Original Research Article

Controllable rogue waves
in the generalized nonlinear Schrödinger equations

Abstract: We obtain the rogue waves with a controllable center in the generalized nonlinear Schrödinger equation by using a direct method. The position of these solutions can be controlled by varying different center parameters. We study the effects of different parameters on rogue waves and hence find that the nonlinearity parameter is responsible for the width of rogue waves. With the increase of the nonlinearity parameter, the rogue wave becomes wider. What is more, the negative nonlinearity parameter can yield some singular rogue waves.

Keywords: Generalized nonlinear Schrödinger equation; Rogue wave; Singular rogue wave

1. Introduction

Rogue waves, are called freak waves, monster waves, killer waves, giant waves or extreme waves, Rogue waves are spontaneous nonlinear waves with amplitudes significantly larger (two or more times higher) than the surrounding average wave crests [1,2], What is more, they appear from nowhere and disappear without a trace.

It is a very meaningful work to search for rogue waves, which has been found in many different systems and has many important applications in some fields since they can signal fascinating stories [3,4].

In this paper, we study the generalized nonlinear cubic Schrödinger equation

$$\mathrm{i}\frac{\partial \psi}{\partial x}+\mathrm{A}\frac{\partial^2 \psi}{\partial t^2}+\psi\left|\psi\right|^2=0, \tag{1}$$

where x is the propagation distance and t is the transverse variable. When A=0, equation (1) becomes the famous nonlinear cubic Schrödinger equation. Some rogue wave solutions of the

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nonlinear cubic Schrödinger equation have been found by taking limit of Akhmediev breather solutions[5,6] and Darboux transformation.[7,8]. The center of these solutions is located at a fixed point (0, 0) on (x, t) plane. Basing on a simple assumption of the rogue wave, WANG and He found larger universality and applicability of rogue waves with a controllable center[9]. More researches on rogue waves can be founded in Ref. [10-13].

In this paper, our interests focus on two aspects:

(1) We want to determine rogue wave solutions of Eq. (1) with an arbitrary coefficient of nonlinearity;

(2) We want to know the role of the nonlinearity coefficient on the formation of rogue waves.

The organization of this paper is as follows. In Section 2, we obtain some special rogue waves with a controllable center by a direct method. In Section 3, we analyze the different controllability by numerical simulation. Conclusion will be given. In appendix, the computational process for the second-order solution is given.

2. Some special rogue waves

By the similar method in Ref. [9], we assume rogue waves as follows

$$\psi_1=\left(1+\frac{p_1+iq_1}{h_1}\right)e^{ix} \tag{2}$$

with

$$p_1\left(x,t\right)=a_0+a_1x+a_2t, \tag{3}$$

$$q_1\left(x,t\right)=b_0+b_1x+b_2t, \tag{4}$$

$$h_1\left(x,t\right)=c_1\left(x-\alpha\right)^2+c_2\left(t-\beta\right)^2+c_3. \tag{5}$$

Here α_i, b_i (i = 0, 1, 2), c_j (j = 1, 2, 3), α, β are real parameters.

Substituting the function ψ_1 into Eq. (1) and setting different coefficient Lists to be zero. We obtain the following possible system of nonlinear algebraic equations with the aid of Maple.

$$-3b_2{c_1}^2=0,$$

$$-3b_2{c_2}^2=0,$$

$$2a_2b_2c_2+a_1{c_2}^2=0,$$

$$8\alpha a_2 c_1 c_2 - 2b_2 c_1 c_2 - 6\alpha^2 b_2 c_1 c_2 - 16\beta a_1 c_2^2 + 12\beta b_0 c_2^2 - 18\beta^2 b_2 c_2^2 - 6b_2 c_2 c_3 = 0, \tag{6}$$

$$-6a_0 c_1 c_2 + 12\alpha a_1 c_1 c_2 + 12\beta a_2 c_1 c_2 - 54\alpha^2 c_1^2 c_2 - 54\beta^2 c_1 c_2^2 - 18 c_1 c_2 c_3 = 0,$$

$$-6a_1 c_1 c_2 + 36\alpha c_1^2 c_2 = 0,$$

$$4a_1 c_2^2 - 3b_0 c_2^2 + 12\beta b_2 c_2^2 = 0,$$

$$12\alpha a_2 c_1 c_2 + 8b_2 c_1 c_2 + 12\beta a_1 c_2^2 - 72\alpha\beta c_1 c_2^2 = 0,$$

……….

From (6), we can have two classes of solutions:

Case 1.

$$c_3 \neq 0, \quad a_0 = -4c_3, \; a_1 = 0, \quad a_2 = 0, \; b_0 = 8c_3\alpha, \quad b_1 = -8c_3 \quad ,$$

$$b_2 = 0, \quad c_1 = 4c_3, \quad c_2 = \frac{2c_3}{m}. \tag{7}$$

Substituting (7) into Eq. (1), we obtain

$$\psi_1 = \left(1 + \frac{-1 + i(2\alpha - 2x)}{(x-\alpha)^2 + \frac{1}{2m}(\mathrm{t} - \beta)^2 + \frac{1}{4}} \right) e^{ix}. \tag{8}$$

Case 2

$$\text{(i)} \quad a_0 = 0, a_1 = 0, \quad a_2 = 0, \quad \alpha = \frac{-b_0 - \beta b_2}{b_1}, \quad m = \frac{-b_1^{\,2}}{2b_2^2}, \quad b_0 \neq 0, \quad c_1 = -b_1,$$

$$m \neq 0, \quad c_2 = -\frac{b_1}{2m}, \quad c_3 = 0. \tag{9}$$

$$\text{(ii)} \quad a_0 = 0, a_1 = 0, \quad a_2 = 0, \quad b_0 \neq 0, \quad \alpha = -\frac{\beta b_2}{b_1}, \quad m = -\frac{\beta^2}{2\alpha^2},$$

$$c_1 = -b_1, m \neq 0, \quad c_2 = -\frac{b_1}{2m}, \quad c_3 = 0. \tag{10}$$

Substituting (9) and (10) into Eq. (1), we obtain

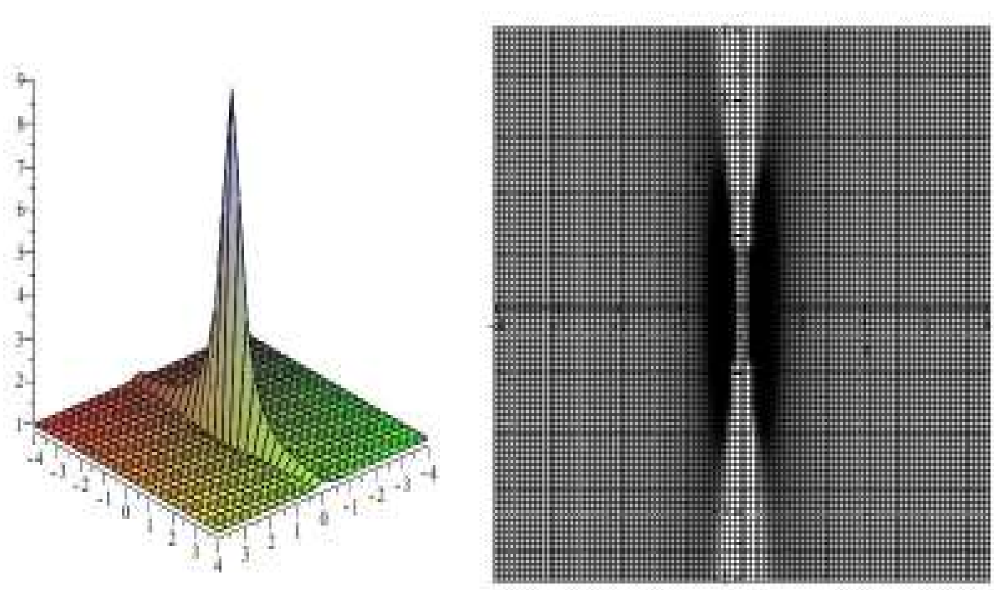
$$\psi_2 = \left(1 + \frac{i(b_0 + b_1 x + b_2 t)}{-b_1 (x - \frac{-b_0 - \beta b_2}{b_1})^2 + \frac{b_2^{\,2}}{b_1}(t - \beta)^2} \right) e^{ix}, \tag{11}$$

$$\psi_3 = \left(1 + \frac{i(b_0 + b_1 x + b_2 t)}{-b_1 (x + \frac{\beta b_2}{b_1})^2 + \frac{b_2{}^2}{b_1}(t - \beta)^2} \right) e^{ix}. \tag{12}$$

3. Some properties of rogue waves

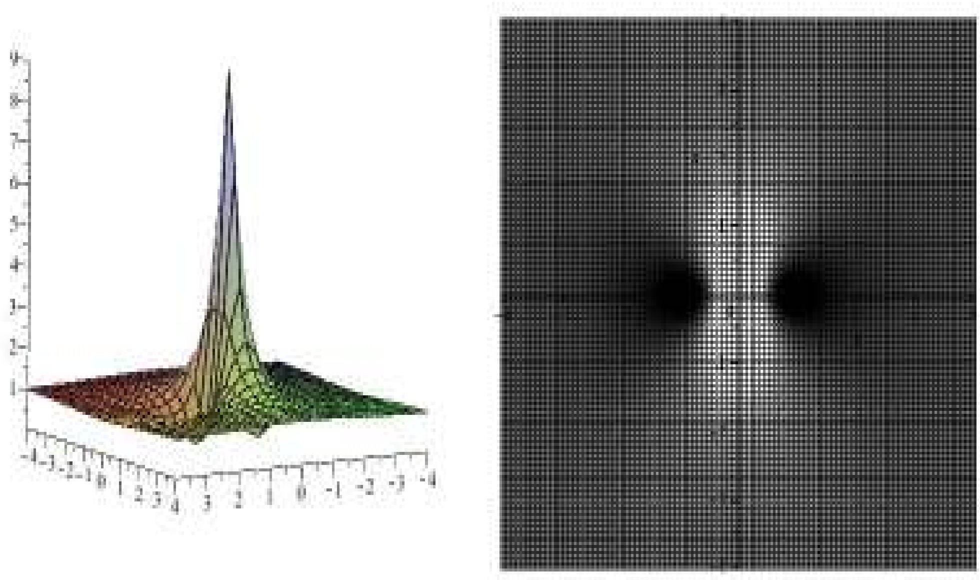
3.1 Width of rogue waves

It is well known that the parameter of the nonlinearity have a major impact on the forms of waves. We herein analyze the impact of rogue waves with the varying parameter of the nonlinearity. Given by different parameters of the nonlinearity, we draw the corresponding rogue waves and density pictures. It is easy to find that: (1) The nonlinearity parameter m has little effect on the height of rogue waves. That is, there is no change the height with different parameters of the nonlinearity. (2) The nonlinearity parameter m has more effect on the width of rogue waves. With the increase of m, the rogue waves becomes wider (in detailed see Figure 1-3).



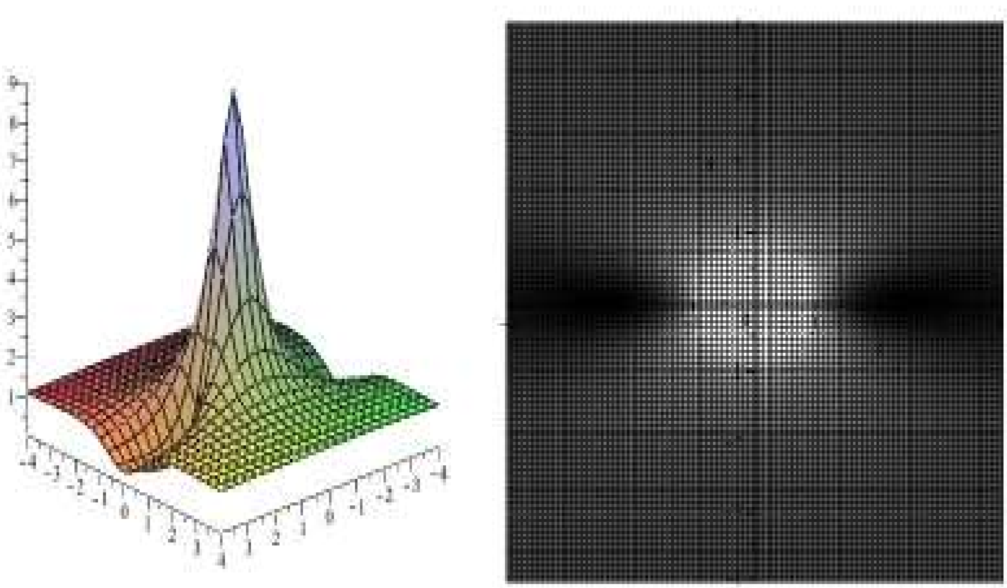
(a) (b)

Fig.1. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m = 0.005, \alpha = 0,$



(a) (b)

Fig.2. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m = 0.5,\ \alpha=0$.

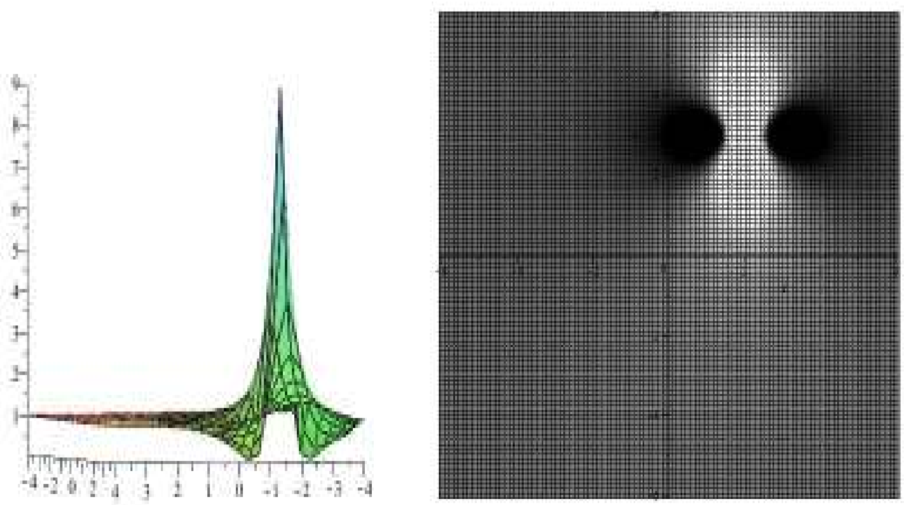


(a) (b)

Fig.3. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m = 4,\ \alpha=0$

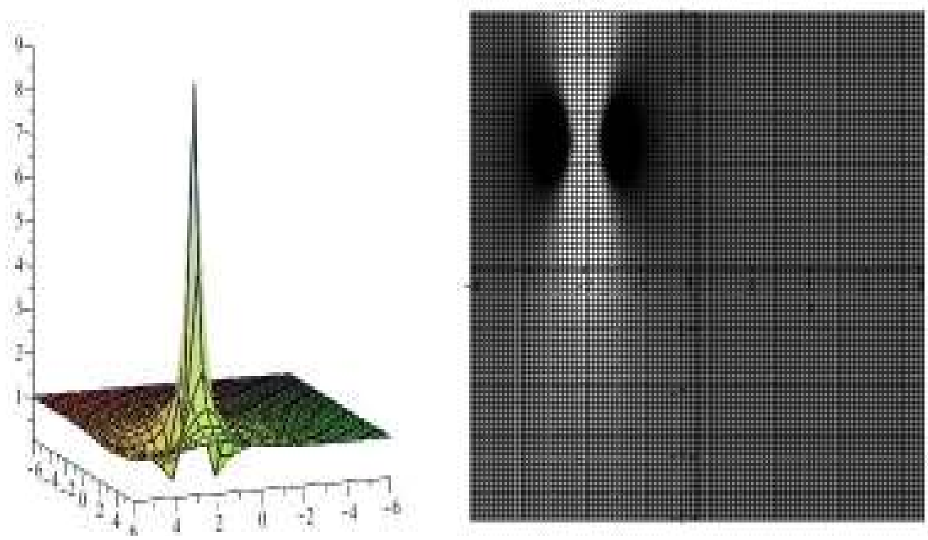
3.2 Rogue Waves with a Controllable Center

From (8), we find rogue waves will move when α, β are given by different values. When we study the situation before taking α definite values for d of m. The following, we consider wave changes under m taking a fixed value 0.5 and α, β varying. We find the following facts: When α, β are given by different values, the central location of the rogue wave are different, namely, the rogue wave center is movable (see Figure 4-7).



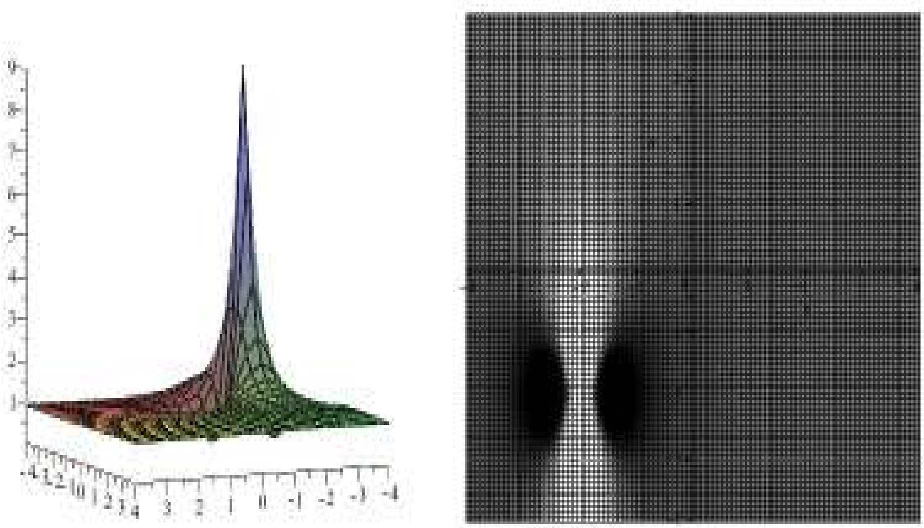
(a) (b)

Fig.4. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m = 0.5$, $\alpha = 3, \beta = 2$.



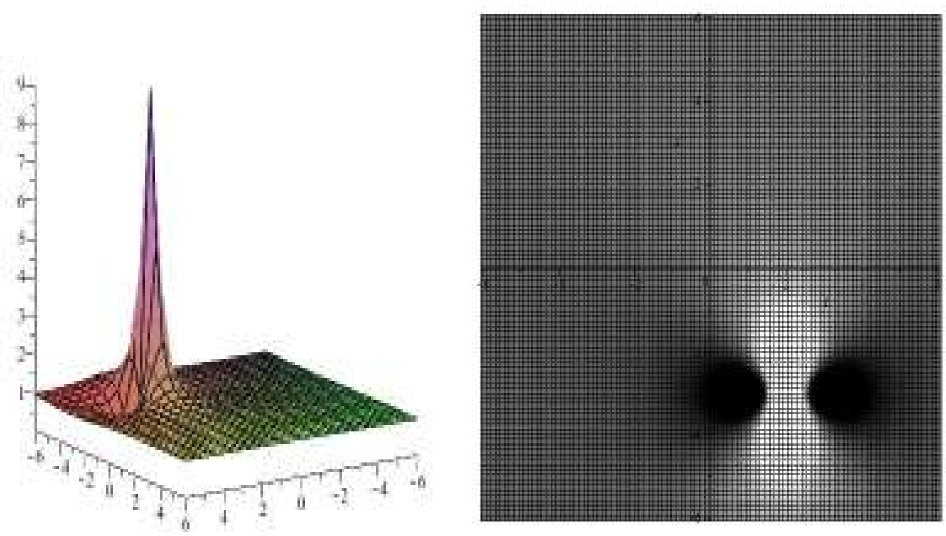
(a) (b)

Fig.5. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m = 0.5$, $\alpha = 2, \beta = -2$



$(a\ )$ $(b\ )b$

Fig.6. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m = 0.5$ $\alpha = -2, \beta = -2$.

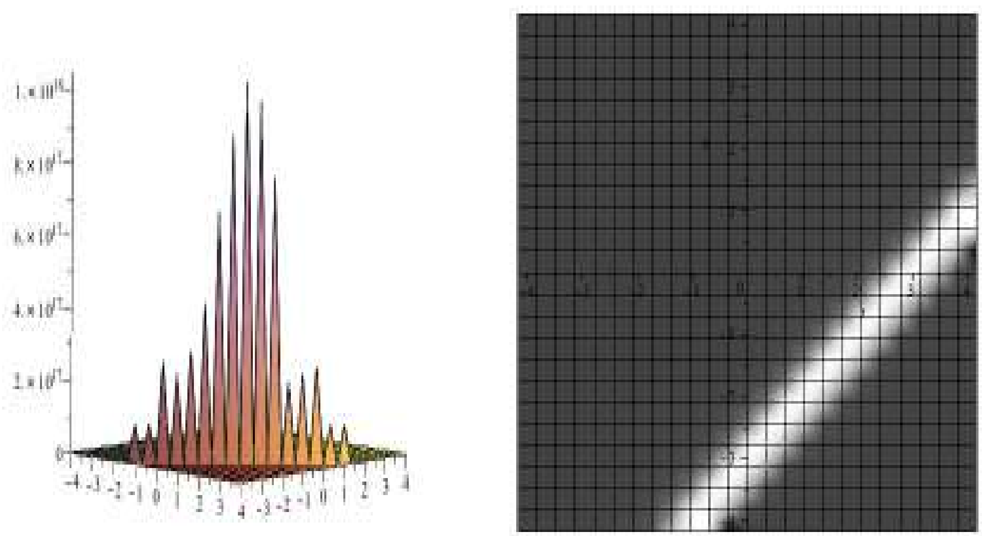


(a) (b)

Fig.7. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $m=4,\ \alpha=0$.

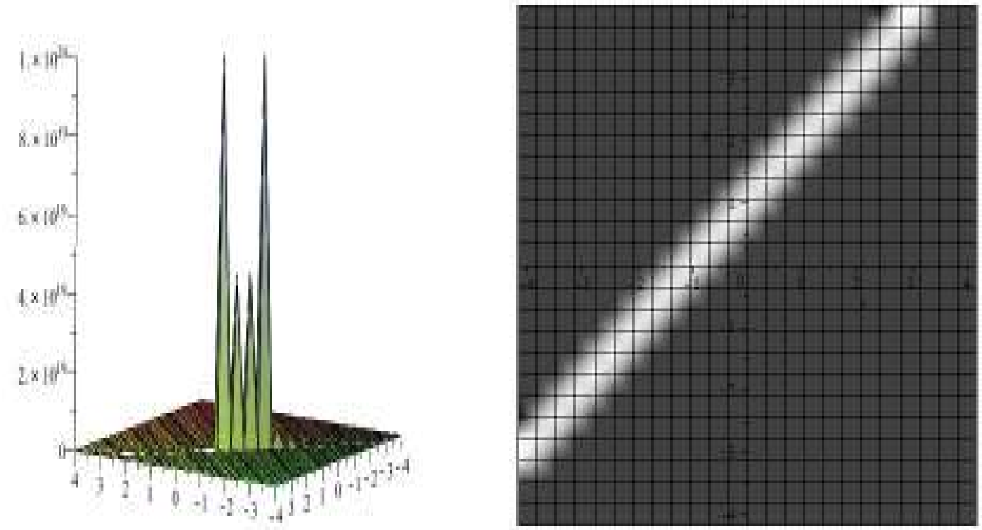
3.3 Singular rogue waves

According to the analysis in Section 2, when m is a negative value, we find that the denominator value of obtained solutions (11)-(12) can be zero under some positions. So we can obtain some singular rogue waves which are shown as follows.



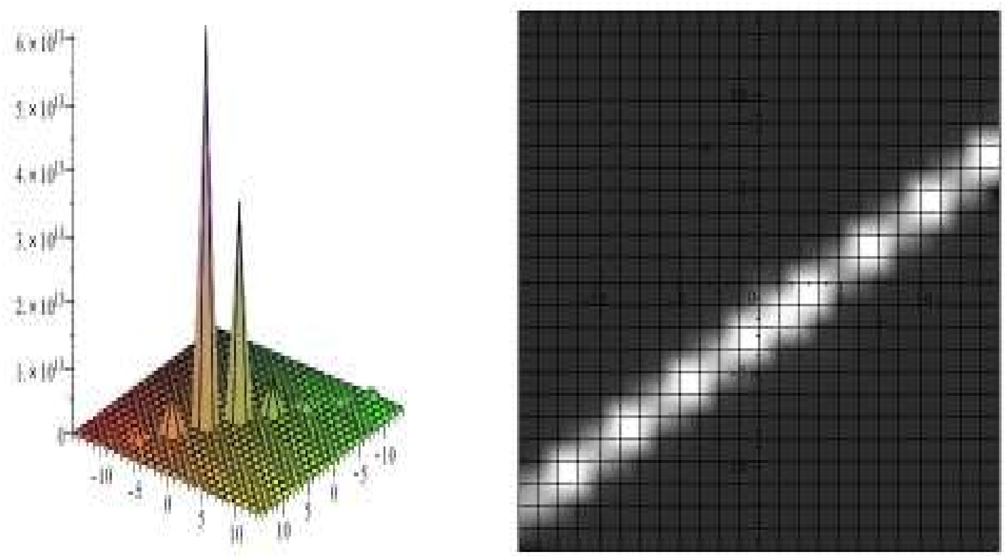
(a) (b)

Fig.8. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $b_0=1,\ b_1=1,\ b_2=1,\ \beta=1$.



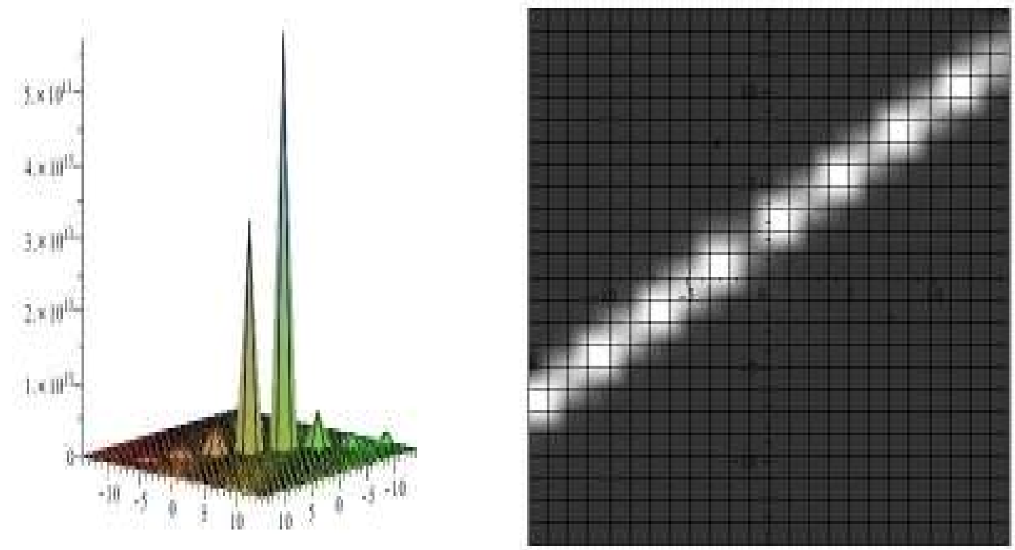
(a) (b)

Fig.9. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $b_0 = 1,\ b_1 = 1,\ b_2 = 1,\ \beta = -1.$



(a) (b)

Fig.10. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $b_0 = -1,\ b_1 = 3,\ b_2 = 2,\ \beta = 2.$



(*a*) (*b*)

Fig.11. Rogue wave propagations (a) and contour plots (b) for the intensity $|\psi_1|^2$ for $b_0 = -1,\ b_1 = 3,\ b_2 = 2,\ \beta = -2.$

References

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