, energy density and string tension for reconstructed $f(R,T)$ theory in FRW space-time as follows

$$p = \frac{\alpha_1 k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{\alpha_1 n^2 (2m+3) \tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} + \frac{m^2 \alpha_2 f_3 + \alpha_1 n^2}{16\pi m^2} - \frac{\alpha_1 n^2 (2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (15)$$

$$\rho = \frac{\alpha_1 n^2 (\alpha_2 (2m-3) f_3' - 24\pi)}{32m^2 \pi (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^2} - \frac{\alpha_1 k (\alpha_2 f_3' + 24\pi)}{8\pi c_1 (\alpha_2 f_3' + 8\pi) \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{\alpha_1 n^2 (\alpha_2 (2m+3) f_3' + 24\pi) \tan\left(\frac{nt}{2}\right)^2}{32m^2 \pi (\alpha_2 f_3' + 8\pi)} - \frac{\alpha_2 m^2 f_3 + 3\alpha_1 n^2}{16\pi m^2} \quad (16)$$

and

$$\lambda = 0 \quad (17)$$

3. RESULTS AND DISCUSSION

In this research, we have investigated new $f(R,T)$ gravitational model with FRW universe. In 2011, Harko et al. proposed new various $f(R,T)$ models as $f(R,T) = R + 2\mu T$, $f(R,T) = \mu R + \mu T$ (here μ is a constant) and $f(R,T) = f_1(R) + f_2(R)f_3(T)$ [3]. For the solutions of modified field equations in this study we proposed new $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model. This model contains first and second models of Harko et al. [3]. For $\alpha_1 = 1$, $\alpha_2 = 2\mu$ and $f_3(T) = T$, we obtain first model of Harko et al. i.e. $f(R,T) = R + 2\mu T$ [3]. If we take $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$, we get second model of Harko et al. i.e. $f(R,T) = \mu R + \mu T$ [3]. Also, for $\alpha_1 = 1$ and $f_3(T) = 0$, our modified $f(R,T)$ gravitation theory transforms to general relativity. In this study, for the solutions of FRW universe model we have used time periodic varying deceleration parameter given by Shen and Zhao [19]. Using this parameter, we have obtained following results.

From equations (15) and (16) it is easily seen that cosmic pressure and energy density are the function of cosmic time. Also, as $t \rightarrow \infty$ cosmic pressure and energy density decrease with time. For $k = 0$ and $t = \frac{\pi}{2n}$, we obtain dark energy ($p = -\rho$) solution in $f(R,T)$ gravity. From equation (17), we found $\lambda = 0$. Kiran and Reddy [30] studied Bianchi III universe with string cosmological model in $f(R,T)$ gravity, and they found zero string tension density. Naidu et al. [31] investigated Bianchi V space-time with string cosmological model. Also they found $\lambda = 0$ in $f(R,T)$ gravity. However, Çağlar and Aygün have obtained zero string tension density for FRW universe in Lyra [32], Creation field [33], Brans-Dicke [34] and self-creation [35] theories. Also, Sen and Aygün have investigated FRW universe model in self creation cosmology with cloud of string with perfect fluid attached strange quark matter and obtained zero string tension density in their study [36]. Our solution agrees with [30 – 36]. For $\lambda = 0$, string cloud with perfect fluid matter distribution turns out to be perfect fluid matter distribution.

the

140 In $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model if we take $\alpha_1 = 1$, $\alpha_2 = 2\mu$ and $f_3(T) = T$, we get the first
 141 model of Harko et al.[3]. Also, from equations (15)- (17) we obtain pressure, energy density
 142 and string tension density as follows
 143

$$p = \frac{n^2(8\pi + 3\mu)\cos(nt)}{2m(2\pi + \mu)(4\pi + \mu)(\cos(2nt) - 1)} - \frac{3n^2}{2m^2(2\pi + \mu)(\cos(2nt) - 1)} + \frac{k\pi}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (18)$$

144

$$\rho = \frac{n^2\mu\cos(nt)}{2m(2\pi + \mu)(4\pi + \mu)(\cos(2nt) - 1)} + \frac{3n^2}{2m^2(2\pi + \mu)(\cos(2nt) - 1)} - \frac{k(3\pi + \mu)}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (19)$$

145

$$\lambda = 0 \quad (20)$$

the

146 Using $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$ in $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we have the second model
 147 of Harko et al. [3]. Using equations (15)- (17) we get pressure, energy density and string tension
 148 density as follows
 149

$$p = \frac{n^2\mu(16\pi + 3\mu)\cos(nt)}{m(4\pi + \mu)(8\pi + \mu)(\cos(2nt) - 1)} - \frac{3n^2\mu}{m^2(4\pi + \mu)(\cos(2nt) - 1)} + \frac{4k\mu\pi}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (21)$$

150

$$\rho = \frac{n^2\mu^2\cos(nt)}{m(4\pi + \mu)(8\pi + \mu)(\cos(2nt) - 1)} + \frac{3n^2\mu}{m^2(4\pi + \mu)(\cos(2nt) - 1)} - \frac{2k\mu(6\pi + \mu)}{c_1^2(2\pi + \mu)(4\pi + \mu)\tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} \quad (22)$$

151

$$\lambda = 0 \quad (23)$$

the

152 If we take $\alpha_1 = 1$ and $f_3(T) = 0$, in $f(R, T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we obtain general
 153 relativity solutions for pressure, energy density and string tension density as follows
 154

$$p = \frac{k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} + \frac{n^2}{16\pi m^2} + \frac{n^2(2m+3)\tan\left(\frac{nt}{2}\right)^2}{32\pi m^2} - \frac{n^2(2m-3)}{32\pi m^2 \tan\left(\frac{nt}{2}\right)^2} \quad (24)$$

155

$$\rho = -\frac{3k}{8\pi c_1 \tan\left(\frac{nt}{2}\right)^{\frac{2}{m}}} - \frac{3n^2}{16\pi m^2} - \frac{3n^2 \tan\left(\frac{nt}{2}\right)^2}{32m^2\pi} - \frac{3n^2}{32m^2\pi \tan\left(\frac{nt}{2}\right)^2} \quad (25)$$

156 and

$$\lambda = 0 \quad (26)$$

157 From equations (18)-(26) we found that cosmic pressure and energy density in $f(R,T)$
 158 gravitation models and general relativity are the function of cosmic time t , and if we take the
 159 limit $t \rightarrow \infty$, cosmic pressure and energy density decrease with cosmic time t . Again $k = 0$
 160 and $t = \frac{\pi}{2n}$, we obtain dark energy ($p = -\rho$) solution in $f(R,T)$ gravity and general relativity
 161 theories. Also, we found that $\lambda = 0$ in all cosmological models. Obtained solutions show that
 162 the source of accelerating of the universe is like from perfect fluid distributions not strings.

4. CONCLUSION

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