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Fuzzy Rule Based Salt & Pepper Noise Removing in Gray Images

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Abstract

This paper presents the application of the diffusion equation for removing the salt & pepper noise in the field of image processing along with the fuzzy logic. The fuzzy logic differentiates the noisy pixels, especially the salt & pepper noise and the edges, then diffusion equation is applied to only noisy pixels and the diffusivity is proportional to the noise. The proposed method is applied to many test images. The simulated results demonstrate the effectiveness of the method based on statistical measures such as MSE and PSNR.

Keywords: The diffusion equation, salt & pepper noise, image denoising, fuzzy logic
2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

1 Introduction

Nowadays, fuzzy sets are extensively used in almost every field of research especially in computer vision and image processing. The concept of fuzzy sets was introduced first in L.Zadeh (1965). The fuzzy sets are used to cope with the uncertainty, vagueness and imprecision. Commonly these sets are used to formulate rules in the development of fuzzy logic. This logic is applied in many fields of image processing, such as object detecting, segmentation, labeling and image understanding modules. It is equally used in image pre-processing applications like image denoising, debluring and edge enhancement techniques.

Image pre-processing is an important step in computer vision and pattern recognition. Noise removal is performed at different intermediate levels of processing. Smoothing the noisy pixels and the edge enhancement are the opposite process to each other. Smoothing a region might destroy an edge and sharpening the edges can create unnecessary noise. To achieve the good performance of

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the noise reducing algorithm, the algorithm should differentiate between the noisy pixel and pixels on the edge or the flat region. Many methods has been proposed in literature such as partial differential equation based models (diffusion models) Alf (1978); Shanmugavadivu and Eliahim Jeevaraj (2011), fractional differential equationsCuesta et al. (2012), adaptive and non-linear filteringNguyen et al. (2010); Thivakaran and Chandrasekaran (2010). These method are applied in frequency domain.

Fuzzy sets are introduced by L.A. Zadeh L.Zadeh (1965) in 1965, now the fuzziness is used in many mathematical theories, such as topology and solution of differential equations. The concepts of the fuzzy derivatives are introduced in the papers Janiš (1999), in the limiting case, fuzzy derivatives resemble with the ordinary derivatives. These derivatives are used to formulate the rules which differentiate the noisy and the pixels on the edge or flat region and preserve the image details. The fuzzy methods for image denoising are, using the fuzzy sets and fuzzy logic Van De Ville et al. (2003); Jampour et al. (2010); Qiu and Shu (2008); Zhou et al. (2008); Esakkirajan et al. (2011); Luo (2006) neuro-fuzzy systems Thanh and Chen (2006), decision based variational techniques Y.Y. Zhou and Huang (2012), cellular automat techniques sad (2012), and bayesian techniques for image denoising C. Jung and Gong (2012) and evidence theoryLin (2011). Commonly fuzzy logical based methods use the median, adaptive median and mean filters and median filters in two stages.

In this paper, we proposed the application of fuzzy logic along with the diffusion equation. Fuzzy logic sorts out the noisy pixels and decide the intensity of the diffusion equation. A derivative in eight cardinal directions with respect to the central pixel is computed, this derivative is used to construct a fuzzy rule and one rule is correspond to one derivative. In this way, eight rules are formulated for each pixel and these rules decide the application of diffusion equation. This also attach a weight to the diffusivity. This method differs from the method proposed in Van De Ville et al. (2003) in two ways. This method is computationally more effective than the method described in Van De Ville et al. (2003) , where one have to construct the 16 rules for each pixel after establishing the fuzzy derivatives. Secondly, in this method, diffusion equation is applied while they proposed the correction terms.

The proposed method is tested for several corrupted images with salt & pepper noise. The quality of the restored images is compared using the statistical measures such as Mean Square Error(MSE) and Peak Signal to Noise Ratio (PSNR). Generally, the value of PSNR more than 25 db is acceptable for the image quality. We also discuss the visual results.

This paper is arranged as follows. The diffusion model is discussed in the forth coming section, Section 2 and fuzzy rules, their development and comparison tools are discussed in Section 3. Section 4 consist of necessary experiments and discussion and Section 5 concludes the paper.

2 Diffusion Model

A number of diffusion models has been proposed, the oldest partial differential equation based model is the application of the heat equation with the certain boundary conditions. This model is mathematically described as

$$\frac{\partial u(x,t)}{\partial t} = c\Delta I(x,t), \qquad I(x,0) = I_0 \tag{2.1}$$

where $I(x,t)$ is the processed image at time slice t, $I_0 = I(x,0)$ is the initial image and c is constant of diffusion, its value is considered as one. In this model, one time slice is equal to the convolution of the Gaussian to the spatial variables. First of all, this model was discussed by Perona and Malik Perona (1990). They imposed certain conditions on c, such that it stops the diffusion process near boundary or edges. When we apply this model directly, image details are also diffused and some fine details are lost.

Many approximations has been proposed for the spatial variables, for example, famous five point formula for uniform spatial gird and uniform diffusion, for the discretization process, the convolution

mask of the five point formula is written as

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This formula has the error order as $O(h^4)$, another version of this formula is described by Thoms and Aplet in Thoms and Aplet (1961). This representation considered the axis with the rotation. A combined version of both the five-point formulas is often used, which results in the nine point formula. Its convolution mask is written as

$$\begin{pmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{pmatrix}$$

This approximation of the filter mask accelerates the convergence of the method, it has error of order $O(h^6)$. This diffusion equation is applied only to the noisy pixels, at the first step and its application is proportional to the noise ratio in that pixel. In the following section (Section 3), we develop the criteria for the selection of noisy pixels and we use the fuzzy inference rules to differentiate the noisy and real pixels.

3 Development of the Fuzzy Rules

The gradients and directional derivatives characterize similarity among pixels. The salt & pepper noise is the certain impulse in the smooth region, mathematically:

$$I_{appr}(x,y) = \begin{cases} 0 & \text{with probability } \frac{p}{2} \\ I(x,y) & \text{with probability } 1-p \\ 255 & \text{with probability } \frac{p}{2} \end{cases} \tag{3.1}$$

where p is the noise probability. At the noisy pixel, the derivative in all cardinal directions is high as compared to the derivative at smooth region. First of all, we calculate the derivative for each cardinal direction (*N, E, S, W, NW, NE, SE, SW*) with respect to the central pixel.

At this step, fuzzy relation *large* is defined in Eq.(3.2), which is further explained in section 3.1. This relation measures the change in pixel value, then the expert knowledge about the derivative is used to establish the rules. Commonly, if all the eight derivatives for the central pixel are high, then the pixel represents the salt & pepper noise otherwise it is a part of the edge or it represent the flat region. Hence, a set of eight fuzzy rules are associated to each pixel to decide that this pixel is a noisy pixel or it represents the edge or flat region. The Figure 1 explains the general application of fuzzy systems in image processing.

The S type membership function is used to define this relation. These rules are used for the application of the diffusion equation. A pixel value is diffused proportional to the value of membership function.

3.1 The Fuzzy Relation large

A fuzzy relation *large* is defined, this relation measures the quantity of change in the derivative and associate this change to the fuzzy value. For this purpose, we used the S type fuzzy membership

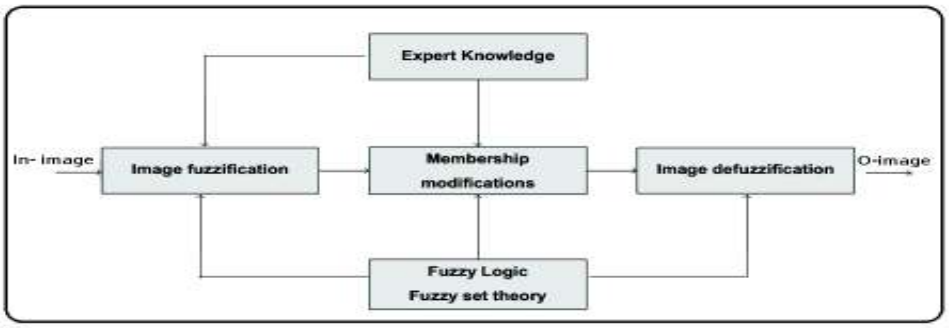


Figure 1: Fuzzy Image Process System

function. This relation is defined as

$$Large(x,a,k) = \begin{cases} 0 & \text{if } \mid x \mid \leq a \\ 2(\frac{|x|}{k})^2 & \text{if } a < \mid x \mid < \frac{k}{2} \\ 1 - 2(\frac{|x|-k}{k}) & \text{if } \frac{k}{2} \leq \mid x \mid < k \\ 1 & \text{if } k \leq x \end{cases} \tag{3.2}$$

Here a is the initial threshold and K represents the adaptive threshold, a is any constant and $\mid a \mid << \mid k \mid$ and x represent the value of the derivative. The absolute value of derivative makes this relation symmetric because the change in derivative may be negative or positive. This relation is expressed

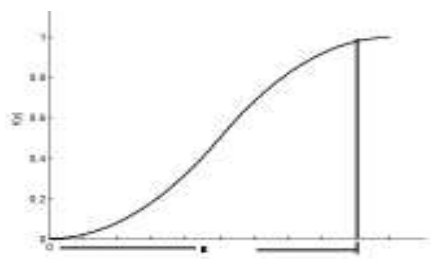


Figure 2: The fuzzy relation *large* with parameter $a = 0$

graphically in Figure 2 . It is a symmetric graph and the figure shows the half portion of the graph. The value of the relation increases with the increase in the derivative.

3.2 Adaptive Threshold

In fuzzy image processing, the fuzzy homogeneity measure is used to estimate that how much a region is homogenous. This measure is defined as

$$\mu = \frac{\max(I(x,y)) - \min(I(x,y))}{L}$$

where μ and L represents the homogeneity measure and number of grey levels in the representation of an image, respectively and $I(x,y)$ stands for the image. There is a linear relationship between the

homogeneity and the standard deviation (for details see Van De Ville et al. (2003)). The threshold is defined as

$$K = \alpha\delta$$

where α is the amplification factor, it is further explained in section 3.6 and the δ here represents the standard deviation for each block separately. The standard deviation is the complement of the fuzzy homogeneity measure, we define this measure for each non overlapping block separately. Instead of treating the each block separately, we used the mean of standard deviations for all the blocks and whole image is processed simultaneously.

3.3 The Fuzzy Rules

In image processing applications, the expert knowledge is implemented through the fuzzy rules. Fuzzy sets and fuzzy logic is a powerful tool to implement the knowledge in the form of *If - Then* rules. These rules are used to serve the quantitative relationship between the variables in linguistic terms. The main aim of the defining the fuzzy rules, is to differentiate the local variations due to noise and image structures.

In order to accomplish this objective, we calculate eight derivatives in eight cardinal directions for each pixel. The relation *large* , using equation (3.2) is defined which assigns a value to the magnitude of the gradient. This relation is derived for each direction and a set of eight rules for every pixel is defined, the decision is based on these rules that a pixel is either belongs to an edge or it is a noisy pixel.

3.4 The Relationship between diffusivity and gradient

The fuzzy rule based system distinguishes the flat region from the highly changing region due to noise, such as edges. These rules set the parameters for each region separately. Commonly, the gradient at the flats region is smaller than the gradient at the noisy pixels in all eight directions and at edges in some directions it is high while in other direction it is large. So, we define the fuzzy relation *large*. This relation defines that how much a gradient is high or low. In this paper, we used the Mamdani inference logic Mamdani (1977), in general this inference system is written as

If	X is	A
and	Y is	B
..	..	..
Z	is	C

This is the single criteria that is used to fire the fuzzy rules. we define eight rules, each rules is associated with the derivative in each direction. Now we define the diffusivity in terms of linguistic variables as

Rule	1	2	3
If gradient is	Low	Medium	High
then diffusivity	Low	Medium	high

Selection criteria for the noisy pixels will stop smoothing inside the homogenous regions and at the edges. It sort out the noisy pixels, then smoothing is performed only on these pixels.

3.5 Comparison Tools

In the theory, there are many tools for the comparison of results and error analysis such as *Mean Square Error (MSE)* and *Peak Signal Noise Ratio (PSNR)*. Mean square error is defined as

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(I_IM(R(i,j) - F_IM(i,j))^2 \tag{3.3}$$

where $M \times N$ is the size of the image and I_IM and F_IM represent the initial and final image, respectively. This measure represents the mean sum of the squared difference in the original and the processed image. Lower the value, better it improves the image quality.

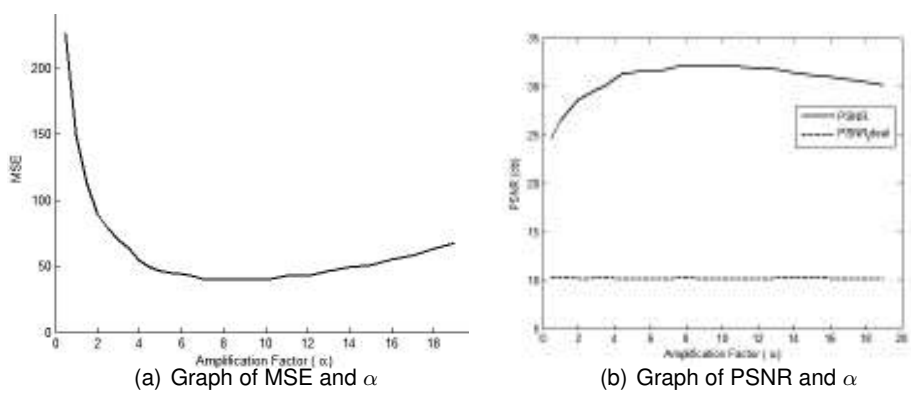
Another measure to compare the image quality is the *PSNR*, represented by the following equation.

$$PSNR = 10\log 10(\frac{Max_i^2}{MSE}) \tag{3.4}$$

Where Max_i represents the maximum brightness level value obtained by a pixel, in gray images, its value is 255. It is a most popular quality metric due to low computational cost. Generally the minimum acceptable value of the *PSNR* is between 20db to 25db. These quantitative measures along with the average execution time are used for comparison of images. These measures are also computed for other image denoising techniques such as mean filters and adaptive median filter.

3.6 Amplification Factor

The mean standard deviation represents the deviation of the pixel value from the mean position. When whole image is divided into sub images, then standard deviation represents the local variation. Its value is very low, hence an amplification factor is used to amplify the deviation of the image. This amplified value is used as a threshold for defining the relation *large*. The Figure 3 shows the effect of



(a) Graph of MSE and α (b) Graph of PSNR and α

Figure 3: Graph of the MSE, and PSNR with the change in amplification factor α for with noise ratio eight percent

amplification factor α on the *MSE* and *PSNR* for the image *lena*, when the image is corrupted with salt & pepper noise with noise ratio eight percent.

4 Experimental Results and Discussion

The proposed method is tested for several test images for removing the salt & pepper noise. The noisy image is divide into the non overlapping blocks. For the calculation of the threshold k, we

first compute the standard deviation of the each non overlapping block. Instead of treating the each block separately, we calculate the threshold using the mean standard deviation of all the blocks. The relation *large* is computed for whole image, then the diffusion equation is applied proportional to the noise. In these experiments we consider the images of size 512×512 and divide the image into sub-blocks.

4.1 Statical Analysis

For he statical analysis, we select the six gray images, which are *lena, Woman_Blonde* (WB), *pirate, liningroom* (LR), *walkbridge* (walkb) and *mandrila* (MR). The MSE measures for these image are computed for the ideal image and the final processed images. These values are depicted in the table (1) (These values are taken as average by repeating five times each experiment and $\alpha = 9$).

Table 1: The *MSE* comparison for P.M (Proposed Method), mean filter (MF) of size 3×3 and Fuzzy adaptive median Filter with weight 7 of selected images

S.No	Image	N. R (%)	MSE_i initial	P. M	SMF	Adaptive Median Filter
1	Lena	20	362.63	20.1	84.60	100.50
		40	744.78	28.82	129.91	102.24
		60	1114.8	32.78	178.30	100.88
		80	1490.7	40.13	222.52	99.63
2	WB	20	365.18	40.60	121.36	189.61
		40	721.02	2.24	164.13	189.88
		60	1087.2	55.35	208.56	191.63
		8 0	1440.1	60.95	253.30	191.65
3	Pirate	2	373.38	30.83	373.11	154.75
		40	748.43	37.67	158.39	155.72
		60	1122.7	44.79	202.76	154.48
		8 0	1508.0.7	25.25	253.96	151.11
4	LR	20	369.12	43.70	136.58	232.90
		40	744.78	28.82	177.77	235.87
		60	1104.5	60.99	224.33	237.14
		8 0	1467.6	71.60	268.98	237.02
5	walkb	20	387.47	57.97	196.45	354.48
		40	781.79	72.04	244.54	355.36
		60	1149.2	84.55	296.81	355.22
		8 0	1537.3	100.16	342.68	354.59
6	Mandrila	20	364.30	49.55	127.55	367.16
		40	715.42	58.82	171.66	365.97
		60	1072.4	69.67	215.88	363.87
		8 0	1413.2	80.83	262.23	361.26

In this table, N.R represents the Noise ratio in percentage, MSE_i shows the mean square error for the initial (ideal) and for the final images respectively. The other columns represent the computation of MSE with the proposed method, mean filter of size 3×3 and with the adaptive median filter with weight 7. The difference between the fourth and fifth column shows the improvement of the image quality. Similarly, the comparison of the values between column fifth to seventh shows the efficiency of the proposed method.

In the same way, the *PSNR* measure is computed for the ideal images and the processed images with the proposed method, mean filter of size 3×3 and adaptive median filter with weight 7. All these values are represented in table (2).

Table 2: The *PSNR* comparison for P.M (Proposed Method), standard mean filter (SMF) of 3×3 and adaptive median filter with weight 7 for selected images

S.No	Image	N. R (%)	$PSNR$ initial	P. M	SMF	Fuzzy adaptive filter
1	Lena	20	22.54	35.9	28.86	28.10
		40	19.41	33.53	26.99	28.04
		60	17.71	32.97	25.62	28.08
		8 0	16.40	32.09	24.65	28.14
2	WB	20	22.05	32.5	22.29	25.35
		40	19.55	31.30	25.98	25.34
		60	17.78	30.70	24.90	25.30
		8 0	16.54	30.28	24.07	25.30
3	Pirate	20	22.41	32.24	27.67	26.24
		40	19.39	32.37	26.13	26.20
		60	17.62	31.62	25.06	26.24
		8 0	16.34	30.70	24.08	26.28
4	LR	20	22.41	31.72	26.87	24.45
		40	19.41	33.53	25.63	24.40
		60	17.70	30.27	24.62	24.38
		8 0	16.46	29.59	23.83	24.38
5	walkb	20	22.24	30.50	25.20	22.62
		40	19.19	29.55	24.25	22.62
		60	17.62	28.86	23.41	22.62
		8 0	16.26	28.12	22.78	22.63
6	Mandrila	20	22.51	31.18	27.07	22.48
		40	19.58	30.48	25.78	22.49
		60	17.82	29.70	24.78	22.52
		8 0	16.62	29.05	23.95	22.55

Notice that the initial *PSNR* ($PSNR_i$), which is for the ideal image, is always less than 23 for each image. The difference between the fourth and fifth column shows the improvement of the image quality with the proposed method. The values in the fifth column is always greater than the values in the sixth and seventh columns, this shows the efficiency of the proposed method over the other methods for the salt & pepper noise.

4.2 Visual Analysis

For the visual analysis, we select the images *lena*, *woman_blond* and *living room*. These images are shown in the Figure (4), where the first row shows the original image and noisy images are shown in the second row. It is evident that the images in second row are corrupted with salt & pepper noise of ratio eight percent.

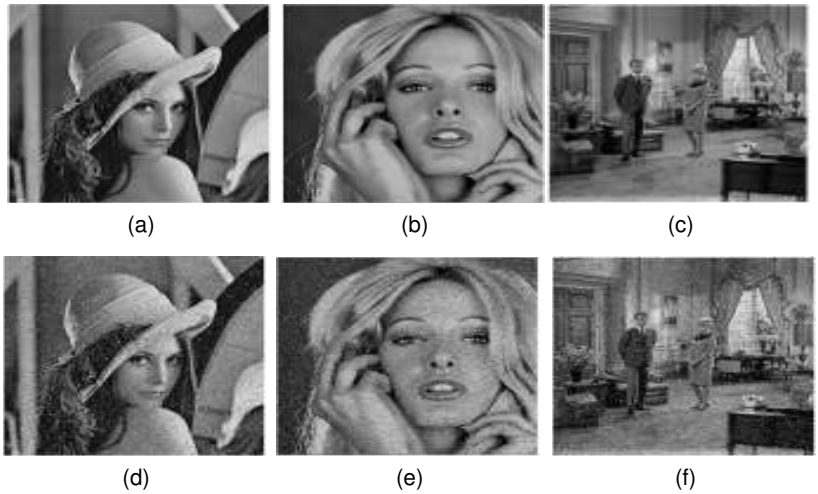


Figure 4: Original images (a) Lena, (b) Woman blond (c) living room and noisy image with noise ratio eight percent (d) lena, (e) woman blond and (f) living room

The results are shown in figure (5). The first column shows the results for the proposed method for the images *lena, woman blond* and *living room*, figure (5(a),5(d) and 5(g)) with the amplification factor $\alpha = 9$. The second column shows results for the same set of images processed with mean filter, figure (5(b),5(e) and 5(e)) and third column shows the results for the adaptive median filters, figure(5(c),5(f) and 5(f). In the mean filter, the structure is preserved but noise is still present there and for the median filter image is well smooth but we loose the small details and fine structure. The proposed method preserve well the fine details and image structure along with the smoothing.

Now we discuss the visual results for the images *pirate*, *mandrila* and *walkbridge*. These images are shown in figure (6), where first row represents the original image and noisy images are shown in the second row. These are the images, where the texture is more complex.

The results for the images are sown in the Figure (7). In this figure, first column shows the results for the proposed method images (7(a),7(d) and 7(g)). The results for the traditional mean filter are shown in the second column images (7(b),7(e) and 7(g)) and results for median filter are shown in the third column (7(c),7(f) and 7(i)).

The algorithm preserves the image structure and local variations due to the texture and edges, however the small black lines can be observed at the boundary of the images. These lines appear due to the treatment of the boundary doesn't depend on the partial implementation of the algorithm and the adjustment of the amplification factor α. These results shows that the image quality improved more and more with the increase in the amplitude factor up to the certain value as discussed in section 3.6. When the amplitude factor α has lower value, the dark line appears at the edge of the image, but with the increase in the value of α, this dark black line disappears and image becomes more closer to the original image.

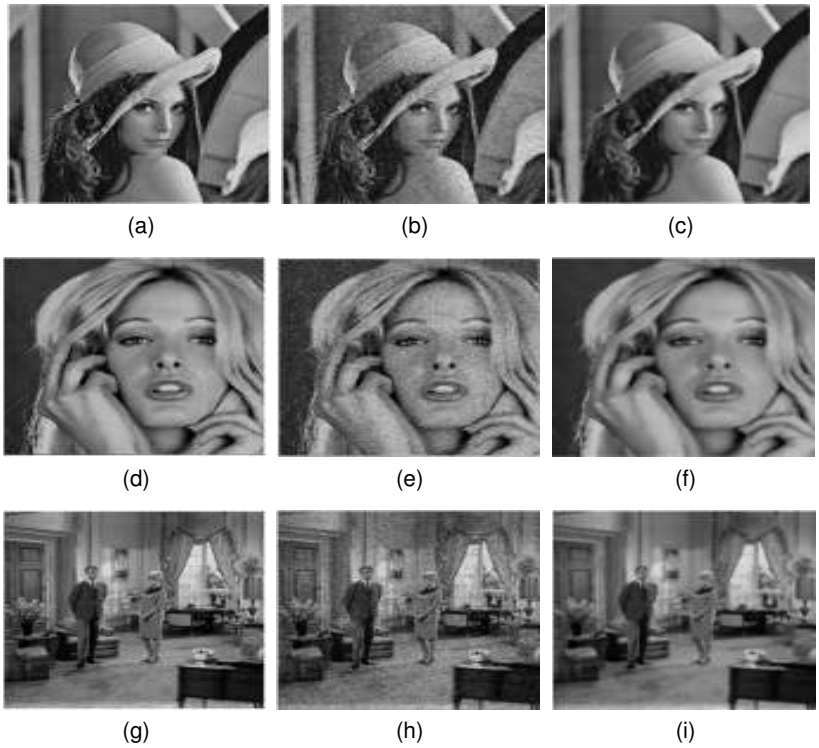
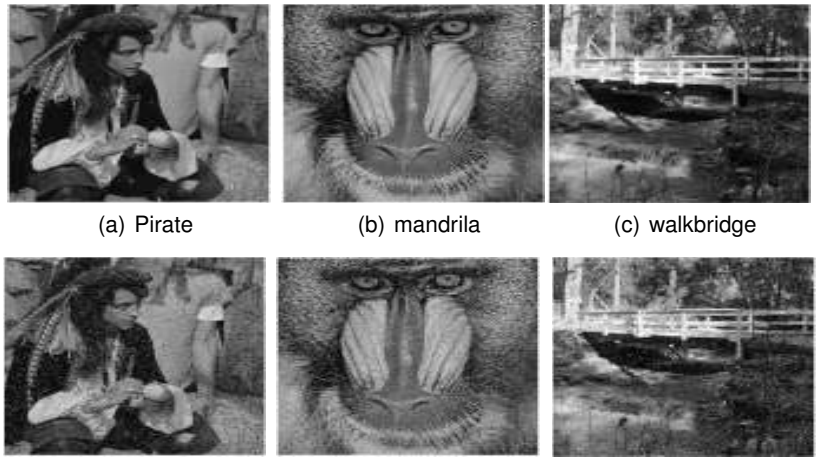


Figure 5: Processed images with proposed method images(a),(d) and (g), mean filter ((b),(e) and (h)) and adaptive median filter, images (c), (f) and (i)



(a) Pirate (b) mandrila (c) walkbridge

(d) (e) (f)

Figure 6: The original images (a) pirate (b) mandrila (c) walk bridge) and noisy images (d) pirate, (e) mandrila and (f) walkbridge with noise ratio eight percent

5 Conclusion

The fuzzy logic is used with the diffusion equation for additive noise removing. The fuzzy rules are used to distinguish the local variations due to noise and the due to the image structures. These rules are fired to decide whether the diffusion equation should be applied to the targeted pixel or not. The proposed method preserve the fine details and texture of the image. The comparison of the statistical measures and visual analysis of method with the traditional filters shows the effectiveness of the method.

The method works well to distinguish the salt & paper noise in images and apply the simple diffusion equation to the targeted pixels. In future work, the method will be extended to apply for the impulsive or gaussian type noise. This method will be further investigated and extended its applications to colored images for all types of noise.

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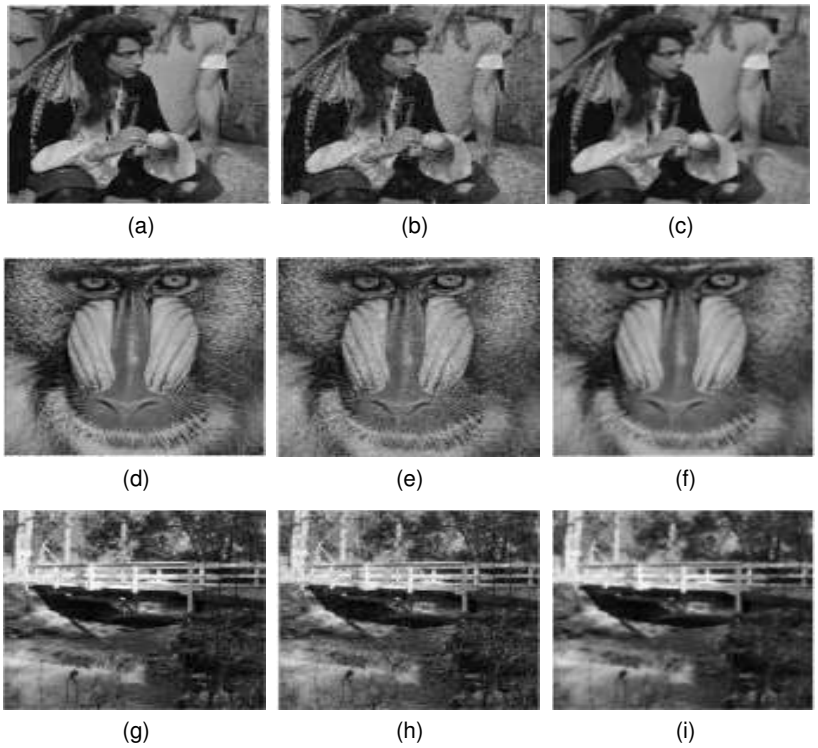

Figure 7: Results for proposed method are shown in the first column, traditional mean filter is shown in the second column and third column shows the images processed with adaptive median filter

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