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Authors contributions

This work is the fruit of a working team. Author AJF and SCK designed the study, wrote the protocol, and wrote the first draft of the manuscript. Author HF and MT managed the analyses of the study and performed simulations. Author MPTD managed the literature searches. Author LCF helped in writing of the final draft. All authors read and approved the final manuscript.

Modified Lee-Low-Pines Polaron in Spherical Quantum Dot in an Electric Field

Part1: Strong Coupling

Abstract

In this paper, we investigated the influence of electric field on the ground state energy of polaron in spherical semiconductor quantum dot (QD) using modified Lee Low Pines (LLP) method. The numerical results show the increase of the ground state energy with the increase of the electric field and the confinement lengths. The modulation of the electric and the confinement lengths lead to the control of the decoherence of the system.

Keywords: *Electric field, modified LLP, Polaron Energy, Quantum Dot*

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1- Introduction

Due to the recent progress achieved in nanotechnology, it has become possible to fabricate low dimensional semiconductor structures. Special interest is being devoted to the quasi zero dimensional structures, usually referred to as quantum dots (QD) [1-9]. In such nanometer QD's, some novel physical phenomena and potential electronic device applications have generated a great deal of interest. They may give theoretical physicists great challenges to develop the theory based on the quantum mechanical regime. Recently, much effort [10-12] has been focused on exploring the polaron effect of QD's. Roussignol et *al*. [10] have shown experimentally and explained theoretically that the phonon broadening is very significant in very small semiconductor QD's. Some have also observed [11-12] that the polaron effect is more important if the dot sizes are reduced to a few nanometers. More recently, the related problem of an optical polaron bound to a Coulomb impurity in a QD has also been considered in the presence of a magnetic field.

The theoretical investigation of the polaron properties was performed by using the standard perturbation techniques [13], by the variational Lee-Low-Pines method [14-15] and by modified LLP approach [16-17], by Feynman path integral method [18], by numerical diagonalization [19], or by Green function methods [20]. The experimental data [21] show, in particular, a large splitting width near the one-phonon and two-phonon resonance in a InAs/GaAs QD. This was accounted for by the theoretical model via a numerical diagonalization of the Fröhlich interaction [19]. The required value of the Fröhlich constant was much larger (by a factor of two [19]), than measured in bulk. In [18] using the Feynman path integral method, the authors observed that the quadratic dependence of the magnetopolaron energy is modulated by a logarithmic function and strongly depend on the Fröhlich electron–phonon coupling constant structure and cyclotron radius. Furthermore the effective electron-phonon coupling is enhanced by high confinement or high magnetic field. In [21] the polaron energy in QD was calculated using a LLP approach and it was found that the polaronic effect is more pronounced for small dot sizes. In [16], using a modified LLP approach, the number of phonons around the electron, and the size of the polaron for the ground state, and for the first two excited states is calculated via the adiabatic approach.

It is important to note that, all works done are not using the modified LLP method to solve the problem of polaron subjected to an electric field. It is also instructive from the works presented above, to recall that polarons are often classified according to the Fröhlich electron-phonon coupling constant. Some authors [18] investigated simultaneously all couplings types characterizing Fröhlich electron-phonon coupling by using the Feynman path integral method. The main feature of the method presented here is the modification of the LLP approach [16] by introducing a new parameter $b_1\ and\ b_2$ in the traditional LLP approach, which permits us to obtain an "all coupling" polaron theory. Here the coupling is weak if $b_1 = b_2 = 1$, strong coupling if $b_1 = b_2 = 0$ and intermediate between these ranges.

In this paper, we study the influence of the electric field on the polaron ground state energy, using the modified LLP method. This paper has the following structure: In section 2, we describe the Hamiltonian of the system while in section 3 the modified LLP method is presented and analytical results of the ground state energy, polaron effective mass are obtained. In section 4, we present results and discussions and finally we end with section 5 where concluding remarks are presented.

2- Hamiltonian of system

The electron under consideration is moving in a polar crystal with three dimensional anisotropic harmonic potential, and interacting with the bulk LO phonons, under the influence of an electric field along the $\rho-$ direction. The Hamiltonian of the electron-phonon interaction system can be written as

$$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{ph} + \mathcal{H}_{e-ph} \qquad (2.1)$$

where $\mathcal{H}_e$ represents the electronic Hamiltonian and is given by

$$\mathcal{H}_e = \frac{p^2}{2m} + \frac{1}{2}m\omega_1^2\rho^2 + \frac{1}{2}m\omega_2^2 z^2 - e^*\mathcal{F}\rho \qquad (2.2)$$

where $\vec{p}$ is the momentum, ω_1 and ω_2 measure the confinement in the $\rho-$ direction and $z-$ direction respectively.

$\mathcal{H}_{ph}$ is the phonon Hamiltonian defined as

$$\mathcal{H}_{ph} = \sum_Q a_Q^\dagger a_Q \qquad (2.3)$$

where $a_Q^\dagger(a_Q)$ are the creation(annihilation) operators for LO phonons of wave vector $\vec{Q} = (\vec{q}, q_z)$, $\mathcal{H}_{e-ph}$ represents the electron-phonon Hamiltonian and is given by

$$\mathcal{H}_{e-ph} = \sum_Q V_Q\left[a_Q e^{i\vec{Q}.\vec{r}} + a_Q^\dagger e^{-i\vec{Q}.\vec{r}}\right] \qquad (2.4)$$

where $V_Q \; and \, \alpha$ are the amplitude of the electron-phonon interaction and the coupling constant respectively given by

$$V_Q = i\left(\frac{\hbar\omega_{LO}}{Q}\right)\left(\frac{\hbar}{2m\omega_{LO}}\right)^{1/4}\left(\frac{4\pi\alpha}{V}\right)^{1/2}, \quad (2.5)$$

$$\alpha = \left(\frac{e^2}{2\hbar\omega_{LO}}\right)\left(\frac{2m\omega_{LO}}{\hbar}\right)^{1/2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right), \quad (2.6)$$

3- Modified LLP method and analytical results of ground state energy and polaron mass

Adopting the mixed-coupling approximation of [23], we propose a modification to the LLP-transformation by inserting two variational parameters $b_1 \; and \, b_2$.

Our new unitary transformation is now

$$\mathcal{U}_1 = \exp\left[i\left[(\vec{P}_\rho - \vec{\mathcal{P}}_\rho)\vec{\rho} b_1 + (P_z - \mathcal{P}_\rho) z b_2\right]\right] \quad (3.1)$$

where

$$\vec{P} = \vec{p} + \sum_Q a_Q^\dagger a_Q \quad (3.2)$$

is the total momentum of the polaron and

$$\vec{\mathcal{P}} = \sum_Q \vec{Q} a_Q^\dagger a_Q \quad (3.3)$$

is the momentum of the phonon.

The two new variational parameters are supposed to trace the problem from the strong coupling case to the weak coupling limit and to interpolate between all possible geometries.

The second transformation has the form [23]

$$\mathcal{U}_2 = \sum_Q u_Q (a_Q^\dagger - a_Q) \quad (3.4)$$

where u_Q is a variational function. This transformation is called the displaced oscillator which is related to the phonon operators via the phonon wave vector and the relation

$$\varphi_{ph} = U_2 \left| 0_{ph} \right\rangle \tag{3.5}$$

where $\left| 0_{ph} \right\rangle$ is the phonon vacuum state since at low temperature there will be no effective phonons.

Applying the transformation in (3.1) on the Hamiltonian (2.1), we obtained

$$\begin{aligned}
\mathcal{H}^{(1)} &= \mathcal{U}_1^{-1} \mathcal{H} \, \mathcal{U}_1 \\
&= \frac{p^2}{2m} + \frac{1}{2} m\omega_1^2 \rho^2 + \frac{1}{2} m\omega_2^2 z^2 - \boldsymbol{e}^* \boldsymbol{\mathcal{F}} \rho + b_1^2 (P_\rho - \boldsymbol{\mathcal{P}}_\rho)^2 + \\
&+ 2b_1 p_\rho (P_\rho - \boldsymbol{\mathcal{P}}) + b_2^2 (P_z - \boldsymbol{\mathcal{P}}_z)^2 + 2b_2 p_z (P_z - \boldsymbol{\mathcal{P}}_z) + \sum_Q a_Q^\dagger a_Q + \\
&+ \sum_Q V_Q \left[a_Q e^{-i(b_1 \vec{q}.\vec{\rho} + b_2 q_z z)} e^{i\vec{Q}.\vec{r}} + a_Q^\dagger e^{i(b_1 \vec{q}.\vec{\rho} + b_2 q_z z)} e^{-i\vec{Q}.\vec{r}} \right]
\end{aligned} \tag{3.6}$$

Applying the transformation (3.4) on (3.6), we obtained

$$\begin{aligned}
\mathcal{H}^{(2)} &= \mathcal{U}_2^{-1} \mathcal{H}^{(1)} \, \mathcal{U}_2 \\
&= \frac{p^2}{2m} + \frac{1}{2} m\omega_1^2 \rho^2 + \frac{1}{2} m\omega_2^2 z^2 - \boldsymbol{e}^* \boldsymbol{\mathcal{F}} \rho + b_1^2 (P_\rho - \boldsymbol{\mathcal{P}}_\rho)^2 + \\
&+ b_1^2 (\boldsymbol{\mathcal{P}}_\rho^{(0)})^2 + 2b_1 p_\rho (P_\rho - \boldsymbol{\mathcal{P}}_\rho + \boldsymbol{\mathcal{P}}_\rho^{(1)} - \boldsymbol{\mathcal{P}}_\rho^{(0)}) + b_1^2 (\boldsymbol{\mathcal{P}}_\rho^{(1)} - 2\boldsymbol{\mathcal{P}}_\rho) \boldsymbol{\mathcal{P}}_\rho^{(1)} + \\
&+ 2b_1^2 (P_\rho - \boldsymbol{\mathcal{P}}_\rho^{(0)}) \boldsymbol{\mathcal{P}}_\rho^{(1)} + 2b_1^2 \boldsymbol{\mathcal{P}}_\rho^{(0)} \boldsymbol{\mathcal{P}}_\rho - 2b_1^2 P_\rho \boldsymbol{\mathcal{P}}_\rho^{(0)} + b_2^2 (P_z - \boldsymbol{\mathcal{P}}_z)^2 + b_2^2 (\boldsymbol{\mathcal{P}}_z^{(0)})^2 \\
&+ 2b_2 p_z (P_z - \boldsymbol{\mathcal{P}}_z + \boldsymbol{\mathcal{P}}_z^{(1)} - \boldsymbol{\mathcal{P}}_z^{(0)}) + b_2^2 (\boldsymbol{\mathcal{P}}_z^{(1)} - 2\boldsymbol{\mathcal{P}}) \boldsymbol{\mathcal{P}}_z^{(1)} + 2b_2^2 (P_z - \boldsymbol{\mathcal{P}}_z^{(0)}) \boldsymbol{\mathcal{P}}_z^{(1)} + \\
&+ 2b_2^2 \boldsymbol{\mathcal{P}}_z^{(0)} \boldsymbol{\mathcal{P}}_z - 2b_2^2 P_z \boldsymbol{\mathcal{P}}_z^{(0)} + \sum_Q u_Q^2 + \sum_Q a_Q^\dagger a_Q + \sum_Q u_Q (a_Q + a_Q^\dagger) + \\
&+ \sum_Q V_Q e^{-i(b_1 \vec{q}.\vec{\rho} + b_2 q_z z)} e^{i\vec{Q}.\vec{r}} (a_Q - u_Q) + \sum_Q V_Q e^{i(b_1 \vec{q}.\vec{\rho} + b_2 q_z z)} e^{-i\vec{Q}.\vec{r}} (a_Q^\dagger - u_Q)
\end{aligned}$$

In Fröhlich units i.e. $2m = \omega_{LO} = \hbar = 1$, this expression takes the form

$$
\begin{aligned}
\mathcal{H}^{(2)} &= \mathcal{U}_2^{-1}\mathcal{H}^{(1)}\mathcal{U}_2 \\
&= p^2 + \frac{1}{4}\omega_1^2\rho^2 + \frac{1}{4}\omega_2^2 z^2 - \boldsymbol{e}^*\mathcal{F}\rho + b_1^2(P_\rho - \mathcal{P}_\rho)^2 + \\
&+ b_1^2(\mathcal{P}_\rho^{(0)})^2 + 2b_1 p_\rho(P_\rho - \mathcal{P}_\rho + \mathcal{P}_\rho^{(1)} - \mathcal{P}_\rho^{(0)}) + b_1^2(\mathcal{P}_\rho^{(1)} - 2\mathcal{P}_\rho)\mathcal{P}_\rho^{(1)} + \\
&+ 2b_1^2(P_\rho - \mathcal{P}_\rho^{(0)})\mathcal{P}_\rho^{(1)} + 2b_1^2\mathcal{P}_\rho^{(0)}\mathcal{P}_\rho - 2b_1^2 P_\rho\mathcal{P}_\rho^{(0)} + b_2^2(P_z - \mathcal{P}_z)^2 + b_2^2(\mathcal{P}_z^{(0)})^2 \\
&+ 2b_2 p_z(P_z - \mathcal{P}_z + \mathcal{P}_z^{(1)} - \mathcal{P}_z^{(0)}) + b_2^2(\mathcal{P}_z^{(1)} - 2\mathcal{P})\mathcal{P}_z^{(1)} + 2b_2^2(P_z - \mathcal{P}_z^{(0)})\mathcal{P}_z^{(1)} + \\
&+ 2b_2^2\mathcal{P}_z^{(0)}\mathcal{P}_z - 2b_2^2 P_z\mathcal{P}_z^{(0)} + \sum_Q u_Q^2 + \sum_Q a_Q^\dagger a_Q + \sum_Q u_Q(a_Q + a_Q^\dagger) + \\
&+ \sum_Q V_Q e^{-i(b_1\vec{q}.\vec{\rho} + b_2 q_z z)} e^{i\vec{Q}.\vec{r}}(a_Q - u_Q) + \sum_Q V_Q e^{i(b_1\vec{q}.\vec{\rho} + b_2 q_z z)} e^{-i\vec{Q}.\vec{r}}(a_Q^\dagger - u_Q)
\end{aligned}
\tag{3.7}
$$

where

$$
\vec{\mathcal{P}}^{(1)} = \sum_Q \vec{Q} u_Q (a_Q + a_Q^\dagger) \tag{3.8}
$$

and

$$
\vec{\mathcal{P}}^{(0)} = \sum_Q \vec{Q} u_Q^2 \tag{3.9}
$$

Applying (3.5) on (3.7), we obtained the ground state energy

$$
\begin{aligned}
\mathcal{E}_g &= \langle 0_e | p^2 + \frac{1}{4}\omega_1^2\rho^2 + \frac{1}{4}\omega_2^2 z^2 - \boldsymbol{e}^*\mathcal{F}\rho | 0_e\rangle + b_1^2 P_\rho^2 - 2b_1^2 P_\rho \mathcal{P}_\rho^{(0)} + b_1^2(\mathcal{P}_\rho^{(0)})^2 + \\
&+ \sum_Q u_Q^2(1 + b_1^2 q^2 + b_2^2 q_z^2) + \langle 0_e | \langle 0_{ph} | 2b_1 p_\rho(\vec{P}_\rho - \vec{\mathcal{P}}_\rho + \vec{\mathcal{P}}_\rho^{(1)} - \vec{\mathcal{P}}_\rho^{(0)}) | 0_{ph}\rangle | 0_e\rangle + \\
&+ \sum_Q V_Q u_Q \langle 0_e | (\exp -i(b_1\vec{q}.\vec{\rho} + b_2 q_z z)\ \exp(i\vec{Q}.\vec{r}) - \exp\ i(b_1\vec{q}.\vec{\rho} + b_2 q_z z)\ \exp(-i\vec{Q}.\vec{r})) | 0_e\rangle + \\
&+ b_2^2 P_z^2 - 2b_2^2 P_z \mathcal{P}_z^{(0)} + b_2^2(\mathcal{P}_z^{(0)})^2 + \langle 0_e | \langle 0_{ph} | 2b_2 p_z(\vec{P}_z - \vec{\mathcal{P}}_z + \vec{\mathcal{P}}_z^{(1)} - \vec{\mathcal{P}}_z^{(0)}) | 0_{ph}\rangle | 0_e\rangle
\end{aligned}
\tag{3.10}
$$

To evaluate this expression, we express the coordinates and momenta of the electron in terms of its creation(annihilation) operators $\sigma^\dagger(\sigma)$ as

$$
\begin{aligned}
p_\mu &= \sqrt{\lambda_1}(\sigma_\mu + \sigma_\mu^\dagger) \\
x_\mu &= i\sqrt{\lambda_1}(\sigma_\mu - \sigma_\mu^\dagger) \\
p_z &= \sqrt{\lambda_2}(\sigma_z + \sigma_z^\dagger) \\
z &= -i\sqrt{\lambda_2}(\sigma_z - \sigma_z^\dagger)
\end{aligned}
$$

where the index μ refers to the $x\ and\ y$ coordinates, and $\lambda_1\ and\ \lambda_2$ are another variational parameters. Performing the required calculations we get for the ground state energy

$$\mathcal{E}_g = \frac{\lambda_1}{2} + \frac{\lambda_2}{4} + \frac{\omega_1^2}{2\lambda_1} + \frac{\omega_2^2}{4\lambda_2} - 2\frac{e^*\mathcal{F}}{\sqrt{\lambda_1}} + b_1^2 P_\rho^2 - 2b_1^2 P_\rho \mathcal{P}_\rho^{(0)} + b_1^2(\mathcal{P}_\rho^{(0)})^2 + \\ + \sum_Q u_Q^2(1 + b_1^2 q^2 + b_2^2 q_z^2) + b_2^2 P_z^2 - 2b_2^2 P_z \mathcal{P}_z^{(0)} + b_2^2(\mathcal{P}_z^{(0)})^2 - 2\sum_Q V_Q u_Q S_Q \tag{3.11}$$

with

$$S_Q = \langle 0_e | (\exp \pm i(b_1 \vec{q}.\vec{\rho} + b_2 q_z z) \exp(\pm i\vec{Q}.\vec{r})) | 0_e \rangle \tag{3.12}$$

this expression can be written as

$$S_Q = \exp\left[-(1-b_1)^2 \frac{q^2}{2\lambda_1}\right] \exp\left[-(1-b_2)^2 \frac{q_z^2}{2\lambda_2}\right] \tag{3.13}$$

Minimizing (3.11) with respect to the variational function u_Q we obtain

$$\left[1 + b_1^2 q^2 + b_2^2 q_z^2 + 2b_1^2 q(\mathcal{P}_\rho^{(0)} - P_\rho) + 2b_2^2 q_z(\mathcal{P}_z^{(0)} - P_z)\right] u_Q = V_Q S_Q \tag{3.14}$$

Solving (3.14) with respect to u_Q, with the assumption that $\vec{\mathcal{P}}^{(0)}$ differ from the total momentum by a scalar factor η $\vec{\mathcal{P}}^{(0)} = \eta \vec{P}$, we get

$$u_Q = \frac{V_Q S_Q}{1 + b_1^2 q^2 + b_2^2 q_z^2 - 2b_1^2 q P_\rho(1-\eta) - 2b_2^2 q_z P_z(1-\eta)} \tag{3.15}$$

Substituting (3.15) into (3.11) we obtain

$$\mathcal{E}_g = \frac{\lambda_1}{2} + \frac{\lambda_2}{2} + \frac{\omega_1^2}{2\lambda_1} + \frac{\omega_2^2}{4\lambda_2} - \frac{2e^*\mathcal{F}\rho}{\sqrt{\lambda_1}} + b_1^2 {P_\rho}^2(1-\eta)^2 + b_2^2 {P_z}^2(1-\eta)^2 \\ \sum_Q \frac{{V_Q}^2 {S_Q}^2 (1 + b_1^2 q^2 + b_2^2 {q_z}^2)}{\left[1 + b_1^2 q^2 + b_2^2 {q_z}^2 - 2b_1^2 q P_\rho(1-\eta) - 2b_2^2 q_z P_z(1-\eta)\right]^2} \\ -2\sum_Q \frac{{V_Q}^2 {S_Q}^2}{\left[1 + b_1^2 q^2 + b_2^2 {q_z}^2 - 2b_1^2 q P_\rho(1-\eta) - 2b_2^2 q_z P_z(1-\eta)\right]} \tag{3.16}$$

But $\mathcal{E}_g(\vec{P})$ may be well represented by the first two terms of a power series expansion in P^2 as [23]

$$\mathcal{E}_g(\vec{P}) = \mathcal{E}_g(0) + \beta \frac{P^2}{2} + 0(P^4) + \ldots \qquad (3.17)$$

with β^{-1} gives the effective mass of the polaron.

Comparing (3.16) and (3.17) we obtain for the ground state energy

$$\mathcal{E}_g = \frac{\lambda_1}{2} + \frac{\lambda_2}{4} + \frac{\omega_1^2}{2\lambda_1} + \frac{\omega_2^2}{4\lambda_2} - \frac{2e^*\mathcal{F}}{\sqrt{\lambda_1}} - \sum_Q \frac{V_Q^2 S_Q^2}{\left[1 + b_1^2 q^2 + b_2^2 q_z^2\right]} \qquad (3.18)$$

and the mass of polaron is given as

$$m_P = \frac{1}{2\left[b_1^2(1-\eta)^2\right]} + \frac{1}{2\left[b_2^2(1-\eta)^2\right]} \qquad (3.19)$$

Substituting (3.13) in the ground state energy (3.18), we obtained

$$\mathcal{E}_g = \frac{\lambda_1}{2} + \frac{\lambda_2}{4} + \frac{\omega_1^2}{2\lambda_1} + \frac{\omega_2^2}{4\lambda_2} - \frac{2e^*\mathcal{F}}{\sqrt{\lambda_1}} - \sum_Q \frac{V_Q^2 \exp\left[-(1-b_1)^2 \frac{q^2}{\lambda_1}\right] \exp\left[-(1-b_2)^2 \frac{q_z^2}{\lambda_2}\right]}{\left[1 + b_1^2 q^2 + b_2^2 q_z^2\right]} \qquad (3.20)$$

re-arranging this expression, we finally obtained the ground state energy

$$\mathcal{E}_g = \frac{\lambda_1}{2} + \frac{\lambda_2}{4} + \frac{1}{2\lambda_1 l_1^4} + \frac{1}{4\lambda_2 l_2^4} - \frac{2e^*\mathcal{F}}{\sqrt{\lambda_1}} - \sum_Q \frac{V_Q^2 \exp\left[-(1-b_1)^2 \frac{q^2}{\lambda_1}\right] \exp\left[-(1-b_2)^2 \frac{q_z^2}{\lambda_2}\right]}{\left[1 + b_1^2 q^2 + b_2^2 q_z^2\right]} \qquad (3.21)$$

where $l_1^2 = \frac{\hbar}{m\omega_1}$ and $l_2^2 = \frac{\hbar}{m\omega_2}$ are the confinement length in $xy-$plane and z direction respectively

4- Numerical results and discussions

For the numerical results, we consider the strong coupling case, i.e. $b_1 = b_2 \to 0$. In this section, we show the numerical results of the ground state energy $\mathcal{E}_0$ versus the electric field strength $\mathcal{F}$, the electron-phonon coupling strength α, and the confinement lengths l_1 and l_2.

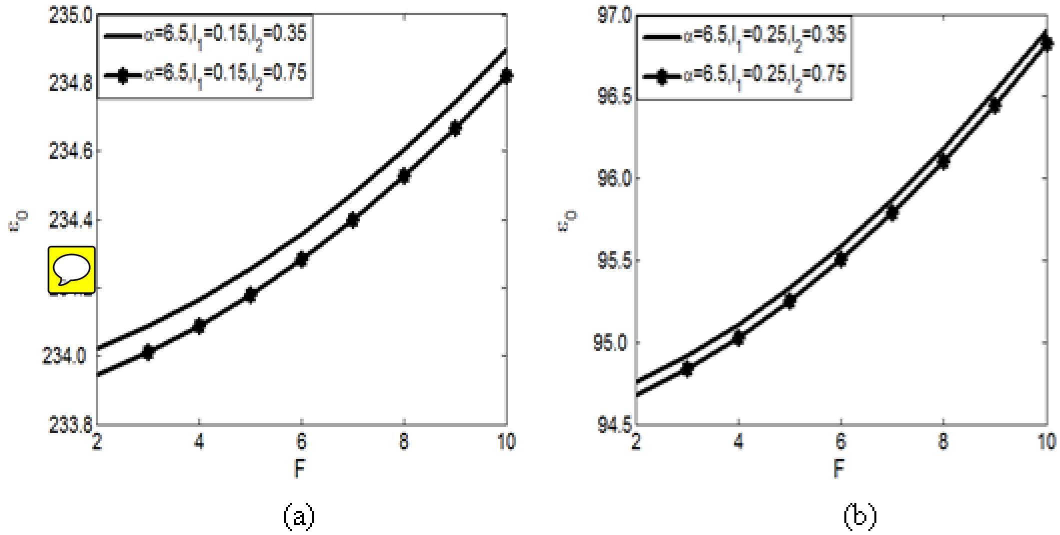

Figure 1: Ground state energy ε_0 as a function of electric field $\mathscr{F}$ with

(a) $\alpha = 6.5, l_1 = 0.15, l_2 = 0.35\, and\, l_2 = 0.75$

(b) $\alpha = 6.5, l_1 = 0.25, l_2 = 0.35\, and\, l_2 = 0.75$

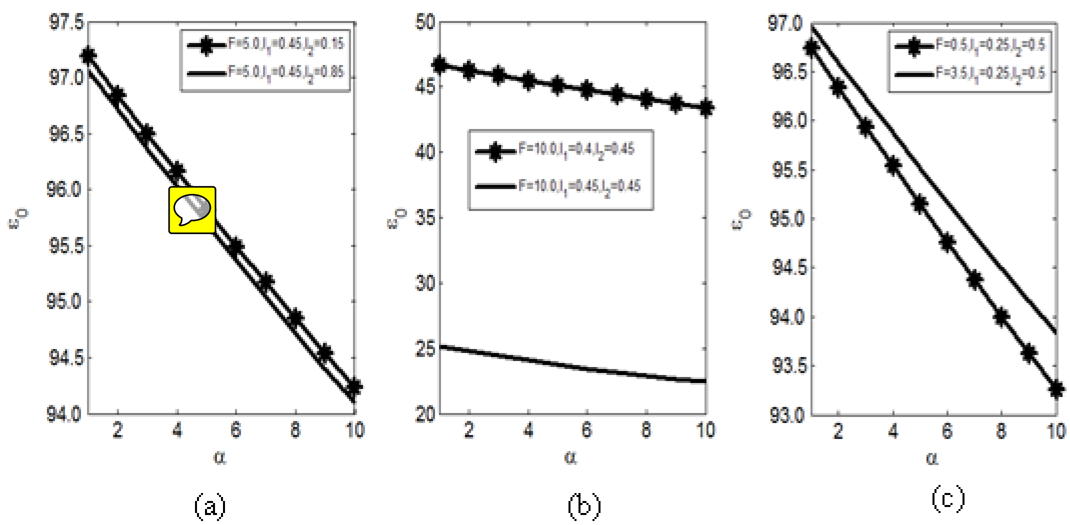

Figure 2: Ground state energy ε_0 as a function of coupling constant α with

(a) $l_1 = 0.25, \mathscr{F} = 5.0, l_2 = 0.25\, and\, l_2 = 0.85$

(b) $l_2 = 0.25,\ \mathscr{F} = 0.0\, l_1 = 0.3\, and\, l_1 = 0.35$

(c) $l_1 = 0.25, l_2 = 0.5\ \ \mathscr{F} = 0.5\, and\, \mathscr{F} = 3.5$

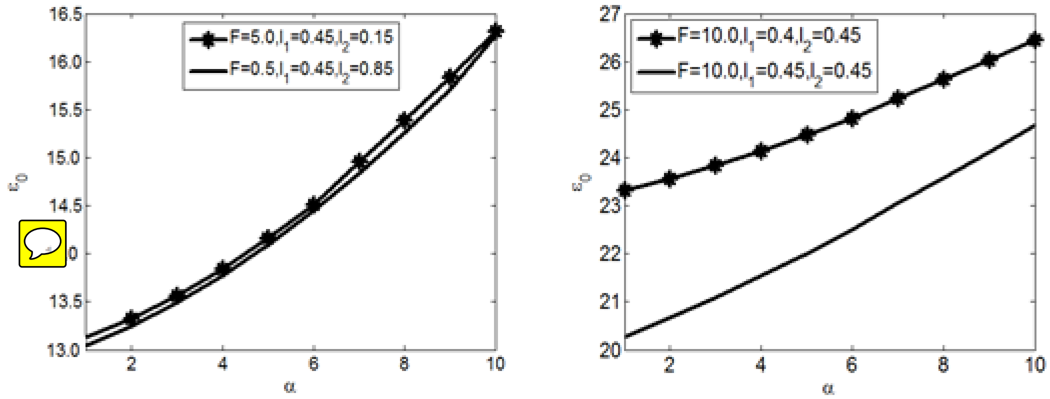

Figure 3: Ground state energy ε_0 as a function of coupling constant α with
(a) $\mathcal{F} = 5.0, l_1 = 0.45, l_2 = 0.15 \, and \, l_2 = 0.85$
(b) $\mathcal{F} = 10.0, l_2 = 0.45 \; and \; l_1 = 0.4 \, and \, l_1 = 0.45$

In figure 1, we have plotted the ground state energy ε_0 of polaron as a function of electric field $\mathcal{F}$ for $\alpha = 6.5, l_1 = 0.15, l_2 = 0.35 \, and \, l_2 = 0.75$ (figure (1a)) and $\alpha = 6.5, l_1 = 0.25, l_2 = 0.35 \, and \, l_2 = 0.75$ (figure (1b)). The ground state energy is an increasing function of electric field. This is because the electric field leads to the electron energy increment and makes the electrons interact with more phonons. In this way, the states' energies are increased. From another point of view, since the presence of the electric field is equivalent to introducing another new confinement to the electron, which leads to greater electron wave-function overlapping with each other, the electron-phonon interaction will be enhanced, resulting in the increase of states' energies with the increase of electric field. This indicates a new way to control the QD energies via the electric field. In fact, the electric field plays an important role in low-dimensional materials. For example, both the quantum decoherence process and the electron's probability density are affected by it. Thus, here we find a suitable two-state system by adjusting the electric field, which is crucial in constructing a qubit [24-25].

In Fig. 2 we plot the ground state energy ε_0 which varies with the electron-phonon coupling strength α for

$l_1 = 0.25, \mathcal{F} = 5.0, l_2 = 0.25\, and\, l_2 = 0.85$ (fig. 2a)

$l_2 = 0.25,\ \mathcal{F} = 0.0, l_1 = 0.3\, and\, l_1 = 0.35$ (fig. 2b)

$l_1 = 0.25, l_2 = 0.5, \mathcal{F} = 0.5\, and\, \mathcal{F} = 3.5$ (fig. 2c)

From the three figures we can see that the ground state energy ε_0 is a decreasing function of the electron-phonon coupling strength. From here, we also see that the ground state is an increasing function of the LO confinement length (fig.2a) and the electric field strength (fig.2c); it is a decreasing function of the transverse confinement length (fig. 2b). With the increase of the harmonic potential ($\omega_1\ and\, \omega_2$), the energy of the electron and the interaction between the electron and the phonons, which take phonons as the medium, are enhanced because of the smaller particle motion range. The presence of the parabolic potential is equivalent to introducing another confinement on the electron, which leads to greater electron wave functions overlapping with each other, and the enhancement of the electron-phonon interactions.

All these figures show the decreasing behavior of the ground state energy as a function of electron-phonon coupling constant α.This is because the larger the electron-phonon coupling strength is, the stronger the electron-phonon interaction. This leads to the increment of the electron's energy and makes the electron interact with more phonons. It is known that the electron-phonon interaction strength is different in different crystal materials. Thus the state energies and the transition frequency of the AQDs can be tuned by changing it [24,26].

In Fig. 3 we plot the ground state energy ε_0 varying with the electron-phonon coupling strength α for

$\mathcal{F} = 5.0, l_1 = 0.45, l_2 = 0.15\, and\, l_2 = 0.85$ (Fig. 3a)

$\mathcal{F} = 10.0, l_2 = 0.45\ \ and\ \ l_1 = 0.4\, and\, l_1 = 0.45$ (Fig. 3b)

From here it is obvious that, the ground state energy increases with the electron-phonon coupling constant.

These results are in agreement with the results of Kervan et al. [27], Ren et al.[28], Kandemir [29] and [30] obtained respectively by using variational, Feynman-Haken path-integral, squeezed-state variational and linear combination operator methods. The transverse and longitudinal lengths of the AQD are equal to the transverse and longitudinal confining lengths of the electrons, which show the property of strong confining strength in the transverse and longitudinal directions.

5- Conclusion

In conclusion, with the use of modified LLP method, we have studied the energy levels of strong polaron in spherical quantum dot (QD) as a strong coupling polaron in an anisotropic QD subjected to an electric field. It is found that the ground state energy of the polaron is an increasing function of the electric field; this is because the presence of electric field makes phonons interact more strongly with the electron. It is also seen that, with the good control of the confinement length and the electron coupling constant we can control the decoherence of the system. The enhancement of the coupling strength is very important in the construction of quantum computers since it leads to the conservation of its internal properties such as its superposition states against the influence of its environment, which can induce the construction of coherent states and cause coherence quenching. Part two of this work is dedicated to the weak and intermediate couplings.

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COMPETING INTERESTS
Authors have declared that no competing interests exist.

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