

Original Research Article

**ADAMS TYPE HYBRID BLOCK METHODS ASSOCIATED WITH
CHEBYSHEV POLYNOMIAL FOR THE SOLUTION OF ORDINARY
DIFFERENTIAL EQUATIONS**

ABSTRACT

The construction of newly Hybrid Adams type Block Methods (HATBM) for step length $k = 2, 3$ and 4 were developed for the solution of first order ordinary differential equations. Collocation and interpolation of chebysehev polynomial approximation are adopted to derive some implicit linear multi-step methods at different values of k . Analysis of all the methods shows that they are consistent, zero stable and convergent. All the newly constructed methods were demonstrated with numerical experiment to ascertain their level convergence.

Keywords: (Chebyshev polynomials, Hybrid Adams type, Block Methods, Ordinary differential equation)

1. Introduction

Numerical analysis is the area of mathematical and computer sciences that creates, analyses and implements algorithms for solving numerically the problems of continuous mathematics. Such problems originates from real world applications of algebra, geometry and calculus, these problems occurs throughout the natural sciences, social sciences, engineering, medicine and business studies. Solutions to ordinary differential equations were derived using analytical or even exact methods but many often cannot be solved analytically because most real life problems are modeled into non linear and involve complex shapes and processes. Hence there is a great need to develop an algorithm to cater for this type of differential equations.

Chebyshev polynomials are special group of polynomials whose properties and applications where discovered a century ago by the Russian mathematician Patnuty Lvovich Chebeshev. Their importance for practical computations was rediscovered 60 years ago by Comelious Lanczos the father of Numerical Analysis.

A simple way of constructing Chebeshev polynomials relies on the recurrence relation of the form

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \quad \forall n \geq 2$$

However, $T_0(x)$ and $T_1(x)$ are constant term with respect to the recurrence relation as 1 and x respectively.

Definition 1: Hybrid Linear Multistep method

The k step generated linear multistep method can be written as

$$\sum_{j=0}^k y_{n+j} = h \sum_{j=0}^k \alpha_j \beta_j f_{n+j} + h \beta_v f_{n+v} \quad (1)$$

where $\alpha_k \neq 0, \alpha_0^2 + \beta_0^2 > 0, u \notin \{0, k\}$

Definition 2: Zero Stable

The method in (1) is said to be zero stable if no root of the first characteristic polynomial

$$\rho(r) = \sum_{j=0}^k \alpha_j r^j$$

has modulus greater than one and if every root with modulus one is simple

2. Background of the Study

Numerical methods for solving first order initial value problems often fall into two large categories, linear multistep and Runge-Kutta methods. A further division could be realised by dividing methods into those that are explicit and implicit methods. For example an implicit linear multistep of Adams Moulton is of the form

$$y_{n+k} + y_{n+k-1} = h \sum_{i=0}^k \beta_i f_{n+i} \quad (2)$$

Most recent researchers have worked in this area among them are ([1-5]) to mention a few, in their propose methods the approximate solution ranges from power series, exponential functions, Chebyshev polynomials. The aim of this research work is to use the Chebyshev polynomials as our trial basis to develop some Adams type hybrid block methods at various values of step length h .

Theorem 1: Well-Posed Condition

Suppose that f and f_y , its first derivative with respect to y , are continuous for $x \in [a, b]$. Then the initial value problem $y' = f(x, y)$, $a \leq x \leq b$, $y(a) = \alpha$ has a unique solution $y(x)$ for $a \leq x \leq b$ and the problem is well posed. (see [6])

Theorem 2: Fundamental Theorem of Dahlquist

The necessary and sufficient condition for linear multistep method to be convergent are that it be consistent and zero- stable. (see [6])

2. Methodology

a) Two step Adams type hybrid block method at ($k = 2$)

Given an approximate solution of the form

$$P(x) = \sum_{n=0}^m a_n T_n \quad (3)$$

$$P'(x) = \sum_{n=0}^m a_n T'_n \quad (4)$$

63 From equation (3) and (4)

$$\sum_{n=1}^m a_n T'_n = f(x, y) \quad (5)$$

64 where a_n are the parameters to be determined and T_n are Chebyshev polynomial. We collocate
 65 (4) at $x = x_{n+j}, j = (u, 1, v, 2)$, specially $m = 5, u = \frac{1}{2}, \text{ and } v = \frac{3}{2}$ and interpolate (3) at
 66 $x = x_{n+1}$ to have the following non linear system of equations of the form

$$\begin{aligned} \sum_{n=0}^5 a_n T_n(x) &= y_{n+1} \\ \sum_{n=1}^5 a_n T'_n(x) &= f_{n+j}, \quad j = (u, 1, v, 2) \end{aligned} \quad (6)$$

67
 68 Our propose continuous formulation of the two step Adams type hybrid block method will be of
 69 the form

$$y(x) = \alpha_{n+1}(x)y_{n+1} + h \sum_{j=0}^2 \beta_j(x)f_{n+j} + h\beta_u(x)f_{n+u} + h\beta_v(x)f_{n+v} \quad (7)$$

70 By using maple 17 mathematical software to determine the values of α and β 's and substituting
 71 to obtain the following continuous formula of the method as

$$\begin{aligned} 72 \quad y(x) &= y_{n+1} + \left[-\frac{29}{180}h + (x - x_n) - \frac{25}{12}\frac{(x-x_n)^2}{h} + \frac{35}{18}\frac{(x-x_n)^3}{h^2} - \frac{5}{6}\frac{(x-x_n)^4}{h^3} + \frac{2}{15}\frac{(x-x_n)^5}{h^4} \right] f_n + \\ 73 \quad &\left[-\frac{3(x-x_n)^2}{h} - \frac{19}{3}\frac{(x-x_n)^3}{h^2} - \frac{4(x-x_n)^4}{h^3} - \frac{2}{15}h + \frac{4}{5}\frac{(x-x_n)^5}{h^4} \right] + \left[-\frac{4}{3}\frac{(x-x_n)^2}{h} - \frac{28}{9}\frac{(x-x_n)^3}{h^2} - \frac{18}{15}\frac{(x-x_n)^5}{h^4} - \right. \\ 74 \quad &\left. \frac{1}{14}h + \frac{7}{3}\frac{(x-x_n)^4}{h^3} \right] f_{n+\frac{3}{2}} + \left[-\frac{1}{4}\frac{(x-x_n)^2}{h} + \frac{1}{180}h + \frac{2}{15}\frac{(x-x_n)^5}{h^4} + \frac{11}{18}\frac{(x-x_n)^3}{h^2} - \frac{1}{2}\frac{(x-x_n)^4}{h^3} \right] f_{n+2} + \\ 75 \quad &\left[-\frac{31}{45}h + \frac{3(x-x_n)^4}{h^3} + \frac{4(x-x_n)^2}{h} - \frac{52}{9}\frac{(x-x_n)^3}{h^2} - \frac{8}{15}\frac{(x-x_n)^5}{h^4} \right] f_{n+\frac{1}{2}} \end{aligned} \quad (8)$$

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Equation (8) is evaluated at $x = x_{n+j}, j = 0, \frac{1}{2}, \frac{3}{2}, 2$ to obtain the following discrete schemes as our two step Adams type hybrid methods (ATHBMs).

$$y_{n+\frac{1}{2}} - y_{n+1} = h \left[\frac{19}{1440} f_n - \frac{19}{60} f_{n+1} - \frac{11}{1440} f_{n+2} - \frac{173}{720} f_{n+\frac{1}{2}} + \frac{73}{720} f_{n+\frac{3}{2}} \right]$$

$$y_n - y_{n+1} = h \left[-\frac{29}{180} f_n - \frac{2}{15} f_{n+1} + \frac{1}{180} f_{n+2} - \frac{31}{45} f_{n+\frac{1}{2}} - \frac{1}{45} f_{n+\frac{3}{2}} \right]$$

$$y_{n+\frac{3}{2}} - y_{n+1} = h \left[\frac{11}{1440} f_n + \frac{19}{60} f_{n+1} - \frac{19}{1440} f_{n+2} - \frac{37}{720} f_{n+\frac{1}{2}} + \frac{173}{720} f_{n+\frac{3}{2}} \right]$$

$$y_{n+2} - y_{n+1} = h \left[-\frac{1}{180} f_n + \frac{2}{15} f_{n+1} + \frac{29}{180} f_{n+2} + \frac{1}{45} f_{n+\frac{1}{2}} + \frac{31}{45} f_{n+\frac{3}{2}} \right]$$

$$(9)$$

Equation (9) is of order $[5, 5, 5, 5]^T$ with error constants of

$$[1.1194 \times 10^{-4}, -1.7361 \times 10^{-4}, 1.1194 \times 10^{-4}, -1.7361 \times 10^{-4}]^T$$

b) Three step Adams type Hybrid block method at ($k = 3$).

Equation (4) is collected at $x = x_{n+j}, j = 0, v, w, 2 \text{ and } 3$, specially $m = 6, v = \frac{3}{2}, w = \frac{5}{2}$

Also equation (3) is also interpolation at $x = x_{n+1}$ to obtain the following system of non linear equation of the form

$$\sum_{n=0}^6 a_n T_n(x) = y_{n+1}$$

$$\sum_{n=1}^6 a_n T'_n(x) = f_{n+j}, \quad j = (u, 1, v, 2, 3) \quad (10)$$

Our propose continuous formula of three step Adams type hybrid block method is of the form

$$y(x) = \alpha_{n+1}(x)y_{n+1} + h \sum_{j=0}^3 \beta_j(x)f_{n+j} + h\beta_v(x)f_{n+v} + h\beta_w(x)f_{n+w} \quad (11)$$

90 By using maple 17 mathematical software to determine the value of α and β 's and substituting
91 in (11) to obtain the continuous formula of the method as

$$\begin{aligned} y(x) = y_{n+1} &+ \left[-\frac{11}{40}h + (\xi) - \frac{29}{20}\frac{(\xi)^2}{h} + \frac{29}{27}\frac{(\xi)^3}{h^2} - \frac{31}{72}\frac{(\xi)^4}{h^3} + \frac{4}{45}\frac{(\xi)^5}{h^4} - \frac{1}{135}\frac{(\xi)^6}{h^5} \right] f_n \\ &+ \left[\frac{15}{2}\frac{(\xi)^2}{h} - \frac{19}{2}\frac{(\xi)^3}{h^2} + \frac{119}{24}\frac{(\xi)^4}{h^3} - \frac{673}{360}h - \frac{6}{5}\frac{(\xi)^5}{h^4} + \frac{1}{9}\frac{(\xi)^6}{h^5} \right] f_{n+1} + \left[-\frac{40}{3}\frac{(\xi)^2}{h} + \frac{136}{45}\frac{(\xi)^5}{h^4} + \frac{104}{45}h - \right. \\ &\frac{104}{9}\frac{(\xi)^4}{h^3} + \frac{536}{27}\frac{(\xi)^3}{h^2} - \frac{8}{27}\frac{(\xi)^6}{h^5} \left. \right] f_{n+\frac{3}{2}} + \left[-\frac{211}{120}h + \frac{45}{4}\frac{(\xi)^2}{h} - \frac{18(\xi)^3}{h^2} + \frac{91}{8}\frac{(\xi)^4}{h^3} - \frac{16}{5}\frac{(x-\xi)^5}{h^4} + \frac{1}{3}\frac{(x-\xi)^6}{h^5} \right] f_{n+2} \\ &+ \left[\frac{32}{45}h - \frac{16}{3}\frac{(\xi)^4}{h^3} - \frac{24}{5}\frac{(\xi)^2}{h} - \frac{8}{45}\frac{(\xi)^6}{h^5} + \frac{8(\xi)^3}{h^2} + \frac{8(\xi)^5}{5h^4} \right] f_{n+\frac{5}{2}} + \left[-\frac{43}{360}h + \frac{71}{72}\frac{(\xi)^4}{h^3} + \frac{1}{27}\frac{(\xi)^6}{h^5} - \frac{77}{54}\frac{(\xi)^3}{h^2} - \right. \\ &\frac{14}{45}\frac{(\xi)^5}{h^4} + \frac{5}{6}\frac{(\xi)^2}{h} \left. \right] f_{n+3} \end{aligned} \quad (12)$$

96 where $\xi = (x - x_n)$

97 Equation (12) is evaluated at $x = x_{n+j}, j = (0, \frac{3}{2}, 2, \frac{5}{2}, \text{and } 3)$ to obtain the following discrete
98 schemes as our three Adams type hybrid block method (ATHBMs)

$$99 \quad y_n - y_{n+1} = h \left[-\frac{11}{40}f_n - \frac{673}{360}f_{n+1} + \frac{104}{45}f_{n+\frac{3}{2}} - \frac{211}{120}f_{n+2} + \frac{32}{45}f_{n+\frac{5}{2}} - \frac{43}{360}f_{n+3} \right]$$

$$100 \quad y_{n+\frac{3}{2}} - y_{n+1} = h \left[-\frac{1}{640}f_n + \frac{1139}{5760}f_{n+1} + \frac{139}{360}f_{n+\frac{3}{2}} - \frac{217}{1920}f_{n+2} + \frac{13}{360}hf_{n+\frac{5}{2}} - \frac{31}{5760}f_{n+3} \right]$$

$$101 \quad y_{n+2} - y_{n+1} = h \left[-\frac{1}{1080}f_n + \frac{7}{40}f_{n+1} + \frac{88}{135}f_{n+\frac{3}{2}} + \frac{7}{40}f_{n+2} - \frac{1}{1080}f_{n+3} \right]$$

$$102 \quad y_{n+\frac{5}{2}} - y_{n+1} = h \left[-\frac{1}{640}f_n + \frac{123}{640}f_{n+1} + \frac{23}{40}f_{n+\frac{3}{2}} + \frac{333}{640}f_{n+2} + \frac{9}{40}hf_{n+\frac{5}{2}} - \frac{7}{640}hf_{n+3} \right]$$

$$y_{n+3} - y_{n+1} = h \left[\frac{7}{45} f_{n+1} + \frac{32}{45} f_{n+\frac{3}{2}} + \frac{4}{15} f_{n+2} + \frac{32}{45} f_{n+\frac{5}{2}} + \frac{7}{45} f_{n+3} \right] \quad (13)$$

Equation (13) is of order $[6, 6, 6, 6]^T$ with error constant of

$$[1.9428(-03), 5.9420(-05), 2.4802(-05), 6.9754(-05), 6.6138(-05)]^T$$

c) Four step Adams type hybrid block method at ($k = 4$).

Equation (4) is collocated at $x = x_{n+j}, j = (1, v, 2, w, 3, 4)$, specially $m = 7, v = \frac{3}{2}$, and $w = \frac{5}{2}$. Also (3) is interpolated at $x = x_{n+1}$ to obtain the following system of linear equation of the form

$$\sum_{n=0}^7 a_n T_n(x_{n+1}) = y_{n+1}$$

$$\sum_{n=1}^7 a_n T'_n(x_{n+j}) = f_{n+j}, \quad j = (1, v, 2, w, 3, 4) \quad (14)$$

Our propose continuous formula of the four step Adams type hybrid block method is of the form

$$y(x) = y_{n+1} + h \sum_{n=1}^4 \beta_j(x) f_{n+j} + h \beta_v(x) f_{n+v} + h \beta_w(x) f_{n+w} \quad (15)$$

By using maple 17 to determine the values of α and β 's substituting them in (15) to obtain the continuous formula of the method as

$$y_n = y_{n+1} + \left[-\frac{63}{40} \frac{(x-x_n)^2}{h} + \frac{1421}{1080} \frac{(x-x_n)^3}{h^2} - \frac{91}{144} \frac{(x-x_n)^4}{h^3} - (x-x_n) + \frac{7}{40} \frac{(x-x_n)^5}{h^4} - \frac{7}{270} \frac{(x-x_n)^6}{h^5} - \right.$$

$$\left. \frac{3923}{15120} h + \frac{1}{630} \frac{(x-x_n)^7}{h^6} \right] f_n + \left[\frac{10(x-x_n)^2}{h} - \frac{16483}{7560} h - \frac{263}{90} \frac{(x-x_n)^5}{h^4} - \frac{2}{63} \frac{(x-x_n)^7}{h^6} + \frac{647}{72} \frac{(x-x_n)^4}{h^3} - \right.$$

$$\left. \frac{43}{3} \frac{(x-x_n)^3}{h^2} + \frac{13}{27} \frac{(x-x_n)^6}{h^5} \right] f_{n+1} + \left[\frac{128}{15} \frac{(x-x_n)^5}{h^4} + \frac{3124}{945} h - \frac{64}{3} \frac{(x-x_n)^2}{h} + \frac{32}{315} \frac{(x-x_n)^7}{h^6} - \frac{40}{27} \frac{(x-x_n)^6}{h^5} + \right.$$

$$\left. \frac{4768}{135} \frac{(x-x_n)^3}{h^2} - \frac{220}{9} \frac{(x-x_n)^4}{h^3} \right] f_{n+\frac{3}{2}} + \left[\frac{59}{2} \frac{(x-x_n)^4}{h^3} - \frac{219}{20} \frac{(x-x_n)^5}{h^4} - \frac{1}{7} \frac{(x-x_n)^7}{h^6} + \frac{45}{2} \frac{(x-x_n)^2}{h} - \frac{159}{4} \frac{(x-x_n)^3}{h^2} + \right.$$

$$\begin{aligned}
 & \left[\frac{2(x-x_n)^6}{h^5} - \frac{221}{70}h \right] f_{n+2} + \left[-\frac{164}{9} \frac{(x-x_n)^4}{h^3} + \frac{64}{9} \frac{(x-x_n)^5}{h^4} + \frac{32}{315} \frac{(x-x_n)^7}{h^6} - \frac{64}{5} \frac{(x-x_n)^2}{h} + \frac{352}{15} \frac{(x-x_n)^3}{h^2} + \right. \\
 & \left. \frac{1612}{945}h - \frac{184}{135} \frac{(x-x_n)^6}{h^5} \right] f_{n+\frac{5}{2}} + \left[\frac{361}{72} \frac{(x-x_n)^4}{h^3} + \frac{10}{3} \frac{(x-x_n)^2}{h} - \frac{2}{63} \frac{(x-x_n)^7}{h^6} - \frac{61}{30} \frac{(x-x_n)^5}{h^4} + \frac{11}{27} \frac{(x-x_n)^6}{h^5} - \right. \\
 & \left. \frac{3253}{7560}h - \frac{169}{27} \frac{(x-x_n)^3}{h^2} \right] f_{n+3} + \left[-\frac{29}{144} \frac{(x-x_n)^4}{h^3} + \frac{31}{360} \frac{(x-x_n)^5}{h^4} - \frac{1}{8} \frac{(x-x_n)^2}{h} + \frac{47}{3024}h + \frac{1}{630} \frac{(x-x_n)^7}{h^6} - \right. \\
 & \left. \frac{1}{54} \frac{(x-x_n)^6}{h^5} + \frac{29}{120} \frac{(x-x_n)^3}{h^2} \right] f_{n+4}
 \end{aligned} \tag{16}$$

Evaluating equation (16) at $x = x_{n+j}, j = (0, \frac{3}{2}, 2, \frac{5}{2}, 3 \text{ and } 4)$ to have the following discrete schemes as our four steps Adams type hybrid block method

$$y_{n+1} - y_n = h \left[\frac{3923}{15120} f_n + \frac{16483}{7560} f_{n+1} - \frac{3124}{945} f_{n+\frac{3}{2}} + \frac{221}{70} f_{n+2} - \frac{1612}{945} h f_{n+\frac{5}{2}} + \frac{3253}{7560} f_{n+3} - \frac{47}{3024} f_{n+4} \right]$$

$$y_{n+\frac{3}{2}} - y_{n+1} = h \left[-\frac{263}{241920} f_n + \frac{22769}{120690} f_{n+1} + \frac{3149}{7560} f_{n+\frac{3}{2}} - \frac{349}{2240} f_{n+2} + \frac{503}{7560} f_{n+\frac{5}{2}} - \frac{1801}{120960} f_{n+3} + \frac{23}{48384} f_{n+4} \right]$$

$$y_{n+2} - y_{n+1} = h \left[-\frac{11}{15120} f_n + \frac{431}{25120} f_{n+1} + \frac{638}{945} f_{n+\frac{3}{2}} + \frac{11}{70} f_{n+2} + \frac{4}{315} f_{n+\frac{5}{2}} - \frac{37}{7560} f_{n+3} + \frac{1}{5040} f_{n+4} \right]$$

$$y_{n+\frac{5}{2}} - y_{n+1} = h \left[-\frac{9}{8960} f_n + \frac{811}{4480} f_{n+1} + \frac{171}{280} f_{n+\frac{3}{2}} + \frac{1053}{2240} f_{n+2} + \frac{73}{280} f_{n+\frac{5}{2}} - \frac{99}{4480} f_{n+3} + \frac{1}{1792} f_{n+4} \right]$$

$$y_{n+3} - y_{n+1} = h \left[-\frac{1}{1890} f_n + \frac{157}{945} f_{n+1} + \frac{128}{189} f_{n+\frac{3}{2}} + \frac{11}{35} f_{n+2} + \frac{128}{189} f_{n+\frac{5}{2}} + \frac{157}{945} f_{n+3} - \frac{1}{1890} f_{n+4} \right]$$

$$y_{n+4} - y_{n+1} = h \left[-\frac{9}{560} f_n + \frac{167}{280} f_{n+1} - \frac{36}{35} f_{n+\frac{3}{2}} + \frac{243}{70} f_{n+2} - \frac{92}{35} f_{n+\frac{5}{2}} + \frac{657}{280} f_{n+3} + \frac{29}{112} f_{n+4} \right] \quad (17)$$

Equation (3.14) is of order $[7, 7, 7, 7, 8, 7]^T$, with error constant $[1.0045(-03), -2.3542(-05), -1.2401(-05), 3.8508(-05), -1.0044(-03)]^T$

3. Convergence Analysis of the Two Step Block Hybrid Adams Type Methods (ATMs)

The two step block hybrid Adams type methods is expressed in the form

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{2}} \\ y_{n+1} \\ y_{n+\frac{3}{2}} \\ y_{n+2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{n-\frac{3}{2}} \\ y_{n-1} \\ y_{n-\frac{1}{2}} \\ y_n \end{bmatrix} + \begin{bmatrix} -\frac{173}{45} & -\frac{19}{15} & \frac{37}{45} & -\frac{11}{180} \\ \frac{720}{31} & \frac{60}{2} & \frac{720}{1} & \frac{1440}{-1} \\ \frac{45}{15} & \frac{15}{45} & \frac{45}{180} & \frac{180}{-19} \\ -\frac{37}{720} & \frac{19}{60} & \frac{173}{720} & -\frac{19}{1440} \\ \frac{1}{45} & \frac{2}{15} & \frac{31}{45} & \frac{29}{180} \end{bmatrix} \begin{bmatrix} f_{n+\frac{1}{2}} \\ f_{n+1} \\ f_{n+\frac{3}{2}} \\ f_{n+2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \frac{19}{1440} \\ 0 & 0 & 0 & \frac{29}{180} \\ 0 & 0 & 0 & \frac{11}{1440} \\ 0 & 0 & 0 & -\frac{1}{180} \end{bmatrix} \begin{bmatrix} f_{n-\frac{3}{2}} \\ f_{n-1} \\ f_{n-\frac{1}{2}} \\ f_n \end{bmatrix} \quad (18)$$

Where $A^{(0)} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, A^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$B^{(0)} = \begin{bmatrix} -\frac{173}{45} & -\frac{19}{15} & \frac{37}{45} & -\frac{11}{180} \\ \frac{720}{31} & \frac{60}{2} & \frac{720}{1} & \frac{1440}{-1} \\ \frac{45}{15} & \frac{15}{45} & \frac{45}{180} & \frac{180}{-19} \\ -\frac{37}{720} & \frac{19}{60} & \frac{173}{720} & -\frac{19}{1440} \\ \frac{1}{45} & \frac{2}{15} & \frac{31}{45} & \frac{29}{180} \end{bmatrix}, B^{(1)} = \begin{bmatrix} 0 & 0 & 0 & \frac{19}{1440} \\ 0 & 0 & 0 & \frac{29}{180} \\ 0 & 0 & 0 & \frac{11}{1440} \\ 0 & 0 & 0 & -\frac{1}{180} \end{bmatrix}$$

We normalized the block method (18) with the inverse of $A^{(0)}$ and applied the condition

$$\rho(\lambda) = \det[\lambda A^{(0)}(A^{(0)})^{-1} - A^{(1)}(A^{(0)})^{-1}] = 0$$

$$\rho(\lambda) = \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 0$$

$$= \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 0$$

$$= \det \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & (\lambda - 1) & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} = 0$$

$$= \lambda^3(\lambda - 1) = 0$$

$$= \lambda^3 = 0 \text{ and } \lambda - 1 = 0$$

Therefore, $\lambda_1 = \lambda_2 = \lambda_3 = 0$, $\lambda_4 = 1$.

From definition (2), the method (a) is zero stable and since the order of the method is $p \geq 1$, the method is consistent, thus convergent.

Similarly methods (b) and (c) were treated in the same manner and displaces their consistencies and convergent.

4. Numerical experiments.

All the three newly constructed Adams type hybrid block method at various values of $k = 2, 3 \text{ and } 4$, were demonstrated with three different linear and one non linear problems

Problem 1

$$y' = -y, \quad y(0) = 1, \quad 0 \leq x \leq 1, \quad h = 0.1$$

Exact solution

$$y(x) = e^{-x}$$

Problem 2

$$y' = xy, \quad y(0) = 1, \quad 0 \leq x \leq 2, \quad h = 0.1$$

164 Exact solution

$$y(x) = e^{\frac{x^2}{2}}$$

165

166 Problem 3

$$167 \quad y' = y - x^2 + 1, \quad y(0) = \frac{1}{2}, \quad h = 0.1$$

168 Exact solution

$$y(x) = (x + 1)^2 - \frac{1}{2}e^x$$

169

170 Problem 4

$$171 \quad y' = 1 - y^2, \quad y(0) = 0, \quad h = 0.1$$

172 Exact solution

$$y(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$$

173

174 Table 1: Approximate Solutions to problem 1

x	Method of k=2	Method of k=3	Method of k=4
0.1000	0.904837417895975	0.904837417881202	0.904837418028227
0.2000	0.818730753110534	0.818730752939751	0.818730753071073
0.3000	0.740818220855484	0.740818220548903	0.740818220675358
0.4000	0.670320046323450	0.670320045918305	0.670320046037833
0.5000	0.606530660091209	0.606530659599218	0.606530659682672
0.6000	0.548811636543486	0.548811635994641	0.548811636211386
0.7000	0.496585304294834	0.496585303694640	0.496585303267860
0.8000	0.449328964660269	0.449328964033219	0.449328966406345
0.9000	0.406569660311169	0.406569659658082	0.406569649706824
1.0000	0.367879441759379	0.367879441100594	0.367879441171442

175

176

177

178 Table 2: Absolute Error of Problem 1

x	Method [2]	Error at k=2	Error at k=3	Error at k=4
0.1000	1.5490(-08)	1.3999(-10)	1.5476(-10)	7.7330(-12)
0.2000	9.3743(-09)	-3.2552(-11)	1.3823(-10)	6.9090(-12)
0.3000	1.6932(-08)	-1.7377(-10)	1.3282(-10)	6.3600(-12)
0.4000	1.6406(-08)	-2.8781(-10)	1.1733(-10)	-2.1940(-12)
0.5000	2.5228(-08)	-3.7858(-10)	1.1342(-10)	3.1970(-12)
0.6000	1.9716(-08)	-4.4946(-10)	9.9385(-11)	2.8320(-12)
0.7000	2.3504(-08)	-5.0342(-10)	9.6770(-11)	2.6370(-12)
0.8000	2.1992(-08)	-5.4305(-10)	8.4003(-11)	2.9400(-12)
0.9000	2.6862(-08)	-5.7057(-10)	8.2517(-11)	8.1400(-13)
1.0000	2.2220(-08)	-5.8794(-10)	7.0848(-11)	6.9600(-13)

179

180 Table 3: Approximate Solutions to problem 2

x	Method k=2	Method k=3	Method k=4
0.1000	1.005012518167810	1.005012524024950	1.005012519628150
0.2000	1.020201340025330	1.020201343118400	1.020201338792430
0.3000	1.046027862950250	1.046027863348370	1.046027858627300
0.4000	1.083287074401830	1.083287071288450	1.083287067665950
0.5000	1.133148464484150	1.133148457147040	1.133148448352280
0.6000	1.197217380713540	1.197217367631370	1.197217379701250
0.7000	1.277621339108210	1.277621318512900	1.277621241114160
0.8000	1.377127803635090	1.377127770522030	1.377128070311000
0.9000	1.499302555226830	1.499302507651910	1.49930211675550
1.0000	1.648721347944550	1.648721280024100	1.648726712852990

181

182 Table 4: Absolute Error of Problem 2

x	Method [2]	Error at k=2	Error at k=3	Error at k=4
0.1000	3.1386(-07)	2.69159(-09)	3.16555(-09)	1.23125(-09)
0.2000	1.3364(-07)	1.43000(-12)	3.17164(-09)	1.23433(-09)

0.3000	3.1819(-07)	3.04153(-09)	3.43965(-09)	1.28142(-09)
0.4000	9.8972(-09)	6.72687(-09)	3.61349(-09)	9.01000(12)
0.5000	6.9521(-07)	1.14173(-08)	4.08021(-09)	3.3949(-09)
0.6000	4.0794(-07)	1.75917(-08)	4.50965(-09)	3.5428(-09)
0.7000	7.7319(-07)	2.59033(-08)	5.30801(-09)	3.8244(-09)
0.8000	7.0821(-07)	3.92991(-08)	6.18607(-09)	3.8275(-10)
0.9000	2.8682(-06)	5.51701(-08)	7.59514(-09)	1.3275(-08)
1.0000	2.0664(-06)	7.72444(-08)	9.32397(-09)	1.4427(-08)

183

184 Table 5: Approximate Solutions to problem 3

x	Method k=2	Method k=3	Method k=4
0.1000	0.657414541069634	0.657414616647980	0.657414540968716
0.2000	0.829298620944187	0.829298629148387	0.829298620927051
0.3000	1.015070596134470	1.015070596059230	1.015070596217970
0.4000	1.214087650978350	1.214087651011410	1.214087651181800
0.5000	1.425639364300310	1.425639480369200	1.425639364675610
0.6000	1.648940599300000	1.648940599593540	1.648940599730980
0.7000	1.883123643700000	1.883123646024930	1.883123646607190
0.8000	2.127229525000000	2.127229535488470	2.127229534342820
0.9000	2.380198432346940	2.381019844412120	2.380198450381810
1.0000	2.640859072215840	2.640859085437560	2.640859060719960

185

186 Table 6: Absolute Error of Problem 3

x	Error at k=2	Error at k=3	Error at k=4
0.1000	1.07459(-10)	1.22237(-10)	-6.54100(-12)
0.2000	2.42720(-11)	1.33398(-10)	-7.13600(-12)
0.3000	7.75300(-11)	1.52770(-10)	-7.97000(-12)
0.4000	2.01010(-10)	1.67950(-10)	-2.44000(-12)
0.5000	3.49630(-10)	1.91450(-10)	-2.56700(-11)
0.6000	5.27260(-10)	2.11200(-10)	7.37600(-11)

0.7000	7.38400(-10)	2.39830(-10)	-3.42430(-10)
0.8000	1.07538(-08)	2.65290(-10)	1.41094(-09)
0.9000	1.20746(-08)	3.00320(-10)	-5.96029(-09)
1.0000	1.35546(-08)	3.32920(-10)	2.50505(-08)

187

188 Table 7: Approximate Solutions to problem 4

x	Method k=2	Method k=3	Method k=4
0.1000	0.099667998229271	0.099667958427121	0.099667983956123
0.2000	0.197375318495844	0.197375285530702	0.197375309998647
0.3000	0.291312602600503	0.291312578047386	0.291312602567245
0.4000	0.379948943860190	0.379948931365855	0.379948964538561
0.5000	0.462117132110030	0.462117128154898	0.462117113959080
0.6000	0.537049538236968	0.537049538236968	0.537049729661565
0.7000	0.604367748161445	0.604367754450766	0.604367039621352
0.8000	0.664036743935206	0.664036751802030	0.664039991411427
0.9000	0.716297848263363	0.716297853189866	0.716283642731265
1.0000	0.761594139111875	0.761594142508150	0.761657006565570

189

190 Table 8: Absolute Error of Problem 4

x	Error at k=2	Error at k=3	Error at k=4
0.1000	-3.60432(-09)	3.61978(-08)	1.2312(-09)
0.2000	1.72906(-09)	3.46942(-08)	1.2343(-09)
0.3000	9.85108(-09)	3.44042(-08)	1.2814(-09)
0.4000	1.83950(-08)	3.08893(-08)	9.0100(-12)
0.5000	2.51500(-08)	2.91051(-08)	3.3949(-09)
0.6000	2.87611(-08)	2.46756(-08)	3.5428(-09)
0.7000	2.89557(-08)	2.26664(-08)	3.8244(-09)
0.8000	2.63326(-08)	1.84658(-08)	3.8293(-10)
0.9000	2.19357(-08)	1.70072(-08)	1.3275(-08)
1.0000	1.68439(-08)	1.34476(-08)	1.4427(-08)

191 5. Conclusion

192 We want to conclude that this research paper demonstrates a successful derivation and
 193 implementation of Adam's type hybrid block linear multi-step method using cheybyshev
 194 polynomial as trial bases functions. Based on the problems solved all the methods at various
 195 values of $k = 2, 3 \text{ and } 4$ performed excellently well than the existing methods particularly
 196 problem 1 and 2. (see Method [2]). All the methods are stable, consistent and convergent.

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